**IN HIS NAME**

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| **The Islamic Institution for Education & Teaching** | | **Mathematics Department** |
| **Al-Mahdi Schools** | | **Scholastic Year**: 2017-2018 |
| | | **Date**: / 02 / 2018 |
| **Class**: Grade 11 (S) | | **Duration**: 150 minutes |
| **Name**: ___________________ | **Mid-Year Exam** | **Mark**: 30 points |

## Answer Key

| | Question I (4 points) | Mark |
|---|---|---|
| **1)** | $x^2-4x+4>0$, then $(x-2)^2>0$; this inequality is always true except for x = 2. **The correct answer is (C).** | **0.75** |
| **2)** | $\lim\limits_{x\to-3}\frac{x^3+6x^2+9x}{(x+3)(x-2)}=0$; **the correct answer is (B).** | **0.75** |
| **3)** | $f'(x)=\frac{x^2-2x-1}{(x-1)^2}$; so $f'(3)=\frac{1}{2}$; **the correct answer is (A).** | **0.75** |
| **4)** | $\lim\limits_{x\to0}\frac{\sin 2x}{x}=\frac{2\sin 2x}{2x}=2$; **the correct answer is (B).** | **0.75** |

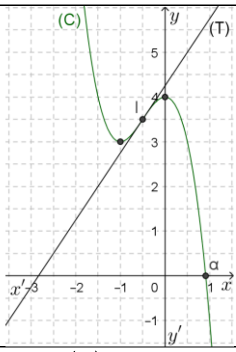
| | | Question II (5 points) | Mark |
|---|---|---|---|
| **1)** | | $x^2+4x+3>0\Rightarrow S_1=]-\infty;-3[\cup]-1;+\infty[$<br>$x^2+x-2<0\Rightarrow S_2=]-2;1[$<br>$\mathbf{S}=S_1\cap S_2=]-1;1[$. | **1.5** |
| **2)** | **a)** | Let $t=x^2$; the equation becomes $8t^2+2t-1=0$ so $t=-\frac{1}{2}$ or $t=\frac{1}{4}$.<br>For $t=-\frac{1}{2}$; $x^2=-\frac{1}{2}$ impossible & for $t=\frac{1}{4}$, $x=\pm\frac{1}{2}$. | **1** |
| | **b)** | Let $t=\frac{1}{1+y}$, the equation becomes $8t^2+2t^2-1=0$ so $t=-\frac{1}{2}$ or $t=\frac{1}{4}$.<br>For $t=-\frac{1}{2}$, y=-3 & for $t=\frac{1}{4}$, y=3. | **1** |
| **3)** | **a)** | $S=x_1+x_2=-\frac{b}{a}$ so $x_2=-\frac{b}{a}-2$. | **0.75** |
| | **b)** | $\frac{1}{x_1}+\frac{1}{x_2}=\frac{S}{P}=-\frac{b}{c}>0$. | **0.75** |

| | | Question III (5 points) | Mark |
|---|---|---|---|
| **1)** | | $U_1=\frac{1}{3}U_0-4$, so $U_1=-3$<br>$U_2=\frac{1}{3}U_1-4$, so $U_2=-5$ | **0.5** |
| **2)** | | $U_2-U_1\neq U_1-U_0$ so, $(U_n)$ is not an arithmetic sequence<br>$\frac{U_2}{U_1}\neq\frac{U_1}{U_0}$ so, $(U_n)$ is not a geometic sequence | **1** |
| **3)** | **a)** | $\frac{V_{n+1}}{V_n}=\frac{\frac{1}{3}U_n+2}{U_n+6}=\frac{1}{3}=r$, so $(V_n)$ is a geometric sequence of first term $V_0=9$ and ratio $r=\frac{1}{3}$ | **1.5** |
| | **b)** | $V_n=V_0\times r^n=9\times\left(\frac{1}{3}\right)^n$ and $U_n=V_n-6=9\times\left(\frac{1}{3}\right)^n-6$. | **1** |
| **4)** | | $S=V_0\times\frac{1-r^{n+1}}{1-r}=\frac{27}{2}\left[1-\left(\frac{1}{3}\right)^{n+1}\right]$ | **0.5** |
| **5)** | | $U_{n+1}-U_n=-6\times\left(\frac{1}{3}\right)^n<0$, so $(U_n)$ is decreasing. | **0.5** |

| Question IV (5 points) | | | Mark |
|---|---|---|---|
| 1) | | $\lim_{x\to0^-}\frac{f(x)-f(0)}{x}=\lim_{x\to0^-}\frac{x(x-1)}{2x(x-1)}=\frac{1}{2}$ and $\lim_{x\to0+}\frac{f(x)-f(0)}{x}=\lim_{x\to0^+}\frac{x(-x-1)}{2x}=-\frac{1}{2}$ then f is not differentiable at 0. | 1.5 |
| 2) | a) | $\tan(x+y)=\frac{30-5\sqrt{3}}{-5+10\sqrt{3}}=\sqrt{3}.$ | 0.75 |
| | b) | $-\frac{\pi}{2}<x+y<\frac{\pi}{2}$ and $\tan(x+y)=\sqrt{3}$ then $x+y=\frac{\pi}{3}$. | 0.5 |
| 3) | a) | $(\sin^2 a+2\sin a\cos a+\cos^2 a)=\frac{49}{25}$, so $\sin a\cos a=\frac{12}{25}$. | 1 |
| | b) | sin a and cos a are roots of the equation $x^2-\frac{7}{5}x+\frac{12}{25}=0$.<br>By calculator $\sin a=\frac{4}{5}$ and $\cos a=\frac{3}{5}$. | 1.25 |

| Question V (8 points) | | | Mark |
|---|---|---|---|
| 1) | | $\lim_{x\to-\infty} f(x)=\lim_{x\to-\infty}\left(-2x^3-3x^2+4\right)=+\infty$; $\lim_{x\to-\infty} f(x)=\lim_{x\to-\infty}\left(-2x^3-3x^2+4\right)=-\infty$ | 0.5 |
| 2) | | $f'(x)=-6x^2-6x$ | 0.25<br>1 |

| x | $-\infty$ | | -1 | | 0 | | $+\infty$ |
|---|---|---|---|---|---|---|---|
| f'(x) | | − | 0 | + | 0 | − | |
| f(x) | $+\infty$ | ↘ | 3 | ↗ | 4 | ↘ | $-\infty$ |

| | | | Mark |
|---|---|---|---|
| 3) | | On ]–∞, 0[: the equation f(x) = 0 has no roots since the minimum of f(x) is 3 > 0.<br>On [0, +∞[: f is continuous, strictly decreasing, and f(x) changes signs from +4 to –∞, then the equation f(x) = 0 admits a root $\alpha$.<br>But $f(0.5)\cdot f(1)=-1(3)<0$, then $\alpha\in\,]0.5\,;1[$. | 0.75 |
| 4) | a) | $f(x)+f(-1-x)=7=2(3.5)$, then $f(x)+f(2a-x)=2b$ and $D_f$ = ]–∞ ; +∞[ is centered at a = –0.5, then I(–0.5 ; 3.5) is a center of symmetry of $(C)$ | 0.75 |
| | b) | $(T): y-f(-0.5)=f'(-0.5)(x-3.5)$. Thus, $(T): y-3.5=1.5(x-3.5)$ | 0.5 |
| 5) | | <br> | 1.5 |
| 6) | | $f(x)<0$ for $x>\alpha$; $f(x)>0$ for $x<\alpha$; $f(x)=0$ for $x=\alpha$ | 0.5 |
| 7) | | $-2x^3-3x^2+0.999=0$; $f(x)-3.001=0$; $f(x)=3.001$; (C) cuts the line of equation y = 3.001 in 3 points; the equation $-2x^3-3x^2+0.999=0$ has 3 roots. | 0.75 |
| 8) | a) | $h(x)=(f(x))^2$. Thus, $\lim_{x\to-\infty}h(x)=\lim_{x\to+\infty}h(x)=+\infty$ | 0.5 |
| | b) | $h(x)=(f(x))^2$. Thus, $h'(x)=2f(x)\cdot f'(x)=2(-2x^3-3x^2+4)(-6x^2-6x)$ | 1 |

| x | $-\infty$ | | -1 | | 0 | | $\alpha$ | | $+\infty$ |
|---|---|---|---|---|---|---|---|---|---|
| h'(x) | | − | 0 | + | 0 | − | | + | |
| h(x) | $+\infty$ | ↘ | 3 | ↗ | 4 | ↘ | $f(a)$ | ↗ | $+\infty$ |

| Question VI (3 points) | | | Mark |
|---|---|---|---|
| 1) | | $(AB)\perp(AC)$; $(AB)\perp(AS)$; $(AB)\cap(AS)=\{A\}$; thus, $(AB)\perp(SAC)$ | 1.25 |
| 2) | a) | $(AH)\perp(SC)$; $(AB)\perp(SC)$; $(AB)\cap(AH)=\{A\}$; thus $(SC)\perp(ABH)$<br>but, H is midpt of [SC]. Thus, (ABH) is the mediator plane of [SC] | 1.25 |
| | b) | b) $(AHB)\perp(SC)$; $(SC)\subset(SBC)$. Thus, $(SBC)\perp(AHB)$ | 0.5 |