IN HIS NAME

The Islamic Institution for Education & Teaching
Al-Mahdi Schools

Mathematics Department
Scholastic Year: 2019-2020
Date: January 2020

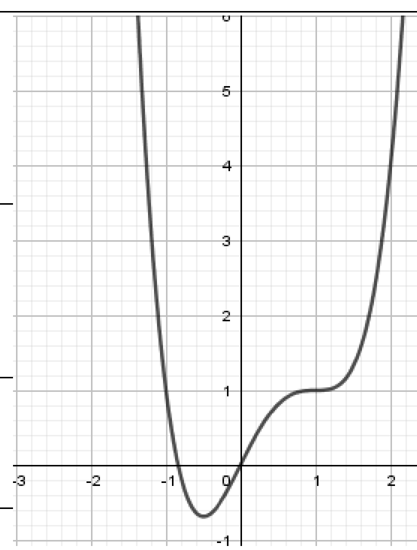
Class: Grade 11 S
Duration: 150 minutes

Name: ____________________

Mid-Year Exam

Mark: 30 points

Question		Answers	Note
I 4pts	**1**	$\lim_{x\to+\infty}\frac{x^2}{4x^2} < \lim_{x\to+\infty} f(x) < \lim_{x\to+\infty} -\left(-\frac{3x^2}{12x^2}\right)$, then $\frac{1}{4} < \lim_{x\to+\infty} f(x) < \frac{1}{4}$ hence by Sandwitch theorem $\lim_{x\to+\infty} f(x) = \frac{1}{4}$, **Thus, the answer is C**	**1**
	2	$V_{n+1} - V_n = (n+1)U_{n+1} - nU_n = nU_n + 4 - nU_n = 4$. **Thus,the answer is C**	**1**
	3	$\lim_{x\to2}\frac{(\sqrt{2x^2+1}-3)(\sqrt{2x^2+1}+3)}{x(x-2)(\sqrt{2x^2+1}+3)} = \lim_{x\to2}\frac{2(x+2)}{x(\sqrt{2x^2+1}+3)} = \frac{2}{3}$. **Thus,the answer is B**	**1**
	4	$x \in]-\infty, \frac{1}{2}[\cup]1,3[$, **Thus,the answer is B**	**1**
II 3pts	**1**	$(x_1-3)(x_2-3) = P - 3S + 9$ but $S = \frac{-b}{a} = 9$ and $P = \frac{c}{a} = 20$ Then $\frac{1}{x_1-3} + \frac{1}{x_2-3} = \frac{S-6}{P-3S+9} = \frac{3}{2}$	**1**
	2	$S' = z_1 + z_2 = \frac{3}{2}$ and $P' = \frac{1}{P-3S+9} = \frac{1}{2}$ **Thus**, the second degree equation is $z^2 - S'z + p' = \mathbf{z^2 - \frac{3}{2} z + \frac{1}{2} = 0}$	**1**
	3	Area of ABC $= \frac{H\times B}{2} = \frac{AB\times AC}{2} = \frac{P^2+s^2-2P+1}{2(P-3S+9)} = \frac{221}{2}U^2$	**1**
III 4pts	**1**	$\cos2x = 2\cos^2x - 1 = 2\left(\frac{1}{9}\right) - 1 = \frac{-7}{9}$; $\cos2y = 1 - 2\sin^2y = 1 - 2\left(\frac{5}{9}\right) = \frac{-1}{9}$	**1**
	2	$\tan(a+b) = \frac{\tan a+\tan b}{1-\tan a\tan b} = \frac{-\frac{4}{3}+(-7)}{1-(-\frac{4}{3})(-7)} = 1$ then $a + b = \frac{\pi}{4}$ with $(0 < a + b < \pi)$	**1.5**
	3	$(\overrightarrow{AB} ; \overrightarrow{BC}) = \pi + (\overrightarrow{BA} ; \overrightarrow{BC}) = \pi - \frac{\pi}{6}\ (2\pi) = \frac{5\pi}{6}\ (2\pi)$ $(\overrightarrow{BA} ; \overrightarrow{CA}) = (\overrightarrow{AB} ; \overrightarrow{AC}) = \frac{\pi}{2}\ (2\pi)$ $(\overrightarrow{AD} ; \overrightarrow{BC}) = (\overrightarrow{AD} ; \overrightarrow{DB}) + (\overrightarrow{DB} ; \overrightarrow{BC}) = \frac{\pi}{2} + \pi - \frac{2\pi}{9} - \frac{\pi}{6}\ (2\pi) = \frac{10\pi}{9}\ (2\pi)$	**0.5** **0.5** **0.5**
IV 3pts	**1**	f(x) is continuous at $x = 4$ then $\lim_{x\to4^+} f(x) = \lim_{x\to4^-} f(x) = f(4)$, then $\mathbf{4a + b = 6}$ f(x) is differentiable at $x = 4$ then $\mathbf{a = 4}$ and $\mathbf{b = -10}$	**1.5**
	2.a	$f'(x) = 4\left(\frac{5}{(3x+2)^2}\right)\left(\frac{x-1}{3x+2}\right)^3 = 20\frac{(x-1)^3}{(3x+2)^5}$	**0.75**
	2.b	$f'(x) = \frac{-1+3\cos3x}{2\sqrt{-x+\sin(3x)}}$	**0.75**
V 4pts	**1**	$U_1 = 3$, $U_2 = -5$, $U_1 - U_0 = 4$ and $U_2 - U_1 = -8$ then $U_1 - U_0 \neq U_2 - U_1$ then (U_n) isn't arithmetic $\frac{U_1}{U_0} = -3$ and $\frac{U_2}{U_1} = \frac{-5}{3}$ then $\frac{U_1}{U_0} \neq \frac{U_2}{U_1}$ then (U_n) isn't geometric	**1**
	2.a	$\frac{V_{n+1}}{V_n} = \frac{-3U_{n+1}+1}{-3U_n+1} = \frac{6U_n-2}{-3U_n+1} = -2 =$ constant then (V_n) is a geometric sequence of common ratio $r = -2$ and first term $V_0 = -3U_0 + 1 = 4$	**0.75**
	2.b	$V_n = V_0r^n = 4(-2)^n$	**0.5**
	2.c	$V_n = -3U_n + 1$ then $3U_n = -V_n + 1$ then $U_n = -\frac{1}{3}V_n + \frac{1}{3}$	**1**

<table>
<tr><td></td><td>2.d</td><td>$S = V_0\left(\frac{1-r^{n+1}}{1-r}\right) = 4\left(\frac{1-(-2)^{n+1}}{1+2}\right) = -\frac{4}{3}(1-(-2)^{n+1})$
$S' = -\frac{1}{3}S + \frac{1}{3}(n+1) = \ S' = -\frac{4}{9}(1-(-2)^{n+1}) + \frac{1}{3}(n+1).$</td><td>0.75</td></tr>
<tr><td rowspan="6">VI
5pts</td><td>1</td><td>$f(0) = 3,\ f(-1) = 0,\ f'(-2) = 0$</td><td>0.75</td></tr>
<tr><td>2</td><td>$\lim\limits_{x\to-\infty} f(x) = -\infty$; $\lim\limits_{x\to+\infty} f(x) = +\infty$</td><td>1</td></tr>
<tr><td>3</td><td>f is not differentiable at $x = -1$ since the curve (C) admits two semi- tangents</td><td>1</td></tr>
<tr><td>4.a</td><td>$x \in]-\infty, -3[$</td><td>0.5</td></tr>
<tr><td>4.b</td><td>$x \in]-3, -2[\cup]0, +\infty[$</td><td>0.75</td></tr>
<tr><td>5</td><td><table>
<tr><td>x</td><td>$-\infty$</td><td>-2</td><td>-1</td><td>$+\infty$</td></tr>
<tr><td>f '(x)</td><td>+</td><td>0</td><td>− ▨</td><td>+</td></tr>
<tr><td>f(x)</td><td>$-\infty$ ↗</td><td>1 ↘</td><td>0 ↗</td><td>$+\infty$</td></tr>
</table></td><td>1</td></tr>
<tr><td rowspan="8">VII
7pts</td><td>1</td><td>$\lim\limits_{x\to-\infty} g(x) = \lim\limits_{x\to-\infty} x^4 = +\infty$ and $\lim\limits_{x\to+\infty} g(x) = \lim\limits_{x\to+\infty} x^4 = +\infty$</td><td>0.5</td></tr>
<tr><td>2</td><td>$g'(x) = 4x^3 - 6x^2 + 2$
But $(x-1)^2(4x+2) = (x^2-2x+1)(4x+2) = 4x^3 - 6x^2 + 2.$
then $g'(x) = 4x^3 - 6x^2 + 2$
$(x-1)^2 > 0$ and $(4x+2) > 0$ if $x > -0.5$ then $g'(x) > 0$ over]–0.5 ; +∞[
Thus, g is strictly increasing over]–0.5 ; +∞[.</td><td>1</td></tr>
<tr><td>3</td><td>$g'(x) = (x-1)^2(4x+2) = 0$ then $x_1 = -\frac{1}{2}$ and $x_2 = 1$.
<table>
<tr><td>x</td><td>$-\infty$</td><td>$-\frac{1}{2}$</td><td>1</td><td>$+\infty$</td></tr>
<tr><td>g '(x)</td><td>−</td><td>0</td><td>+ 0</td><td>+</td></tr>
<tr><td>g(x)</td><td>$+\infty$ ↘</td><td>$-\frac{11}{16}$ ↗</td><td>1 ↗</td><td>$+\infty$</td></tr>
</table></td><td>1.5</td></tr>
<tr><td>4.a</td><td>g is cont. str. increasing over$]-\frac{1}{2}$; +∞ [from < 0 to > 0 then g(x) = 0 has one root in this interval but $g(0) = 0$ then 0 is the root of $g(x) = 0$ in this interval.
g is cont. str. decreasing over $]-\infty, -\frac{1}{2}[$ from > 0 to < 0 then g(x) = 0 has one root α in this interval</td><td>0.5</td></tr>
<tr><td>4.b</td><td>$g(-0.84) = 0.0032$ and $g(-0.83) = -0.041$
Then $g(-0.84) \times g(-0.83) < 0$
and $]-0.84, -0.83[\subset]-\infty, -\frac{1}{2}[$
Then $\alpha \in]-0,84\ ;\ -0,83[$.</td><td>0.75</td></tr>
<tr><td>5</td><td>g ''(x) = 12x² – 12x = 12x(x–1)
then g ''(x) vanishes at x = 0 and x =1
by changing its signs, so g admits
two points of inflection (0,0) and (1,1)</td><td>1</td></tr>
<tr><td>6</td><td>(t): $y - g(1) = g'(1)(x-1)$
but $g(1) = 1$ and $g'(1) = 0$.
Thus, (t): $y = 1$</td><td>0.75</td></tr>
<tr><td>7</td><td>Figure

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