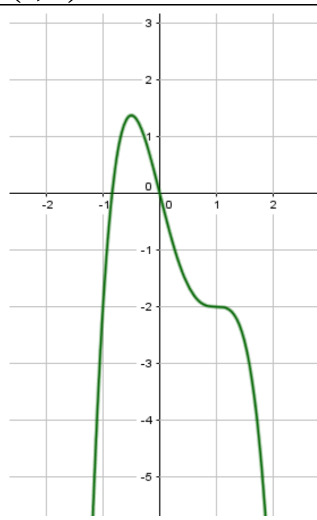
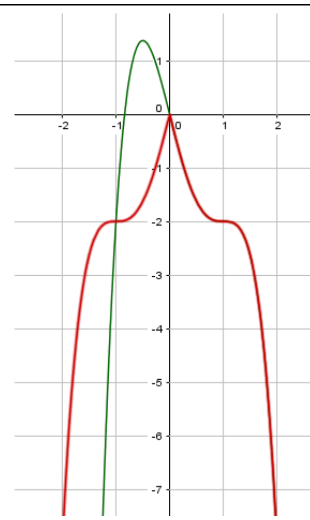
The Islamic Institution for Education & Teaching	**IN HIS NAME**	**Mathematics Department**
Al-Mahdi Schools		**Scholastic Year** :2016- 2017
		Date:
Grade 11(Sc)		**Duration:** 150 min
Name:	**Mid-Year Exam**	**Mark:** 30 points

Questions		Answers	Note
I-(3pts)	**1)**	$\lim\limits_{x\to+\infty} 1-\frac{5}{x}=1$; $\lim\limits_{x\to+\infty}\frac{4x+1}{4x-3}=1$; so, $\lim\limits_{x\to+\infty} g(x)=1$ (sandwich theorem) **(A)**	¾
	2)	Let $t=x^2\,(t\geq 0)$, $t^2-3t-4=0$, $t_1=-1$ rejected and $t_2=4$ accepted then $x^6-63=(x^2)^3-63=4^3-63=1$ **(B)**	¾
	3)	f admits at x = 1 a local minimum, then f '(1) =0 f '(3) = slope of tangent (T) to (C) = $\frac{2+1}{3-0}=1$; so, $f(-2)+f'(1)+f'(3)=3+0+1=4$ **(C)**	¾
	4)	$f'(0)=\lim\limits_{x\to 0}\frac{f(x)-f(0)}{x-0}=\lim\limits_{x\to 0}\frac{\frac{\sin x}{x}-0}{x}=\lim\limits_{x\to 0}\frac{\sin x}{x}\times\frac{1}{x}=\pm\infty$ So f is not differentiable at x=0 **(B)**	¾
II-(2pts)	**1)**	$\Delta=0,\ \ 25+8k=0;\ \ \ k=\frac{-25}{8}$.	½
	2)	for $x=-1;\ \ 2+5-k=0;\ \ k=7$. $P=\frac{7}{2}=x_1.x_2$; $\frac{-7}{2}=-1\times x_2$; $x_2=\frac{7}{2}$.	½
	3)	$S=y_1+y_2=\frac{x_1^{\,2}+x_2^{\,2}}{x_1.x_2}=\frac{s^2-2p}{p}=\frac{\frac{25}{4}+\sqrt{3}}{-\frac{\sqrt{3}}{2}}=-\frac{25\sqrt{3}+12}{6}$. $P=y_1.y_2=1$; $y^2+(\frac{25\sqrt{3}-12}{6})y+1=0$.	½ ½
III-(4pts)	**1)**	$U_2=\frac{3}{2}$ & $U_3=3$	½
	2)	$U_1-U_0\neq U_2-U_1$ & $\frac{U_1}{U_0}\neq\frac{U_2}{U_1}$ then (U_n) is neither arithmetic, nor geometric	1
	3.a)	$\frac{V_{n+1}}{V_n}=\frac{(n+1)U_{n+1}}{nUn}=\frac{(n+1)\frac{3n}{n+1}Un}{nUn}=3$, then (V_n) is a geometric sequence with common ratio r=3 and the first term $V_1=1.U_1=1$	1
	3.b)	$V_n=V_1\times r^{n-1}=3^{n-1}$, $U_n=\frac{V_n}{n}=\frac{3^{n-1}}{n}$	1
	3.c)	$S_n=V_1\times\frac{1-3^n}{1-3}=\frac{3^n-1}{2}$	½

IV-(4pts)		**1)**	$\frac{\sin a-\sin b}{\cos b-\cos a}=\frac{2\cos(\frac{a+b}{2})\sin(\frac{a-b}{2})}{-2\sin(\frac{a+b}{2})\sin(\frac{b-a}{2})}=\frac{\cos(\frac{\pi}{6})}{\sin(\frac{\pi}{6})}=\cot(\frac{\pi}{6})=\sqrt{3}$	1
		2)	$\sin x+\sqrt{3}\cos x=\frac{\sin x\cos\frac{\pi}{3}+\cos x\sin\frac{\pi}{3}}{\cos\frac{\pi}{3}}=2\sin(x+\frac{\pi}{3})$;so a=2,b=1 and c=$\frac{\pi}{3}$	1
		3)	sin8x=2sin4x.cos4x=2(2sin2x.cos2x).cos4x=8sinx.cosx.cos2x.cos4x	¾
V-(4pts)		**1)**	$g'(x)=\frac{1}{2\sqrt{\frac{x^2+1}{x-1}}}\times\frac{(x^2-2x-1)}{(x-1)^2}$	1
		2.a)	$\lim_{x\to0^-}f(x)=\lim_{x\to0^-}\frac{2-\sqrt{4-x}}{x}\times\frac{2+\sqrt{4-x}}{2+\sqrt{4-x}}=\lim_{x\to0^-}\frac{x}{x(2+\sqrt{4-x})}=\frac{1}{4}$; $\lim_{x\to0^+}f(x)=\lim_{x\to0^+}\frac{x^2-3x+2}{x^2-4}=\frac{-1}{2}$; $\lim_{x\to0^-}f(x)\neq\lim_{x\to0^+}f(x)$,so f is not continuous at x=0.	1½
		2.b)	$\lim_{x\to2^-}f(x)=\lim_{x\to2^+}f(x)$; $\lim_{x\to2^-}\frac{x-1}{x+2}=\lim_{x\to2^+}\frac{ax-3}{x+2}$; $\frac{1}{4}=\frac{2a-3}{4}$; a = $\frac{4}{2}=2$	½
VI-(13pts)	**A)**	**1)**	$f(0)=1$; $c=1$; $f(1)=0$; $a+b=-1$; $f'(1)=0$; $f'(x)=3ax^2+2bx$. So, $3a+2b=0$; $\begin{cases}a+b=-1\\3a+2b=0\end{cases}$ so, a=2 & b= -3	1½
		2)	$f(\frac{-1}{2})=2(\frac{-1}{2})^3-3(\frac{-1}{2})^2+1=0$ for $x\in]-\infty,\frac{-1}{2}[$; $f(x)<0$; for $x\in]\frac{-1}{2},1[\cup]1,+\infty[$; $f(x)>0$; for $x=\frac{-1}{2}$; or $x=1$; $f(x)=0$	1
		3)	$f'(x)=6x^2-6x$; $f'(\frac{-1}{2})=\frac{9}{2}$ $y-f(\frac{-1}{2})=f'(\frac{-1}{2})(x+\frac{1}{2})$; $y-0=\frac{9}{2}(x+\frac{1}{2})$; $y=\frac{9}{2}x+\frac{9}{4}$	1
	B)	**1)**	F is defined on $\mathbb{R}$.	½
		2)	$F'(x)=3(f(x))^2\times f'(x)$.	½
		3)	$F'(x)$ and $f'(x)$ have the same sign since $(f(x))^2$ is positive.	1½

x	$-\infty$		$-\frac{1}{2}$		0		1		$+\infty$
F'(x)		+	0	+	0	−		+	
F(x)	$-\infty$	↗	0	↗	1	↘	0	↗	$+\infty$

		1)	$\lim_{x\to-\infty} g(x) = -\infty;\ \lim_{x\to+\infty} g(x) = -\infty$	½
		2)	$g'(x) = -8x^3 + 12x^2 - 4 = -4f(x)$	½
		3)	x: $-\infty$; $-\frac{1}{2}$; 1 ; $+\infty$ $g'(x)$: + 0 − 0 − $g(x)$: $-\infty$ ↗ $\frac{11}{8}$ ↘ -2 ↘ $-\infty$	1
		4)	● For $x \in]-\infty, \frac{-1}{2}[$, g is continuous, strictly increasing, and g(x) changes signs from $-\infty$ to $\frac{11}{8}$; then the equation g(x) = 0 has a first root root. ●For $x \in]\frac{-1}{2}, +1[$, g is continuous…… $g(-0.9) \times g(-0.8) < 0$, then $-0.9 < \alpha < -0.8$	½ ½ ½
	C)	**5)**	$g''(x) = -24x^2 + 24x = 24x(-x+1)$ g''(x) vanishes at x=0 and x=1 by changing its signs ,so g admits two points of inflection (0,0) and (1,-2)	1
		6)	 	1
		7.a)	$D_g = \mathbb{R}$ centered at zero and h(-x)=h(x)	½
		7.b)	If $x \geq 0$: (H) ≡ (C) If $x < 0$: (H) is the symmetric of the first drawn part with respect to the ordinate axis. curve above	½ ½