Name:

Grade:

Section:

Score

Date:

Duration:

Subject:

Parent's Signature:

1^{st} - exercise: 1) a)

$a = \frac{4^7 - 8^4}{4^6 - 4^5}$

$= \frac{2^{14} - 2^{12}}{2^{12} - 2^{10}}$

$= \frac{2^{12}(2^2 - 1)}{2^{10}(2^2 - 1)}$

$\boxed{a = 2^2 = 4}$

$b = \frac{7}{3+\sqrt{2}} - \sqrt{(3-2\sqrt{2})^2}$ $\quad 3 - 2\sqrt{2} > 0$

$= \frac{7(3-\sqrt{2})}{(3+\sqrt{2})(3-\sqrt{2})} - (3 - 2\sqrt{2})$

$= \frac{7(3-\sqrt{2})}{7} - 3 + 2\sqrt{2}$

$\boxed{b = \sqrt{2}}$

b) a + b are dimensions of ABCD (given)

AD > AB (given)

+ $a = 4$ + $b = \sqrt{2}$ (proved)

So, $a > b$

Thus, $AD = 4$ cm and $AB = \sqrt{2}$ cm

c) Area of AEB = P = $\frac{\text{height} \times \text{base}}{2}$

$= \frac{AB \times AE}{2}$

$= \frac{x\sqrt{2}}{2}$ cm^2.

$Q = \frac{leg_1 \times leg_2}{2}$ (E is on [AD] (given)

So, $ED = AD - AE$
$= 4 - x$

$= \frac{ED \times DC}{2}$

$= \frac{(4-x)\sqrt{2}}{2}$ cm^2

2) $Q = \frac{(4-x)\sqrt{2}}{2}$

$= -\frac{\sqrt{2}}{2}x + 2.$

which is not of the form $y = ax$.

$y = ax$ represents a linear function

Thus, Q doesn't represent a linear function.

Since slope determines sense of variation of a linear eqn and Slope $= -\frac{\sqrt{2}}{2} < 0$, then the eqn is decreasing.

3) $Area_{ADCB} = length \times width$

$= AD \times AB$

$= 4\sqrt{2}\ cm^2.$

Now, area $BEC = Area_{ADCB} - (P+Q)$

$= 4\sqrt{2} - \left[\frac{\sqrt{2}x}{2} + \frac{(4-x)\sqrt{2}}{2}\right]$

$= \frac{8\sqrt{2} - \sqrt{2}x - 4\sqrt{2} + \sqrt{2}x}{2}$

$= 2\sqrt{2}\ cm^2.$

4) a) $J(x) = (x-2)^2 - 2$

$= x^2 - 4x + 4 - 2$

$J(x) = x^2 - 4x + 2$

b) $J(x) = (x-2)^2 - 2$

$= (x-2)^2 - \sqrt{2}^2$

$= (x-2+\sqrt{2})(x-2-\sqrt{2})$

To find roots of $J(x)$ we solve $J(x) = 0$

So, $(x-2+\sqrt{2})(x-2+\sqrt{2}) = 0$

$x = 2-\sqrt{2}$ or $x = 2+\sqrt{2}$

c) ΔABE is right at A.

So, use pythagorean theorem

$hyp^2 = leg_1^2 + leg_2^2$

$BE^2 = AE^2 + AB^2$

Thus, $BE = \sqrt{x^2 + 2}$ cm.

d) If ΔBEC is right at E,

then, we can apply pythagorean theorem

So, $hyp^2 = leg_1^2 + leg_2^2$

$BC^2 = BE^2 + EC^2$

$16 = x^2 + 2 + (4-x)^2 + 2.$

$16 = x^2 + 2 + 16 - 8x + x^2 + 2$

$0 = 2x^2 - 8x + 4$

So, $2(x^2 - 4x + 2) = 0.$

but $J(x) = x^2 - 4x + 2$

$= (x-2+\sqrt{2})(x-2-\sqrt{2})$

then, $2(x-2+\sqrt{2})(x-2-\sqrt{2}) = 0$

hence, $x = 2-\sqrt{2}$ or $x = 2+\sqrt{2}$

In right ΔEDC. use pyth. theo to get. $EC^2 = (4-x)^2 + (\sqrt{2})^2$

but $0 < x < 2$ given

Thus, $x = 2-\sqrt{2}$ is accepted

& $x = 2+\sqrt{2}$ is rejected.

5) In Δ's FAE and FBC sharing same vertex F we have:

ADCB is a rectangle (given)

So, (AD) // (BC) (opp. sides of a rect.)

but E is on [AD] (given)

then (AE) // (BC)

& pts F, A & B } are collinear in this order.
pts F, E & C }

So, using Thales' property: A st. line drawn parallel to one side of a triangle cuts other sides proportionaly

Ratios: $\frac{FA}{FB} = \frac{FE}{FC} = \frac{AE}{BC} = cst.$

① ② ③

From ratios ① & ③

$$\frac{FA}{FB} = \frac{AE}{BC}.$$

$$\frac{FA}{(FA+\sqrt{2})} = \frac{(2-\sqrt{2})}{4}$$

then, $4FA = 2FA - \sqrt{2}FA + 2\sqrt{2} - 2$

So, $2FA + \sqrt{2}FA = 2\sqrt{2} - 2$

$FA(2+\sqrt{2}) = 2\sqrt{2} - 2$

$FA = \frac{(2\sqrt{2}-2)}{(2+\sqrt{2})}$

So, $FA = \frac{(2\sqrt{2}-2)}{(2+\sqrt{2})} \frac{(2-\sqrt{2})}{(2-\sqrt{2})}$

$= \frac{4\sqrt{2}-4-4+2\sqrt{2}}{4-2}$

$= \frac{6\sqrt{2}-8}{2}$

Thus, $FA = 3\sqrt{2} - 4$ cm

2nd exercise 1) $\begin{cases} (2x+y=20) \times (-9) & \text{①} \\ 22x+9y=204 & \text{②} \end{cases}$

$\begin{cases} -18x - 9y = -180 \\ 22x + 9y = 204 \end{cases}$ add

$4x = 24$

So, $x = 6$.

replace $x=6$ in eqn ①

$2x + y = 20$

$12 + y = 20$

$y = 8$.

Thus, the couple $(6;8)$ is a solution of the given system.

2) The couple $(6;8)$ verifies $(m+2)^2 x - 3my = 48$ (given)

So, $(m+2)^2(6) - 24m = 48$

$6m^2 + 24m + 24 - 24m = 48$

$6m^2 - 24 = 0$

$6(m^2-4) = 0$

$6(m-2)(m+2) = 0$

So, $m = 2$ or $m = -2$.

3) a) Statement-1: perimeter of $\triangle ABC = 20$

Sum of sides $= 20$

$AB + AC + BC = 20$.

but $\triangle ABC$ is isosceles at A (given)

So, $AB = AC$ (legs of an isosceles $\triangle$)

& let $AB = \ell$

& $BC = b$.

So, $2\ell + b = 20$ - - - ①.

Statement-2: Each leg is increased by 10%.

then new length of leg $= \ell(1+10\%)$

$= 1.1\ell$.

Length of base is decreased by 10%

Then new length of base $= b(1-10\%)$

$= 0.9b$.

So, new perimeter is 20.4cm

hence, $2(1.1)\ell + 0.9b = 20.4$

$2.2\ell + 0.9b = 20.4$ - - - ②

Thus, system: $\begin{cases} 2\ell + b = 20 \\ 2.2\ell + 0.9b = 20.4 \end{cases}$

b) $\begin{cases} 2\ell + b = 20 \\ (2.2\ell + 0.9b = 20.4)10 \end{cases}$

So, $\begin{cases} 2\ell + b = 20 \\ 22\ell + 9b = 204 \end{cases}$

which is equivalent to system solved in part 1), for $x = \ell$ & $y = b$.

So, they admit same solution.

Thus, $\ell = AB = AC = 6$ cm.

and $b = BC = 8$ cm.

3rd exercise: 2) a) If $x=3-\sqrt{7}$ is a root of $r(x)$

Then, $r(3-\sqrt{7})=0$.

So, $(3-\sqrt{7})^2-6(3-\sqrt{7})+n=0$

$9-\underline{6\sqrt{7}}+7-18+\underline{6\sqrt{7}}+n=0$

Thus, $\boxed{n=2}$

b) $x=\frac{2}{3-\sqrt{7}}\times\frac{(3+\sqrt{7})}{(3+\sqrt{7})}$

$=\frac{2(3+\sqrt{7})}{9-7}$

So, $x=3+\sqrt{7}$.

Now, $r(3+\sqrt{7})=(3+\sqrt{7})^2-6(3+\sqrt{7})+2$

$=9+6\sqrt{7}+7-18-6\sqrt{7}+2$

$=0$

c) $[AC)$ & $[AB)$ are tangents (λ) (given)

So, $AC=AB$ (tangent theorem: exterior pt from which tangents are drawn is equidistant from pts of tangencies)

So, $x^2=6x-2$

Then $x^2-6x+2=0$

which is $r(x)$.

& $r(x)$ admits $x=3+\sqrt{7}$ & $x=3-\sqrt{7}$ as roots

& both are greater than $0.\bar{3}$

Thus, both are accepted.

1) $A=\frac{5^{248}-3^2\times 25^{123}}{4\times 125^{82}}-2$

$=\frac{5^{248}-3^2\times 5^{246}}{4\times 5^{246}}-2$

$=\frac{5^{246}(5^2-3^2)}{4\times 5^{246}}-2$

$=\frac{25-9}{4}-2$

$=\frac{16}{4}-2$

Thus, $A=2=2(1)$.

$B=\left(\frac{a^2-1}{a^2+1}\right)^2+\left(\frac{2a}{a^2+1}\right)^2$

$=\frac{a^4-2a^2+1+4a^2}{(a^2+1)^2}$

$=\frac{a^4+2a^2+1}{(a^2+1)^2}$

$=\frac{(a^2+1)^2}{(a^2+1)^2}$

$B=1$

$$D=\left(\frac{3}{5}-\frac{5}{2}\times\frac{-1}{10}\right)\div\left(1-\frac{23}{40}\right)$$
$$=\left(\frac{3}{5}+\frac{1}{4}\right)\div\left(\frac{17}{40}\right)$$
$$=\frac{17}{20}\times\frac{40}{17}$$
$$\boxed{D=2}$$

$$C=\frac{7^3\times5^{-4}}{49\times5^{-6}\times7}-21$$
$$=\frac{7^3\times5^{-4}}{7^2\times5^{-6}\times7}-21$$
$$=5^2-21$$
$$\boxed{C=4}$$

but $\frac{B}{A}=\frac{1}{2}$ & $\frac{D}{C}=\frac{2}{4}=\frac{1}{2}$.

hence, $\frac{B}{A}=\frac{D}{C}$, thus, table is a table of proportionality

3) i) $\begin{cases}(3(x+1)^2+2(y-3)^2=11)\times(3) & \text{--- (1)}\\ (5(x+1)^2-3(y-3)^2=-7)\times(2) & \text{--- (2)}\end{cases}$

$$\begin{cases}9(x+1)^2+6(y-3)^2=33\\ 10(x+1)^2-6(y-3)^2=-14\end{cases}\text{ add}$$

$19(x+1)^2=19$

$(x+1)^2=1$

So $(x+1)^2-1^2=0$

$(x+1-1)(x+1+1)=0$

then $x=0$ or $x=-2$

replace $(x+1)^2=1$ in eqn (1).

$3+2(y-3)^2=11$

$2(y-3)^2=8$

$(y-3)^2-2^2=0$

$(y-3-2)(y-3+2)=0$

$y=5$ or $y=1$

ii) If $x<-1$; then $x=-2$ is accepted

& $y>4$; then $y=5$ is accepted.

So, $e=\frac{\frac{1}{x}+y}{(x+y)^2}$

$=\frac{\frac{-1}{2}+5}{(-2+5)^2}$

$=\frac{\frac{-1+10}{2}}{(3)^2}$

$=\frac{-\frac{9}{2}}{9}$

$e=-\frac{1}{2}$

since 2 is a divisor of 10 then, e is a decimal fraction.

Exercise-4.

1a) Drawn.

b) F belongs to (D) if its coordinates satisfy eqn of (D).

$$\frac{16}{5} \stackrel{?}{=} \frac{1}{2}\left(\frac{22}{5}\right) + 1$$

$$\frac{16}{5} \stackrel{?}{=} \frac{11}{5} + 1$$

$$\frac{16}{5} = \frac{16}{5} \checkmark$$

Thus, F belongs to (D).

2) a) E is on x-axis

So, $y_E = 0$

Thus, $E(-2, 0)$

C is on y-axis

So, $x_C = 0$

Thus, $C(0, 1)$.

b) $EC = \sqrt{(x_C - x_E)^2 + (y_C - y_E)^2}$

$= \sqrt{(0+2)^2 + (1-0)^2}$

$EC = \sqrt{5}$ cm

3a) O is midpt of [EB] if

$x_O \stackrel{?}{=} \frac{x_E + x_B}{2}$ & $y_O \stackrel{?}{=} \frac{y_E + y_B}{2}$

$0 \stackrel{?}{=} \frac{-2+2}{2}$ $\quad$ $0 \stackrel{?}{=} \frac{0+0}{2}$

$0 = 0 \checkmark$ $\quad$ $0 = 0$

Thus O is midpt of [EB].

3) In Δ AEB we have

$AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$

$= \sqrt{(2-0)^2 + (0-2\sqrt{3})^2}$

$= \sqrt{4+12}$

$AB = 4$ cm

$EB = x_B - x_E$

$= 2 - (-2)$

$= 4$ cm

O is mid pt of [EB] (proved)
so, (AO) is a median to [EB]
by x'ox & y'oy are perp. (ortho.)
then, (AO) is a height t [EB]
hence, (AO) is a perp. bisector of [EB]
so, $AE = AB = 4$ cm

Thus, Δ AEB is equilateral having 3 equal sides.

4a) In quadrilateral ABRE we have

R is on (C) of radius [AB] (given)

so $RB = AB$.

& $AB = EA$

[AO) is the perp. bisector of [EB] (proved)

& R, O & A are collinear.

then $RB = RE$

Thus, ABRE is a rhombus having 4 equal sides.

b) $BF = \sqrt{(x_F - x_B)^2 + (y_F - y_B)^2}$

$= \sqrt{\left(\frac{22}{5} - 2\right)^2 + \left(\frac{16}{5} - 0\right)^2}$

$= \sqrt{\left(\frac{12}{5}\right)^2 + \left(\frac{16}{5}\right)^2}$

$= \sqrt{\frac{144 + 256}{25}}$

$= \sqrt{\frac{400}{25}}$

$= \frac{20}{5}$

$BF = 4cm$

but $BA = 4cm$

Thus, F is on (C).

5) a) In Δ's ACH & ECR having same vertex C we have:

$(AH) \parallel (ER)$.

pts: H, C & E and A, C & R are collinear in this order

So, by Thales' property (Statement ——)

Ratio:

$\frac{CA}{CR} = \frac{CH}{CE} = \frac{AH}{ER}$ (1) (2) (3) but $ER = 4cm$

Thus, $AH \times CR = 4CA$.

Using ratios (1) & (3)

$\frac{CA}{CR} = \frac{AH}{ER}$

b) In Δ's ECA & CRG of same vertex C we have:

$(EA) \parallel (RG)$

pts E, C & G and pts A, C & R are collinear in this order

Then, using Thales' property.

Ratios:

$\frac{CA}{CR} = \frac{CE}{CG} = \frac{AE}{RG}$ (a) (b) (c)

(1) & (a) is a common ratio:

Using ratios: (2) & (b) we get

$\frac{CH}{CE} = \frac{CE}{CG}$

Thus, $CE^2 = CH \times CG$.

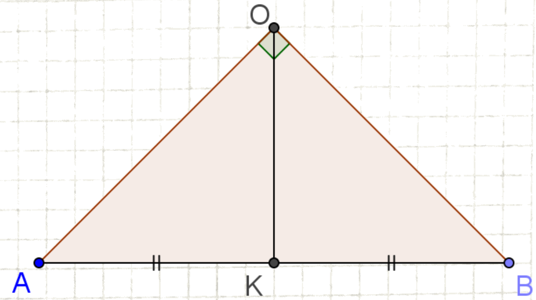
c) $CE^2 = CH \times CG$ (proved)

$CE = \sqrt{5}$ proved

Thus, $CH \times CG = 5 = cst$.

5^{th} - Exercise:

1) Drawn



2) Δ AOB is isosceles at O (given)
+ AB = 6cm

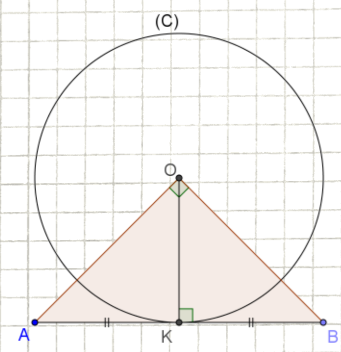
So, $OA = \frac{\sqrt{2}}{2} \times hyp$ (leg of iso. Δ)

$OA = \frac{\sqrt{2}}{2} \times AB$

Thus, $OA = 3\sqrt{2}$ cm.

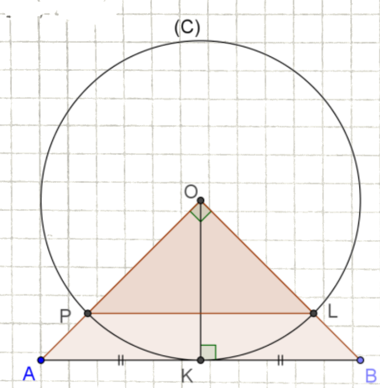
• K is midpt of [AB] (given)
So, [OK] is a median relative to hyp [AB].
then, $OK = \frac{1}{2}$ hyp.
$= \frac{1}{2}(AB)$
Thus, OK = 3cm.

3).



(c) is a circle of center O + radius [OK] (given)
+ [OK] is a median relative to base [AB] given)

but, Δ AOB is right iso at O (given)
then, [OK] is a height to [AB] at K. (median issued from main vertex of an iso Δ)
Thus, (AB) is tangent to (C) at K (tangent theorem: tangent + radius are perpendicular at pt. of tangency.



4) a) In Δ OPL we have:
(C) is a circle of center O (given).
(C) cuts [OA] + [OB] in P + L resp. (given)
So, OP = OL (radii of (C))
but, Δ AOB is right iso. at O (given)
So, $A\hat{O}B = 90°$.
then, $P\hat{O}L = A\hat{O}B = 90°$ (angles sharing same vertex + same arms)

Thus, Δ OPL is right isosceles at O (having a pair of equal legs + 90°).

b) ΔOAB is right isosceles at O (given)
So, $O\hat{A}B = 45°$ (base angle of a right iso Δ) (a)
ΔOPL is right isosceles at O (proved)
So, $O\hat{P}L = 45°$ (rule (a))
then, $O\hat{A}B = O\hat{P}L = 45°$.
Thus, (AB) is parallel to (PL) (equal corresponding angles are held by parallel lines)

c) In Δ's OAB & OPL sharing same vertex O we have:
O, A & P
O, B & L } are collinear in this order.
and $(AB) \parallel (PL)$ (proved)
then, using Thales' property: (st. line drawn parallel to one side of a Δ cuts other sides proportionally).

Thales' ratios:

$$\frac{OA}{OP} = \frac{OB}{OL} = \frac{AB}{PL}.$$

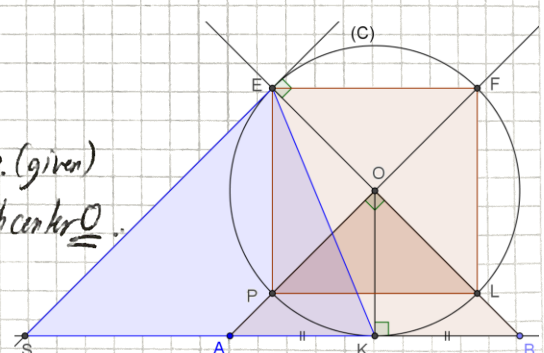
(1) (2) (3)

Using ratios (1) & (3)

$$\frac{OA}{OP} = \frac{AB}{PL}.$$

$$\frac{3\sqrt{2}}{3} = \frac{6}{PL}.$$

Thus, $PL = \frac{3\sqrt{2}}{2}$ cm.



5) In quadrilateral $PEFL$ we have:
$[PO)$ & $[LO)$ cut (C) at F & E resp. (given)
So, $[PF]$ & $[EL]$ are diameters of (C) with center O.
So, $PF = EL$.
but, $P\hat{O}L = 90°$ (proved)
Thus, $PEFL$ is a square (equal and perp diagonal)

6) In ΔSEK we have:
$[SE)$ & $[SK)$ are tangents to (C) at E & K resp. (given)
So, $SE = SK$. (tangent theorem: ext. pt. from which tangents are drawn is equidistant from pts of tangencies)
Thus, ΔSEK is isosceles at S.