In His Name
Mathematics Midyear Exam
Answer Key
Grade 10
2015-2016

Questions			Answers	Marks
I 4pts	**1)**		**B**: $\alpha = -\frac{27\pi}{7} = \frac{-28\pi+\pi}{7} = -4\pi + \frac{\pi}{7}$. The principle measure of α is $\frac{\pi}{7}$	**1**
	2)		**B**: $K = \frac{2}{\sqrt[3]{4}} \times \frac{\sqrt[3]{2}}{\sqrt[3]{2}} = \frac{2\sqrt[3]{2}}{2} = \sqrt[3]{2}$	**1**
	3)		**B**: $\lvert -(x+1)^2 + 2x \rvert = \lvert -x^2 - 2x - 1 + 2x \rvert = \lvert -x^2 - 1 \rvert = x^2 + 1$	**1**
	4)		**C**: If $2 \le \frac{1}{x-1} \le 3$ then $\frac{1}{3} \le x - 1 \le \frac{1}{2}$ so $\frac{4}{3} \le x \le \frac{3}{2}$ then $x \in \left[\frac{4}{3}; \frac{3}{2}\right]$	**1**
II 3pts	**1)**		$\lvert x - 5 \rvert \le 1$ then $-1 \le x - 5 \le 1$, so $4 \le x \le 6$ and $1 < \sqrt{y+1} < 2$ then $1 < y + 1 < 4$ and $0 < y < 3$	½ ½
	2)		$4 < x + y < 9$ $-3 < -y < 0$, so $1 < x - y < 6$ $4 < (x+y)(x-y) < 36, 1 < x^2 - y^2 < 36$	½ ½ ½
	3)		$x^2 - y^2 > 1$, so $\sqrt{x^2 - y^2} > \frac{1}{x^2 - y^2}$	½
III 4pts	**1)**		$A = \frac{2 \times 3^{1/3} \times 2 \times 2^{1/2} \times 2^{1/12} \times 3^{1/12}}{2^{5/12} \times 3^{5/12} \times 2^{7/12}} = 2^{19/12} = 2\sqrt[12]{2^7}$	**2**
	2)		$B = \left(\sqrt[4]{5} - 1\right)\left(\sqrt[4]{5} + 1\right)\left(\sqrt[4]{25} + 1\right) = \left(\sqrt[4]{25} - 1\right)\left(\sqrt[4]{25} + 1\right)$ $= \left(\sqrt{5} - 1\right)\left(\sqrt{5} + 1\right) = 4$	**1**
	3)		$C = \sqrt[6]{\left(\sqrt{3} - 2\right)^6} + \sqrt[3]{\left(\sqrt{3} - 2\right)^3} + \left\lvert -2\sqrt{3} - 1 \right\rvert = 2 - \sqrt{3} + \sqrt{3} - 2 +$ $2\sqrt{3} + 1 = 2\sqrt{3} + 1$	**1**
IV 4pts	**1)**	**a)**	$E = (2x - 1)(x + 3) - (x + 3) = (x + 3)(2x - 1 - 1) = (x + 3)(2x - 2) =$ $2(x + 3)(x - 1)$	½
		b)	$F = \frac{E}{x+3} = 2(x - 1);\ x \ne -3$ $\lvert F \rvert > 2$ then $\lvert x - 1 \rvert > 1$ $x - 1 > 1$ or $x - 1 < -1$ then $x > 2$ or $x < 0$; $x \in]-\infty; 0[\cup]2; +\infty[$	¼ **1** ¼
	2)		$\frac{x^2 + 9}{3 - x} < 0$ is reduced to $3 - x < 0$, so $x > 3$ and $S_1 =]3; +\infty[$ $x^2 \ge (2x - 1)^2$ then $(-x + 1)(3x - 1) \ge 0$ and $S_2 = [\frac{1}{3}; 1]$ $S = S_1 \cap S_2 = \emptyset$	¾ ¾ ½

V 5pts	**1)**	$\overrightarrow{AB}(-1\ ;\ 7);\ \overrightarrow{AC}(1\ ;\ 5)$ $\det(\overrightarrow{AB}, \overrightarrow{AC}) = -12 \neq 0$, then A , B and C are non collinear or A, B, and C form a triangle.		**1**
	2)	ABCJ is a parallelogram then $\overrightarrow{AJ} = \overrightarrow{BC}$ so J(4 ; –5)		**1**
	3)	**a)**	$x_M - x_A = 2a - 1$ so $x_M = 2a - 1 + 2 = 2a + 1$ $y_M = -b + 2 - 3 = -b - 1$	**1**
		b)	a = –2 and b = 0	**1**
	4)	$\overrightarrow{AE}(4\cos\alpha\,;\,4\sin\alpha)$ so $AE = \sqrt{(4\cos\alpha)^2 + (4\sin\alpha)^2} = 4$		**½**
	5)	The coordinates of A in $(C; \vec{i}, \vec{j})$ are (–1 ; –5)		**½**
VI 5pts	**1)**	Figure		**½**
	2)	$\overrightarrow{BF} = \overrightarrow{BA} + \overrightarrow{AF} = \overrightarrow{CD} + \frac{3}{2}\overrightarrow{AE}$		**1**
	3)	$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AE} + \overrightarrow{EC} = \overrightarrow{CD} + \overrightarrow{AE} - \frac{1}{3}\overrightarrow{CD} = \frac{2}{3}\overrightarrow{CD} + \overrightarrow{AE}$		**1**
	4)	$\overrightarrow{BC} = \frac{2}{3}\overrightarrow{CD} + \overrightarrow{AE} = \frac{2}{3}\left(\overrightarrow{CD} + \frac{3}{2}\overrightarrow{AE}\right) = \frac{2}{3}\overrightarrow{BF}$, then B, C, and F are collinear		**1**
	5)	**a)**	$BF = \frac{3}{2}BC = \frac{3}{2} \times 3 = 4.5$ cm	**½**
		b)	MD = 9 cm, then M moves on the circle of center D and radius 9 cm	**1**
VII 5pts	**1)**	**a)**	$\cos^2\alpha = \frac{1}{1+\tan^2\alpha} = \frac{4}{5}$ so $\cos\alpha = \frac{2\sqrt{5}}{5}$ $\sin\alpha = -\frac{\sqrt{5}}{5}$	**1 ½**
		b)	$E = \tan\left(\frac{\pi}{2} + \alpha\right) \times \cos(\pi + \alpha) = -\cot\alpha \times (-\cos\alpha) = 2 \times \frac{-2\sqrt{5}}{5} = -\frac{4\sqrt{5}}{5}$	**1**
	2)	**a)**	$F = -\sin^2 x$	**1**
		b)	$G = -2\sin x$	**1 ½**