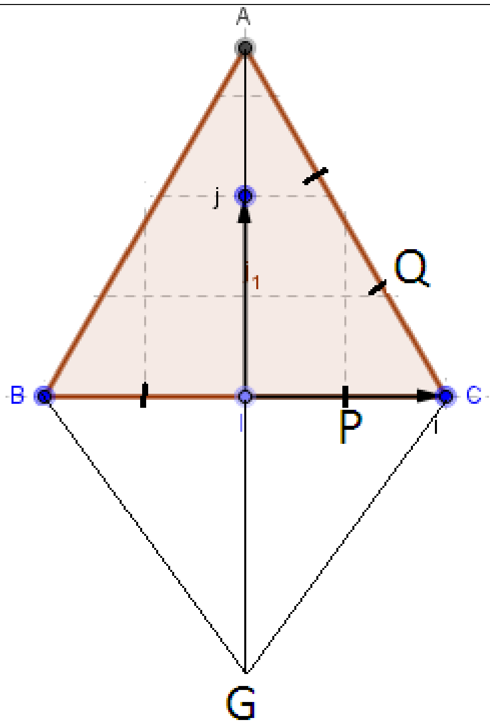
Bareme grade 10 mid year 2016-2017

Nº		Solution	Note
I	1	$31\frac{\pi}{4}=8\pi-\frac{\pi}{4}\Rightarrow P.M.\left(31\frac{\pi}{4}\right)=\frac{-\pi}{4}\quad (B)$	1
	2	$\lvert 1-2t\rvert\leq 3\Rightarrow -3\leq 1-2t\leq 3$ $-4\leq -2t\leq 2$ $-1\leq t\leq 2\quad\Rightarrow (A)$	1
	3	$\lVert\overrightarrow{EF}+\overrightarrow{EG}\rVert=\lVert 2\overrightarrow{EI}\rVert$ $or\ 2EI\quad\Rightarrow (C)$	1
	4	$\vec{u}=3\overrightarrow{AB}-2\overrightarrow{AC}\Rightarrow\vec{u}(3,-2)$ $\vec{v}=3\overrightarrow{AB}+2\overrightarrow{BA}+2\overrightarrow{AC}$ $=\overrightarrow{AB}+2\overrightarrow{AC}\Rightarrow\vec{v}(1,2)\quad\Rightarrow (C)$	1
	5	$\frac{2x}{x^2+1}-\frac{2x-1}{x^2}=\frac{2x^3-2x^3-2x+x^2+1}{x^2\left(x^2+1\right)}=\frac{\left(x-1\right)^2}{x^2\left(x^2+1\right)}\geq 0\ \ \forall x\in\mathbb{R}^*\ \Rightarrow\ (B)$	1
II Part A	1	$\overrightarrow{PB}+3\overrightarrow{PC}=\vec{0}$ $\overrightarrow{PC}+\overrightarrow{CB}+3\overrightarrow{PC}=\vec{0}$ $\overrightarrow{CB}=4\overrightarrow{CP}$ $\overrightarrow{CP}=\frac{1}{4}\overrightarrow{CB}$ $\overrightarrow{QA}+2\overrightarrow{QC}=\vec{0}$ $\overrightarrow{QC}+\overrightarrow{CA}+2\overrightarrow{QC}=\vec{0}$ $\overrightarrow{CA}=3\overrightarrow{CQ}$ $\overrightarrow{CQ}=\frac{1}{3}\overrightarrow{CA}$	1.5
	2	$\overrightarrow{GA}-\overrightarrow{GB}-\overrightarrow{GC}=\vec{0}$ $\overrightarrow{GA}-\overrightarrow{GA}-\overrightarrow{AB}-\overrightarrow{GA}-\overrightarrow{AC}=\vec{0}$ $\overrightarrow{AG}=\overrightarrow{AB}+\overrightarrow{AC}$	0.75

	3		1
	4	$\overrightarrow{QG}=\overrightarrow{QC}+\overrightarrow{CG}$ $\overrightarrow{QG}=\frac{1}{3}\overrightarrow{AC}+\overrightarrow{AB}$ $\overrightarrow{QP}=\overrightarrow{QC}+\overrightarrow{CP}$ $=\frac{1}{3}\overrightarrow{AC}+\frac{1}{4}\overrightarrow{CB}=\frac{1}{3}\overrightarrow{AC}+\frac{1}{4}\overrightarrow{CA}+\frac{1}{4}\overrightarrow{AB}$ $=\frac{1}{3}\overrightarrow{AC}-\frac{1}{4}\overrightarrow{AC}+\frac{1}{4}\overrightarrow{AB}=\frac{1}{12}\overrightarrow{AC}+\frac{1}{4}\overrightarrow{AB}$	1.5
	5	$4\overrightarrow{QP}=\frac{1}{3}\overrightarrow{AC}+\overrightarrow{AB}=\overrightarrow{QC}\ \Rightarrow \overrightarrow{QC}$ and $\overrightarrow{QP}$ are collinear and since Q is common, then Q, P, and G are collinear.	0.5
	1	$AI=AB\frac{\sqrt{3}}{2}=2\frac{\sqrt{3}}{2}=\sqrt{3}$, then $A\left(0,\sqrt{3}\right)$ in $\left(I,\ \vec{i}\ ,\vec{j}\right)$ $B\left(-1,0\right), C\left(1,0\right), P\left(\frac{1}{2},0\right), G\left(0,-\sqrt{3}\right)$ $\overrightarrow{CQ}=\frac{1}{3}\overrightarrow{CA}\ \Rightarrow\left(x-1,\ y\right)=\frac{1}{3}\left(-1,\sqrt{3}\right)=\left(-\frac{1}{3},\frac{\sqrt{3}}{3}\right)$, where $Q\left(x,y\right)$ *then,* $x-1=-\frac{1}{3}\ \Rightarrow x=\frac{2}{3}$, *and* $y=\frac{\sqrt{3}}{3},\Rightarrow Q\left(\frac{2}{3},\frac{\sqrt{3}}{3}\right)$	2

	2	$\left.\begin{array}{l} x_{\overline{QG}} = x_G - x_Q = 0-\frac{2}{3} = -\frac{2}{3} \\ y_{QG} = y_G - y_G = -\sqrt{3}-\frac{\sqrt{3}}{3} = \frac{-4\sqrt{3}}{3} \end{array}\right\} \Rightarrow \overrightarrow{QG}\left(-\frac{2}{3}, \frac{-4\sqrt{3}}{3}\right)$ $\left.\begin{array}{l} x_{\overline{PG}} = x_G - x_P = 0-\frac{1}{2} = -\frac{1}{2} \\ y_{PG} = y_G - y_P = -\sqrt{3}-0 = -\sqrt{3} \end{array}\right\} \Rightarrow \overrightarrow{QG}\left(-\frac{1}{2}, -\sqrt{3}\right)$	1.25
	3	$\left.\begin{array}{l} x_{\overline{QG}} \times y_{\overline{PG}} = \frac{2\sqrt{3}}{3} \\ y_{\overline{QG}} \times x_{\overline{PG}} = \frac{2\sqrt{3}}{3} \end{array}\right\} x_{\overline{QG}} \times y_{\overline{PG}} = y_{\overline{QG}} \times x_{\overline{PG}}$, then $\overrightarrow{QG}$ and $\overrightarrow{PG}$ are collinear, and G is common, then Q, P, and G are collinear	0.5
	4	$\overrightarrow{CM}(x-1, y)$ and $\overrightarrow{AB}(-1, -\sqrt{3})$ for (CM) and (AB) to be parallel, $\overrightarrow{CM}$ and $\overrightarrow{AB}$ must be collinear , then $-\sqrt{3}(x-1) = -y \quad \Rightarrow y = \sqrt{3}x - \sqrt{3}$.	1
III	1	$\frac{\sqrt[3]{4}\,\sqrt{3}}{\sqrt{\sqrt[3]{2^5}}} = \frac{2^{\frac{2}{3}}.3^{\frac{1}{2}}}{2^{\frac{5}{6}}} = \frac{3^{\frac{1}{2}}}{2^{\frac{1}{6}}}$	0.75
	2	$E^2 = \left(3+2\sqrt{2}\right)\left(3-2\sqrt{2}\right) = 9-8 = 1$ $\Rightarrow E = \pm 1$, *but* $E < 0$, *then* $E = -1$	1.25
	3	$\left.\begin{array}{ll} 2 \le a \le 3 & 1 \le b \le 4 \\ 6 \le 3a \le 9 & -4 \le -b \le -1 \end{array}\right\} \Rightarrow \quad 2 \le 3a - b \le 8$ $4 \le a^2 \le 9$ $\left.\begin{array}{l} 8 \le 2a^2 \le 18 \\ 1 \le b^2 \le 16 \end{array}\right\} \Rightarrow 9 \le 2a^2 + b^2 \le 34$ *and* $\frac{1}{34} \le \frac{1}{2a^2+b^2} \le \frac{1}{9}$, *then* $\frac{2}{34} \le (3a-b).\frac{1}{2a^2+b^2} \le \frac{8}{9} \Rightarrow \frac{1}{17} \le F \le \frac{8}{9}$ Since $0 < F < 1$, then $F^2 < F < \sqrt{F}$	2

IV	1	$\cos\left(\frac{7\pi}{3}\right)=\cos\left(2\pi+\frac{\pi}{3}\right)=\cos\left(\frac{\pi}{3}\right)=\frac{1}{2}$ $\sin\left(\frac{7\pi}{4}\right)=\sin\left(2\pi-\frac{\pi}{4}\right)=\sin\left(-\frac{\pi}{4}\right)=-\sin\left(\frac{\pi}{4}\right)=-\frac{\sqrt{2}}{2}$ $\cos\left(\frac{5\pi}{6}\right)=\cos\left(\pi-\frac{\pi}{6}\right)=-\cos\left(\frac{\pi}{6}\right)=-\frac{\sqrt{3}}{2}$ $then\ A=\frac{1}{2}+\sqrt{2}.\left(-\frac{\sqrt{2}}{2}\right)-\sqrt{3}.\left(-\frac{\sqrt{3}}{2}\right)=\frac{1}{2}-1+\frac{3}{2}=1$ $\sin\left(x-\frac{9\pi}{2}\right)=\sin\left(x-4\pi-\frac{\pi}{2}\right)=\sin\left[-\left(\frac{\pi}{2}-x\right)\right]=-\sin\left(\frac{\pi}{2}-x\right)=-\cos x$ $\cos(\pi+x)=-\cos x$ $\sin(7\pi-x)=\sin(6\pi+\pi-x)=\sin(\pi-x)=\sin x$ $\cos\left(x-\frac{\pi}{2}\right)=\cos\left[-\left(\frac{\pi}{2}-x\right)\right]=\cos\left(\frac{\pi}{2}-x\right)=\sin x\ ,\ then$ $B=-\cos x(-\cos x)+\sin x(\sin x)=\cos^2 x+\sin^2 x=1$	3.5
	2	$2\sin^2\alpha+\cos^2\alpha=\frac{14}{9}$ $2\sin^2\alpha+1-\sin^2\alpha=\frac{14}{9}$ $\sin^2\alpha=\frac{5}{9}\Rightarrow\sin\alpha=-\frac{\sqrt{5}}{3},\ \ (\text{since }\alpha\in QIV,\ \sin\alpha<0)$ $also: 2\left(1-\cos^2\alpha\right)+\cos^2\alpha=\frac{14}{9}$ $2-2\cos^2\alpha+\cos^2\alpha=\frac{14}{9}$ $\cos^2\alpha=\frac{4}{9}\Rightarrow\cos\alpha=\frac{2}{3},\ (\text{since }\alpha\in QIV,\ \cos\alpha>0)$ $then\ \ \tan\alpha=\frac{\sin\alpha}{\cos\alpha}=\frac{-\frac{\sqrt{5}}{3}}{\frac{2}{3}}=-\frac{\sqrt{5}}{2}$ $C=3\left(-\frac{\sqrt{5}}{3}\right)+3\left(\frac{2}{3}\right)-2\left(-\frac{\sqrt{5}}{2}\right)-2$ $=-\sqrt{5}+2+\sqrt{5}-2=0$	1.25

	3	B+C=1+0=1=A	0.25
V	1	(see below)	2.5

$$\frac{(2-x)(-4x)}{x-1}<0 \quad \text{(has a solution } S_1\text{)}$$

$2-x=0 \Rightarrow x=2$

$-4x=0 \Rightarrow x=0$

$x-1=0 \Rightarrow x=1$

x	$-\infty$	0	1	2	$+\infty$
2-x	+	+	+	-	
-4x	+	-	-	-	
x-1	-	-	+	+	
$\frac{(2-x)(-4x)}{x-1}$	-	+	‖ -	+	

$S_1 : x \in \left]-\infty , 0\right[\cup \left]1, 2\right[$

$(x-3)^2-(x-3)\geq 0$

$(x-3)(x-3-1)\geq 0$

$(x-3)(x-4)\geq 0$

x	$-\infty$	3	4	$+\infty$
x-3	-	0 +	+	
x-4	-	-	0 +	
(x-3)(x-4)	+	0 -	0 +	

$S_2 : x \in \left]-\infty , 3\right] \cup \left[4 , +\infty\right[$

The solution of the system $S = S_1 \cap S_2$

$S : x \in \left]-\infty , 0\right[\cup \left]1 , 2\right[$

2		$M=\dfrac{2(x-1)}{(x-1)(x-1-3-2x)}=\dfrac{-2}{x+4}$ $\lvert M\rvert>2\Rightarrow\dfrac{2}{\lvert x+4\rvert}>2$ $\Rightarrow\dfrac{1}{\lvert x+4\rvert}>1$ $\Rightarrow\lvert x+4\rvert<1$ ($if\ \dfrac{1}{b}>\dfrac{1}{a}\Rightarrow b<a$, both a and b are positives) $\Rightarrow -1<x+4<1$ $\Rightarrow -5<x<-3\quad or\ x\in\,]-5\,,-3[$	1.5