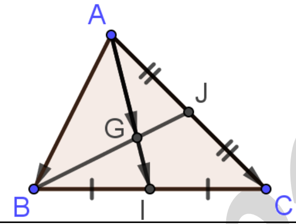
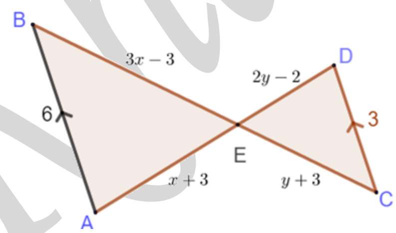
أسس التصحيح

<table>
<tr><th>Q</th><th>Parts</th><th colspan="2">Elements of answer</th><th>Notes</th></tr>
<tr><td rowspan="6">I.</td><td>1)</td><td colspan="2">I is the midpoint of $[BC]$ (given)
So, $\overrightarrow{AB}+\overrightarrow{AC}=2\overrightarrow{AI}$ (Median and vectors)
But, G is the centroid of triangle ABC (given)
So, $\overrightarrow{AI}=3\overrightarrow{GI}$
Thus, $\overrightarrow{AB}+\overrightarrow{AC}=2(3\overrightarrow{GI})=6\overrightarrow{GI}$ (by substitution)

</td><td></td></tr>
<tr><td>2)</td><td colspan="2">In $\Delta's\ ABE$ & EDC we have:
$(AB)\parallel(DC)$ (given)
The points $\left.\begin{matrix}A, E \& D\\ B, E \& C\end{matrix}\right\}$ are collinear in this order
By Thales' property: (Any line drawn parallel to a side of a triangle cuts other sides proportionally)
Ratios: $\frac{EC}{EB}=\frac{ED}{EA}=\frac{DC}{AB}$
$then\ \frac{y+3}{3x-3}=\frac{2y-2}{x+3}=\frac{3}{6}$


Using ratios: $\frac{y+3}{3x-3}=\frac{3}{6}$ and $\frac{2y-2}{x+3}=\frac{3}{6}$
We get: $\begin{cases}-3x+2y=-9 & (I)\\ (-x+4y=7)\quad\times(-3) & (II)\end{cases}$
By elimination we get: $y=3$
Replace value of $y=3$ in (I) to get $x=5$ (accepted greater than 1)</td><td>`</td></tr>
<tr><td>3)</td><td>$\left(\frac{3x+2}{5}-\frac{2x+1}{3}\le\frac{x+4}{3}\right)\times 15$
Then, $3(3x+2)-5(2x+1)\le 5(x+4)$</td><td>So, $(-6x\le 19)\times(-1)$
Hence, $x\ge -\frac{19}{6}$.
Thus, the negative integer solutions are: $\{-3,-2,-1,0\}$.</td><td></td></tr>
<tr><td>4)</td><td>$\overline{X}=\frac{\sum_{i=1}^{5}x_in_i}{N}$</td><td>$\overline{X}=\frac{1\times6+3\times5+5\times2+7\times4+8\times3}{20}$
Thus, $\overline{X}=\frac{83}{20}$</td><td></td></tr>
<tr><td>5)</td><td colspan="2">$m\times n=\frac{2^{12}-2^{10}}{2^8-2^6}=\frac{2^{10}(2^2-1)}{2^6(2^2-1)}=2^4=16.$</td><td></td></tr>
<tr><td>6)</td><td>$x=\frac{7+\sqrt{125}+\sqrt{20}}{20}$
$=\frac{7+\sqrt{5^2\times5}+\sqrt{2^2\times5}}{14}$
$=\frac{7+7\sqrt{5}}{14}$</td><td>Hence, $x=\frac{1+\sqrt{5}}{2}$
$x=1+\sqrt{5}$ is a solution of $x^2-ax+1=0$
$\left(\frac{1+\sqrt{5}}{2}\right)^2-a\left(\frac{1+\sqrt{5}}{2}\right)+1$
Hence, $10+2\sqrt{5}-2a(1+\sqrt{5})=0$
Thus, $a=\sqrt{5}$.</td><td></td></tr>
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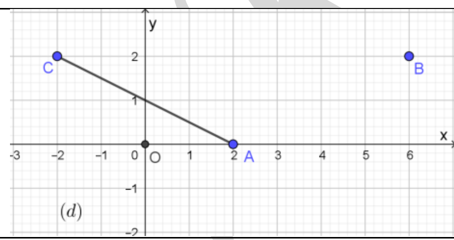
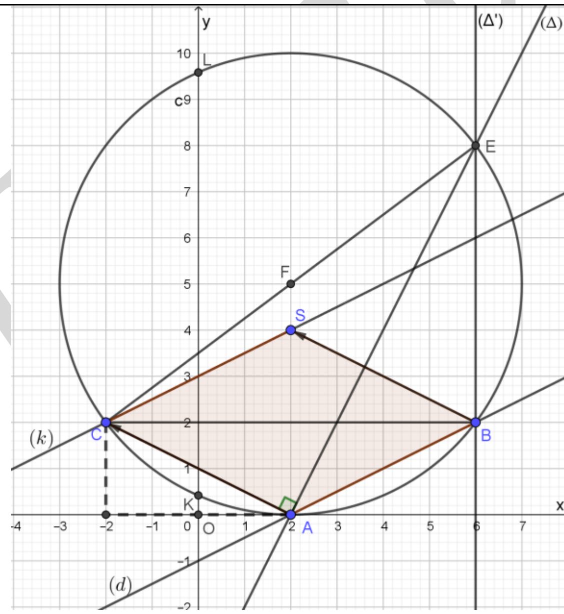
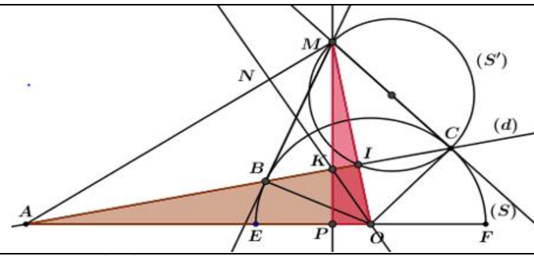
II.	1)		- Population is the set of 115 000 voters in the parliament elections in Beirut. - Variable: Names of the lists competed in the 2018 parliament elections. - Nature: qualitative since the variable is not a numeral		
	2)	a.	The central angle corresponding to List-A is $\alpha_A = 90^o$ (using the pie-chart) So, $\frac{\alpha_A}{360^o} = \frac{\% f_A}{100}$ (central angles and percentage frequencies are proportional) Then, $\% f_A = 25\%$ But, any list that scored more than 20% wins (given) Thus, list-A wins. $\% f_A = \frac{f_A}{N} \times 100$ (rule or frequencies and their percentages are propotional) Thus, the number of voter for list-A is $f_A = \frac{115000 \times 25}{100} = 28750$ voter.		
		b.	n is the central angle associated with the percentage of list E then, $n = 360 - (90 + 60 + 70 + 80)$. (sum of central angles) thus, $n = 58^0$		
	3.		No, we can't calculate the average value of given data since character is qualitative.		
III.	1	a.			
		b.	A & B belong to (d) if their coordinates satisfy the equation of $(d): x - 2y - 2 = 0$ • $x_A - 2y_A - 2 = 2 - 0 - 2 = 0.$ Verified • $x_B - 2y_B - 2 = 6 - 4 - 2 = 0.$ Verified		
		c.	$AB = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2}$ $= \sqrt{16 + 4}$ $AB = 2\sqrt{5}$ $units.$	$AC = \sqrt{(x_A - x_C)^2 + (y_A - y_C)^2}$ $= \sqrt{16 + 4}$ $AC = 2\sqrt{5}$ $units.$ Hence, $AB = AC = 2\sqrt{5}$ Thus, triangle ABC is isosceles at A.	
	2		$a_{(AC)} = \frac{y_A - y_C}{x_A - x_C} = -\frac{1}{2}$ $(\Delta) \perp (AC)$ (given) So, $a_{(\Delta)} \times a_{(BA)=} - 1$	$a_{(\Delta)} = 2.$ But, (Δ) pass through A (given) Hence, $(\Delta): \frac{y - y_A}{x - x_A} = a_{(\Delta)}$ Thus, $(\Delta) y = 2x - 4$	
	3	a	$(\Delta') \parallel y - axis$ (given) So, $(\Delta'): x = cst$ But, So, $B(6;2)$ belongs to (Δ') (given) Hence, $(\Delta'): x = 2$	To get E the intersection point of (Δ') & (Δ) we solve the system: $\begin{cases} y = 2x - 4 \\ x = 6 \end{cases}$ Then, $y = 8.$ Thus, $E(6; 8)$	

		b	$(CA)\perp(AE)$ at A (proved) So, $\widehat{CAE}=90^0$ But, (C) is a circle of diameter $[CE]$ (given) Thus, A is on (C) (inscribed angle facing diameter)	B & E have same abscissas $x_E=x_B=6$ So, $(BE)\parallel y-axis$ B & C have same ordinates $y_E=y_B=6$ So, $(BC)\parallel x-axis$ But coordinate axes are perpendicular. Hence, $\widehat{CBE}=90^0$ Thus, B is on (C) (inscribed angle facing diameter)	
	4	a	F is the center of (C) with diameter $[CE]$ So, F is the midpoint of $[CE]$	$x_F=\frac{x_C+x_E}{2}=2$ and $y_F=\frac{y_C+y_E}{2}=5$ Thus, F (2; 5)	
		b	L & K on $y-axis$ (given) So, $x_L=x_K=0$ L & K are on (C) (given) So, $FL=R$ Then, $FL^2=R^2$ But, $R=FA=\sqrt{(x_A-x_F)^2+(y_A-y_F)^2}$ $R=5cm$ $FL^2=(x_F-x_L)^2+(y_F-y_L)^2$ $25=4+(y-5)^2$ So, $(y-5)^2-21=0$ Hence, $(y-5-\sqrt{21})(y-5+\sqrt{21})=0$ Thus, $K(0;5-\sqrt{21})$ & $L(0;5+\sqrt{21})$		
	5	a	$\overrightarrow{AC}(-4;2)$ **(Graphically)**		
		b	S is translate of B by $\overrightarrow{AC}$ **(given)** **So,** $\overrightarrow{BS}=\overrightarrow{AC}$ **So,** $ABSC$ is a parallelogram But, $AB=AC=2\sqrt{5}cm$ (proved) Thus, $ABSC$ is a rhombus	$\overrightarrow{BS}=\overrightarrow{AC}$ Equal vectors admit equal coordinates. So, $x_{\overrightarrow{AC}}=x_{\overrightarrow{BS}}$ and $y_{\overrightarrow{AC}}=y_{\overrightarrow{BS}}$ $x_C-x_A=x_B-x_S$ & $y_C-y_A=y_B-y_S$ Hence, $x_S=2$ and $y_S=4$ Thus, $S(2;4)$	
		c	(k) is the image of (AB) by $\overrightarrow{AC}$ **(given)** **So,** $(k)\parallel(AB)$ So, $a_{(AB)}=a_{(k)}=\frac{1}{2}$ But, A is on (AB)	So, its image C is on (k) Then, $(k): \frac{y-y_C}{x-x_C}=a_{(k)}$ Thus, $(k): y=\frac{1}{2}x+3$	
	6	a	Let α be acute angle between (AB) & $x'Ox$ So, $\boldsymbol{a_{(AB)}=tan\alpha}$	So, $\tan\alpha=0.5$ $\alpha=\tan^{-1}(0.5)$ Thus, $\alpha\cong 27^o$	
		b	Let β be acute angle between (AB) & $y'Oy$ So, $\alpha+\beta=90^0$.	Hence, $\beta=90^0-27$ Thus, $\beta\cong 63^0$	

IV.

1



$[MB)$ & $[MC)$ are tangents to (C), issued from M at points B & C respectively (given)
O is center of (C) (given)
Thus, (OM) is the perpendicular bisector of $[BC]$ (Tangent theorem: line joining exterior point from which two tangents are drawn and center is perpendicular bisector of segment joining points of tangencies)

2

a

In triangles AIO & OMP we have:
$(OM)\perp$ bisector of $[BC]$ (proved)
So, $\widehat{AIO}=90^0$
P is the orthogonal projection of M on (OA) (given)
So, $\widehat{MPO}=90^0$

- Hence, $\widehat{AIO}=\widehat{MPO}=90^0$
- $\widehat{MOP}=\widehat{IOA}$ **« common angle »**

Thus, AIO & OMP are similar by angle-angle postulate.

b

AIO & OMP are similar (proved)

Ratio of similitude: $\dfrac{AIO}{MPO}\Big|\ \overset{1}{\dfrac{AI}{MP}}=\overset{2}{\dfrac{IO}{PO}}=\overset{3}{\dfrac{AO}{MO}}$

Using ratios: 2 & 3 we get:
Thus, $MO\times IO=AO\times PO$

c

In right triangle OMP:
$\cos M\hat{O}P=\dfrac{adj}{hyp}=\dfrac{OP}{OM}$
So, $OM=\dfrac{OP}{\cos M\hat{O}P}$

In right triangle OMC:
$\sin O\hat{M}C=\dfrac{opp}{hyp}=\dfrac{OC}{OM}$
So, $OM=\dfrac{OC}{\sin O\hat{M}C}$

Thus,
$\dfrac{OP}{\cos M\hat{O}P}=\dfrac{OC}{\sin O\hat{M}C}$
(by comparison)

3

a

CMI being a right-angled triangle at I (perpendicular bisector) from which it is inscribed in the circle (S ') of diameter [MC]. And (OC) ⊥ (MC) (because of the tangent (MC) to the circle (S)).
So (OC) is the tangent to (S) as being perpendicular to the diameter [MC].

b

In the triangles CIO and MCO we have:
<u>(A)</u> : $\widehat{OCM}=\widehat{CIO}=90^0$ **« perpendicular »**
<u>(A)</u> : $\widehat{COI}=\widehat{COM}$ **« Common angle»**
Hence, triangles CIO and MCO are similar by using AA property.
$\dfrac{CIO}{MCO}\Big|\rightarrow\dfrac{CI}{MC}=\dfrac{IO}{CO}=\dfrac{CO}{MO}$ *so* $\dfrac{IO}{CO}=\dfrac{CO}{MO}$ *then* $MO\times IO=CO^2$
therefore $MO\times IO=R^2$

c

We have:
$MO\times IO=AO\times PO$;
$MO\times IO=R^2$

$AO\times PO=R^2$; $2R\times OP=R^2$
therefore $OP=\frac{R}{2}$ **unit**

4

In the triangle OAM we have: (AI) ⊥ (MO) (where (AI) is the height proved)
And likewise (MP) ⊥ (AO) (because of the height)
Hence (MP) is a height.
The two heights intersect in K so K is the orthocenter of the triangle OAM.
Thus, (ON) ⊥ (MA) because (ON) passes through vertex O and orthocenter K