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Grade 8C

Monday, March, 14th, 2011.

Math Correction for the Mid-Term Exam

-I-

1) $E = \frac{(-7)^3 \times (-5)^{-4}}{49 \times 5^{-6} \times (-7)} = \frac{\cancel{\ominus} 7^3 \times 5^{-4}}{7^2 \times 5^{-6} \times (\cancel{\ominus} 7)} =$

$= \frac{7^3 \times 5^{-4}}{7^2 \times 5^{-6} \times 7} = \frac{\cancel{7^3} \times 5^{-4}}{\cancel{7^3} \times 5^{-6}} = 5^{-4-(-6)} = 5^{-4+6} = 5^2$

True

2) $f(x) = \frac{-x+1}{x+2}$

Cond: $x + 2 \neq 0 \Rightarrow \boxed{x \neq -2}$

True since $x = -2$ is not a natural integer.

3) $f(x) = x^2 - 2x - 2$

$f(1-\sqrt{3}) = (1-\sqrt{3})^2 - 2(1-\sqrt{3}) - 2$

$= 1^2 - 2(1)(\sqrt{3}) + (\sqrt{3})^2 - 2 + 2\sqrt{3} - 2$

$= 1 - \cancel{2\sqrt{3}} + \sqrt{3}^2 - 2 + \cancel{2\sqrt{3}} - 2$

$= \cancel{1 - 1} + 3 - 2 - 2 + 1$

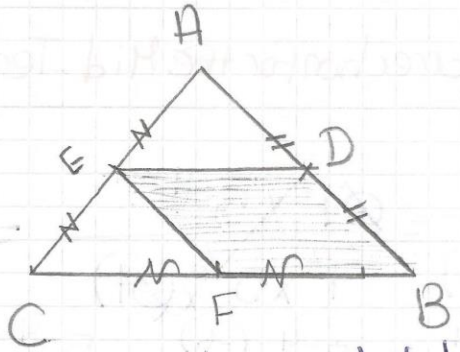
$= 3 + 1 - 2 - 2$

$= \cancel{4 - 4}$

$= 0$

False

4)



In ΔABC and in quadrilateral EDBF we have:
Since E is the midpoint of [AC] (given).
& D is the midpoint of [AB] (given)
then, $(ED) // (CB)$ (midpoint theorem in any Δ).
but, C, F, and B are collinear (F is the midpoint of [CB]) (given)
$\Rightarrow (ED) // (FB)$.

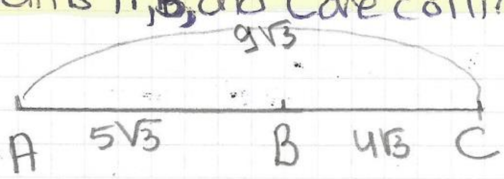
but, $ED = \frac{1}{2} BC$ (midpoint theorem in any Δ)
and since F is the midpoint of [BC] (given).
so, $ED = CF = FB$
Thus, EDBF is a parallelogram (having 1 pair of parallel and equal sides). True.

5) $AB = \sqrt{75}$ $AC = 3\sqrt{27}$ $BC = 2\sqrt{12}$

$AB = \sqrt{3 \times 5^2} = 5\sqrt{3}$
$AC = 3\sqrt{3^2 \times 3} = 9\sqrt{3}$
$BC = 2\sqrt{12} = 2\sqrt{3 \cdot 2^2} = 4\sqrt{3}$

$\Rightarrow AB + BC = AC$
$5\sqrt{3} + 4\sqrt{3} = 9\sqrt{3}$
$9\sqrt{3} = 9\sqrt{3}$.

Thus, points A, B, and C are collinear. True



-II-

1) $A = \frac{8}{3} + 5 \div \left(1 - \frac{2}{5}\right)$

$= \frac{8}{3} + 5 \div \left(\frac{5-2}{5}\right)$

$= \frac{8}{3} + 5 \div \left(\frac{3}{5}\right)$

$= \frac{8}{3} + 5 \cdot \frac{5}{3}$

$= \frac{8}{3} + \frac{25}{3} = \frac{33}{3} = \boxed{\frac{11}{1}}$

$B = \frac{55 \times 10^3 \times 2^{10}}{10^4 \times 2^9}$

$= \frac{5 \times 11 \times 10^3 \times 2^{10}}{10^4 \times 2^9} = \frac{5 \times 11 \times \cancel{2}}{10} = \frac{5 \times 11}{5}$

$= \frac{55}{5} = \boxed{\frac{11}{1}}$

$\boxed{\text{Thus, } A = B = 11}$

2) $C = (3\sqrt{5} - 1)(\sqrt{5} + 1) - (\sqrt{5} + 1)^2$

$= 3\sqrt{5}^2 + 3\sqrt{5} - \sqrt{5} - 1^2 - \left[(\sqrt{5})^2 + 2(\sqrt{5})(1) + 1^2\right]$

$= 3\sqrt{5}^2 + 3\sqrt{5} - \sqrt{5} - 1 - \left[5 + 2\sqrt{5} + 1\right]$

$= 15 + \cancel{3\sqrt{5}} - \cancel{\sqrt{5}} - 5 - \cancel{2\sqrt{5}} - 1 - 1$

$= 15 - 5 - 2$

$= 10 - 2$

$\boxed{C = 8}$ This nb. is a natural nb. since it is a no. that we can count with.

3) $D = 36 \times 10^{-6} \times (2 \times 10^{-1})^{-2}$

$= 6^2 \times 10^{-6} \times 2^{-2} \times 10^2$

$= 6^2 \times 10^{-4} \times 2^{-2} = \cancel{2^2} \times 3^2 \times 2^{-4} \times 5^{-4} \times \cancel{2^{-2}}$

$= 2^{-4} \times 3^2 \times 5^{-4} = \frac{9}{2^4 \times 5^4} = \frac{9}{(2 \times 5)^4} = \frac{9}{10^4} = 0.0009$

$= 9 \times 10^{-4} = \boxed{9.0 \times 10^{-4}} \rightarrow$ scientific notation

-III-

Part A:

1) a) $P(x) = (5x-4)^2 - (1-3x)^2$ (Form $a^2 - b^2 \equiv (a-b)(a+b)$)

$= [(5x)^2 - 2(5x)(4) + (4)^2] - [1^2 - 2(1)(3x) + (3x)^2]$

$= [25x^2 - 40x + 16] - [1 - 6x + 9x^2]$

$= \cancel{25x^2} - 40x + 16 - 1 + 6x - \cancel{9x^2}$

$P(x) = 16x^2 - 34x + 15$ where $a = 16$, $b = -34$ & $c = 15$

b) $P(x) = 14x^2 - 34x + 23$

$16x^2 - 34x + 15 = 14x^2 - 34x + 23$

$16x^2 - 14x^2 \cancel{- 34x} \cancel{+ 34x} = 23 - 15$

$\frac{2x^2}{2} = \frac{8}{2}$

$x^2 = 4$

$x^2 - 2^2 = 0$

$(x-2)(x+2) = 0$

product of two factors equals zero then at least one of them is zero

So, $x - 2 = 0$ or $x + 2 = 0$

Thus, $\boxed{x = \pm 2}$

2) a) $4x^2 - 12x + 9$

$= (2x)^2 - 12x + (3)^2$

$\boxed{= (2x-3)^2}$ which is a perfect square.

$Q(x) =$

$= 4x^2 - 12x + 9 + 3(2x-3)(2-x) - 4x + 6$

$= (2x-3)^2 + 3(2x-3)(2-x) + 2(-2x+3)$

$= (2x-3)^2 + 3(2x-3)(2-x) - 2(2x-3)$

$= (2x-3)[(2x-3) + 3(2-x) - 2]$

$= (2x-3)[2x - 3 + 6 - 3x - 2]$

$Q(x) = (2x-3)(-x+1)$

b) $P(x) = (5x - 4)^2 - (1-3x)^2$

So, $P(\frac{3}{2}) = \left(5\left(\frac{3}{2}\right) - 4\right)^2 - \left(1 - 3\left(\frac{3}{2}\right)\right)^2$

$= \left(\frac{15}{2} - 4\right)^2 - \left(1 - \frac{9}{2}\right)^2$

$= \left(\frac{15-8}{2}\right)^2 - \left(\frac{2-9}{2}\right)^2$

$= \left(\frac{7}{2}\right)^2 - \left(\frac{-7}{2}\right)^2$

$P(\frac{3}{2}) = \frac{49}{4} - \frac{49}{4} = \frac{49-49}{4} = \frac{0}{4} = 0$

hence $x = \frac{3}{2}$ is a root for $P(x)$.

$Q(x) = (2x - 3)(-x+1)$

so, $Q(\frac{3}{2}) = \left(\cancel{2}\left(\frac{3}{\cancel{2}}\right) - 3\right)\left(-\left(\frac{3}{2}\right) + 1\right)$

$= (3-3)\left(\frac{-3}{2} + 1\right)$

$= (0)\left(\frac{-3+2}{2}\right)$

$= (0)\left(\frac{-1}{2}\right)$

$Q(\frac{3}{2}) = 0$

hence, $x = \frac{3}{2}$ is a root for $Q(x)$.

Thus $x = \frac{3}{2}$ is a common root for $P(x)$ + $Q(x)$.

3) $T(x) = \frac{P(x)}{Q(x)}$ $\quad P(x) = (5x-4)^2 - (1-3x)^2$

a) $P(x) = (5x-4)^2 - (1-3x)^2$

$= [(5x-4)-(1-3x)][(5x-4)+(1-3x)]$

$= [5x-4-1+3x][5x-4+1-3x]$

$P(x) = [8x-5][2x-3]$

b) $T(x) = \frac{(8x-5)(2x-3)}{(2x-3)(-x+1)}$

Condition: $2x-3 \neq 0 \Rightarrow 2x \neq 3 \Rightarrow \boxed{x \neq \frac{3}{2}}$ and

$-x+1 \neq 0 \Rightarrow (-x \neq -1) \Rightarrow \boxed{x \neq 1}$

This fraction is defined for all real values of x such that $x \neq \frac{3}{2}$ and $x \neq 1$.

$T(x) = \frac{(8x-5)\cancel{(2x-3)}}{\cancel{(2x-3)}(-x+1)} = \boxed{\frac{8x-5}{-x+1}}$

Part B:

$AD = x+3$, $DC = 3x-1$ $\quad AB = x+9$

$A_{ABCD} = h\left(\frac{b_1+b_2}{2}\right) = (x+3)\left(\frac{3x-1+x+9}{2}\right) = (x+3)\left(\frac{4x+8}{2}\right)$

$= \frac{\cancel{2}(x+3)(2x+4)}{\cancel{2}} = (x+3)(2x+4) = 2x^2 + 4x + 6x + 12$

$A_{ABCD} = 2x^2 + 10x + 12$

2) So, $x+3 = 3x-1$ (For AHCD to be a square)
then, $DC = AD$

$x - 3x = -1 - 3$

$\frac{-2x}{-2} = \frac{-4}{-2}$

$\boxed{x = 2}$

Now, AHCD is a square after being a rectangle + 2 consecutive equal sides.

3) $x = 2$

$A_{ABCD} = 2x^2 + 10x + 12$

$= 2(2)^2 + 10(2) + 12$

$= 2(4) + 20 + 12$

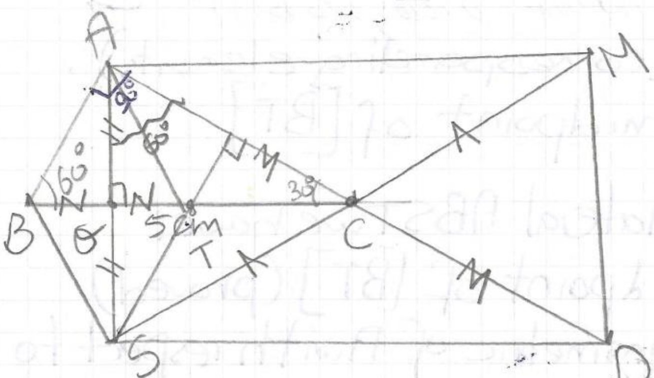
$= 8 + 20 + 12$

$\boxed{A_{ABCD} = 40}$ square unit.

-IV-

$BC = 5cm$, $A\hat{B}C = 60°$, $A\hat{C}B = 30°$, $AO = OS$, $B\hat{A}C = 90°$,

1)



2) $B\hat{A}C = ?$

$B\hat{A}C + A\hat{B}C + A\hat{C}B = 180°$ (sum of angles in a $\Delta = 180°$)

$B\hat{A}C + 60° + 30° = 180°$

$B\hat{A}C + 90° = 180°$

$B\hat{A}C = 180°$
$-90°$

$\boxed{B\hat{A}C = 90°}$

-7-

$B\hat{A}C = 90°$ (proved)

then, $(BA) \perp (AC)$

$(ST) \perp (AC)$ (given).

Thus, $(ST) \mathbin{//} (AB)$ (2 straight lines $\perp$ to the same straight line are parallel).

3) a) In Δ's ABO and OTS we have;

1) $A\hat{O}B = T\hat{O}S = 90°$ (S is the symmetric of A with respect to [BC]).

2) S is the symmetric of A with respect to [BC]

$\Rightarrow OS = OA$

3) $(AB) \mathbin{//} (ST)$ (proved)

$\Rightarrow B\hat{A}O = O\hat{S}T$ (Alternating Interior angles between 2 parallel straight lines).

Thus Δ's are congruent by ASA property.

O, B and T are collinear (O belongs to [BT])

$OB = OT$ (corresponding elements).

Thus, O is the midpoint of [BT]

b) In quadrilateral ABST we have:

O is the midpoint of [BT] (proved)

& S is the symmetric of A with respect to [BC] (given)

$\Rightarrow$ O is the midpoint of [SA]

So, ABST is a parm (having its diagonals bisect each other at the same midpoint)

Since S is the symmetric of A with respect to [BC] (given)

So, $(AS) \perp (BC)$

and, O is the midpoint of [SA] (proved)

and, O is the the midpoint of [BT] (proved).
$\Rightarrow$ (AS) $\perp_{bt}$ of (BT)
$\therefore$ parm ABST is a rhombus having a parm + diagonals which are $\perp_{bt}$ of each other)

4)a) $S\hat{A}C + A\hat{C}O + A\hat{O}C = 180°$ (sum of angles in $\Delta = 180°$).

$S\hat{A}C + 30° + 90° = 180°$

$S\hat{A}C + 120° = 180°$

$S\hat{A}C = 180° - 120°$

Finally, $\boxed{S\hat{A}C = 60°}$

b) (BC) $\perp$ (AS) (diagonals of a rhombus).
+ O is the midpoint of [AS] (proved).
$\therefore$ (CO) represent the perp. bisector for [AS].

c) In Δ ASC we have:
(CO) is a height relative to [AS] (given)
but (CO) is the $\perp_{bt}$ [AS] (proved)
$\Rightarrow$ ASC is an isosceles Δ having a height which is also a $\perp_{bt}$.
so, $C\hat{A}S = C\hat{S}A = 60°$
then, $C\hat{A}S + C\hat{S}A + A\hat{C}S = 180°$ (sum of angles in a $\Delta = 180°$).

$60° + 60° + A\hat{C}S = 180°$

$120° + A\hat{C}S = 180°$

$A\hat{C}S = 180° - 120°$

$\boxed{A\hat{C}S = 60°}$

Thus, ~~the isosceles~~ ΔASC is an equilateral Δ having 3 equal angles $= 60°$.

b) In quadrilateral ASDM we have:
C is the midpoint of [AD]:
D is the symmetric of A with respect to C.
C is the midpoint of [SM]
M is the symmetric of S with respect to C.
so, ASDM is a parm having its diagonals bisect each other at the same midpoint.
but, AC = CS (sides of an equilateral Δ) (proved).
and AC = CD
$\Rightarrow$ AC = CS = CD
but, SC = CM
and, CA = CS
$\Rightarrow$ SC = CM = CA
$\Rightarrow$ SC = CM = CA = CD.
as a result, AD = SM (sum of equals are equal).
$\therefore$ parm ASDM is a rectangle having a parm + equal diagonals.

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