Lycée Des Arts. Correction standards Mid-Term 8th Grade 2012-2013.

1st exercise:

1a) $AO = \frac{2^{11}+4^5}{2^{11}-4^5}$

$= \frac{2^{11}+2^{10}}{2^{11}-2^{10}}$

$= \frac{2^{10}(2+1)}{2^{10}(2-1)}$

$\boxed{AO = 3}$ which is a natural no. (since we can count with)

b) $OD = \dfrac{\frac{2}{10}+\frac{2}{5}}{\frac{2}{3}\times\frac{9}{10}-\frac{2}{5}}$ (Use: Bedmas & sm)

$= \dfrac{\frac{1}{5}+\frac{2}{5}}{\frac{3}{5}-\frac{2}{5}}$

$= \dfrac{\left[\frac{3}{5}\right]}{\left[\frac{1}{5}\right]}$

$\boxed{OD = 3}$

$BD = \frac{0.24\times 9}{5^{-2}\times 36\times 2^{-2}}$ (Use: powers of 10 & prime bases)

$= \frac{24\times 10^{-2}\times \cancel{3^2}}{5^{-2}\times 2^2\times \cancel{3^2}\times 2^{-2}}$

$= \frac{24\times 5^{-2}\times 2^{-2}}{5^{-2}} = \frac{24}{2^2}$

$BD = \frac{24}{4}$

$\boxed{BD = 6}$

Now, $\frac{BD}{OD} = \frac{6}{3}$

So, $\frac{BD}{OD} = 2$

Thus, $\boxed{OD = \frac{1}{2} BD}$

c). In quadrilateral ABCD we have

- [AC] and [BD] intersect at O (given)

- $OD = \frac{1}{2} BD$ (proved)

then, O is midpoint of [BD].

$AC = 6$ cm (given)
$AO = 3$cm (proved) } So, $AO = \frac{AC}{2}$

then, O is midpoint of [AC].

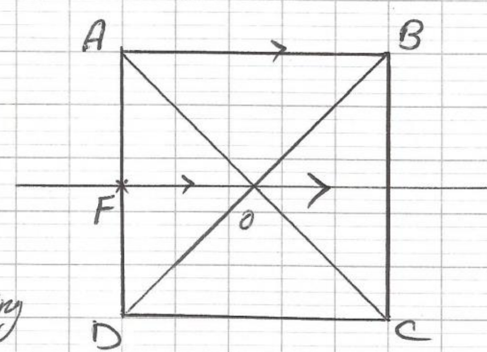
Thus, O is center of quadrilateral ABCD.

d) In quadrilateral ABCD we have:

O is center of ABCD (proved)
So, ABCD is a parallelogram (quadrilateral having a center of symmetry).

- [AC] perpendicular [BD] (given)
$AC = 6$ cm (given)
$BD = 6$ cm proved } So, $AC = BD$.
Thus, ABCD is a square (parm having equal & perp diagonals).

2) a) In $\triangle ABD$ we have
O is midpt of [BD] (proved)
(OF) parallel to (AB) (given)



then, F is mid pt of [AD]
(by converse of mid pt theorem: A st line passing through mid pt of a side, parallel to another side should pass through mid pt of third side)

Now, O & F are respective midpts of [BD] & [AD]

Then, $OF = \frac{1}{2} AB$ (by midpt theorem: statement)

but $AB = AD$ (sides of square ABCD)

Thus, $\boxed{OF = \frac{1}{2} AD}$.

c). Area of AOD = 4.5 cm² (given).

O is center of square ABCD.
then $\triangle$'s formed by diagonals of ABCD are congruent.

hence, $Area_{ABCD} = 4 \times Area_{AOD}$

Thus, $Area_{ABCD} = 4\,(4.5)$
$= 18\ cm^2$.

2^{nd} exercise: part A:

1a). $n = \dfrac{\sqrt{18} \times \sqrt{20}}{\sqrt{45} \times \sqrt{2}}$

$= \sqrt{\dfrac{18\div 9}{45 \div 9} \times \dfrac{20 \div 2}{2 \div 2}}$ (simplify)

$= \sqrt{\dfrac{2}{5 \div 5} \times 10 \div 5}$

$\boxed{n = 2}$.

b). For $x = n$

so, $x = 2$.

$u = \sqrt{3(2) - 8}$

$= \sqrt{6-8}$

$u = \sqrt{-2}$.

so, u is not defined in set of real no,s since radicand should be a positive real no,.

c) u is defined for which radicand is a positive real no,

then $3x - 8 \geqslant 0$

$3x \geqslant 8$.

$\boxed{x \geqslant 8/3}$ Thus u is defined for all real values of $x \geqslant \frac{8}{3}$.

2). $u = n$

$\sqrt{3x-8} = 2$. square both sides

$3x - 8 = 4$

$3x = 4 + 8$

$3x = 12$

$x = 4$.

which is accepted included in domain of u.

Part-B:

1a) $m = 2\sqrt{75} - 3\sqrt{48} + 2\sqrt{12} + \sqrt{1}$
$= 2\sqrt{5^2\times 3} - 3\sqrt{4^2\times 3} + 2\sqrt{2^2\times 3} + 1$
$= 10\sqrt{3} - 12\sqrt{3} + 4\sqrt{3} + 1$

$\boxed{m = 1 + 2\sqrt{3}}$ where $a = 1$ & $b = 2$.

b) $m = 1 + 2\sqrt{3}$
$= 4.4641\ldots$
$\simeq 4.465$ (approximated by ~~default~~ excess to nearest 10^{-3}).

2)a) $m = 2\sqrt{3} + 1$
$m^2 = (2\sqrt{3} + 1)^2$ $\qquad (a+b)^2 = a^2 + 2ab + b^2$
$m^2 = 13 + 4\sqrt{3}$

$(\sqrt{3} + 2)^2 = 7 + 4\sqrt{3}$ $\qquad$ (Square of sum: $(a+b)^2$)

b) $t = m^2 - (\sqrt{3} + 2)^2 + 3$
$= 13 + 4\sqrt{3} - (7 + 4\sqrt{3}) + 3$
$= 13 + 4\sqrt{3} - 7 - 4\sqrt{3} + 3$.

$t = 9$.

c) Area of square ABCD $= t$ given

then, $A_{square} = 9$
but, $A_{square} = \text{Side}^2$.
So, $\text{Side}^2 = 9$
Thus, Side $= 3$ units of length.

3^{rd}-exercise:

1a). $P(x) = (3x-5)^2 + (1+x)(5-3x)$

$= (3x-5)^2 - (1+x)(3x-5)$

$= (3x-5)[3x-5-(1+x)]$

$= (3x-5)[3x-5-1-x]$

$= (3x-5)(2x-6)$

$\boxed{P(x) = 2(3x-5)(x-3)}$

b). $P(x) = 0$

$2(3x-5)(x-3) = 0$ product of two or more factors equals to zero, indicates that either of the factors is zero

$3x-5=0$ OR $x-3=0$

$3x=5$

$\boxed{x=\frac{5}{3}}$ OR $\boxed{x=3}$.

but, $x=\frac{5}{3}$ is not a natural n°, so, it is rejected

and $x=3$ is a " " so " " accepted.

2a) To factorize $Q(x)$ is to write it in product form

$Q(x) = (x^2-9) - (3x-9)(x-5)$

$= (x^2-3^2) - 3(x-3)(x-5)$

$= (x-3)(x+3) - 3(x-3)(x-5)$

$= (x-3)[(x+3) - 3(x-5)]$

$= (x-3)[x+3-3x+15]$.

$\boxed{Q(x) = (x-3)(-2x+18)}$

b). To write $Q(x)$ in form ax^2+bx+c is expand and reduce $Q(x)$.

$Q(x) = (x-3)(-2x+18)$ (proved)

$Q(x) = -2x^2 + 24x - 54$. (Now FOIL)

where $a = -2$; $b = 24$ and $c = -54$.

c). $Q(x) = -54$

$-2x^2 + 24x - 54 = -54$

$-2x^2 + 24x = 0$

$-2x(x-12) = 0$.

$-2x = 0$ OR $x - 12 = 0$

$x = \frac{0}{-2}$ $\quad$ $x = 12$

Thus $\boxed{x = 0}$ OR $\boxed{x = 12}$

d). If $x = 9$ is a root of $Q(x)$ the $Q(9) = 0$ / OR. In words: For $x = 9$ to be a root of $Q(x)$, then $Q(x)$ vanishes at $x = 9$.

1st way

$Q(x) = (x-3)(-2x+18)$

$Q(9) = (9-3)(-2(9)+18)$

$= 3(-18+18)$

$= 3(0)$

$Q(9) = 0$

Thus $x = 9$ is a root of $Q(x)$

2nd way!

$Q(x) = (x-3)(-2x+18)$

$= -2(x-3)(x-9)$

Since $x - 9$ is a factor of $Q(x)$

Thus $x = 9$ is a root of $Q(x)$.

3)a) $F(x) = \frac{P(x)}{Q(x)}$ (Use product form)

$F(x) = \frac{2(3x-5)(x-3)}{-2(x-3)(x-9)}$.

$Q(x)$ or the denominator of $F(x)$ vanishes for $Q(x) = 0$

$-2(x-3)(x-9)=0 \quad (-2\neq 0)$

$x-3=0$ OR $x-9=0$

$\boxed{x=3}$ OR $\boxed{x=9}$

Thus, $Q(x)$ vanishes for $x=3$ or $x=9$.

→ Domain of $F(x)$ includes all values of x for which denominator of $F(x)$ (which is $Q(x)$) is different than zero. that is, $Q(x)\neq 0$.

Now $Q(x)\neq 0$.

for $x\neq 3$ & $x\neq 9$

Thus, the domain of $F(x)$ is all real values of x. except for $x=3$ & $x=9$.

b) $F(x)=\dfrac{2\cancel{(x-3)}(3x-5)}{-2\cancel{(x-3)}(x-9)}$

$F(x)=\dfrac{-(3x-5)}{(x-9)}$

c). $F(\sqrt{2})=\dfrac{-[3(\sqrt{2})-5]}{\sqrt{2}-9}$

$=\dfrac{(-3\sqrt{2}+5)}{(\sqrt{2}-9)}\times\dfrac{(\sqrt{2}+9)}{(\sqrt{2}+9)}$

$=\dfrac{-3\sqrt{2}^2-27\sqrt{2}+5\sqrt{2}+45}{(\sqrt{2})^2-(9)^2}$

$F(\sqrt{2})=\dfrac{39-22\sqrt{2}}{-79}=\dfrac{22\sqrt{2}-39}{79}$.

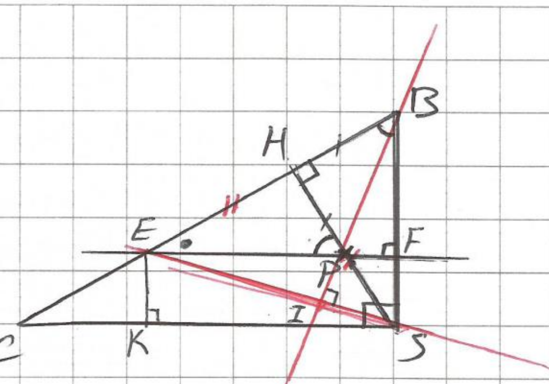
note that:

- $F(\sqrt{2})$ is simplified if denominator is free of radical
- To eliminate radical from denominator we rationalize
- To rationalize to multiply by the conjugate of the denominator

4^{th} exercise:

1) Drawn ✓

a) In $\triangle BEF$ we have:



$\triangle BSC$ is right at S (given)
So, (BS) is perp. to (SC)

And (EF) is parallel to (SC) (given)
then, (EF) is perpendicular to (BS) (A st. line parallel to one of two perp st. lines is perp. to the other)

Thus, $\triangle EBF$ is right at F (having a pair of adj. sides perp).

b)

[SH) is perp. [BC] (given).
but, P & E belong to [SH] & [CH] respectively (given)
So, $E\hat{H}P = 90°$.
Then, $\triangle EHP$ is right at H (having a right angle.

Hence, $H\hat{E}P + H\hat{P}E = 90°$ (Sum of base angles of right $\triangle HEF$)
But $E\hat{B}F + B\hat{E}F = 90°$ (" " " " " " $\triangle BEF$)

Thus, $H\hat{P}E = E\hat{B}F$ (Angles having same complement)
(OR by comparison)

c) In $\triangle$'s HEP & BHS we have

$H\hat{P}E = E\hat{B}F$ (proved)
$E\hat{H}P = B\hat{H}S = 90°$ (proved)
HB = HP (given)

Thus, $\triangle$'s are equal by ASA property.

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