Lycée Des Arts

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Mark/ La note

Test in/ Examen de : Mathematics.

Time / La durée :

Name/Le nom :

Class/ La Classe: 8^{th} Grade

Date / La date: 2014-2015

~ Correction-Standards ~

Ex.-1 1) $Area_{ABCD} = Side^2$ (area of a square)

$= AB^2$

$= 36\ cm^2$

$A_{MBC} = A_{NDC} = \dfrac{leg_1 \times leg_2}{2}$ (Δ's MBC + NDC are equal)

$= \dfrac{MB \times BC}{2}$

$= 12\ cm^2.$

hence $Area_{ANCM} = Area_{ABCD} - 2\,Area_{MBC}$

$= 36 - 2(12)$

$= 12\ cm^2.$

Thus, $\dfrac{Area_{ANCM}}{Area_{ABCD}} = \dfrac{12}{36} = \dfrac{1}{3}$ A.

2) $A = \left(\dfrac{4}{3}\right)^{-1} - \dfrac{\frac{3}{1} + \frac{5}{4}}{\frac{5}{1} - \frac{1}{7}}$

$= \dfrac{3}{4} - \dfrac{\left[\frac{17}{4}\right]}{\left[\frac{34}{7}\right]}$

$= \dfrac{3}{4} - \dfrac{\cancel{17} \times 7}{4 \times \cancel{34}_2}$

$= \dfrac{3 \times 2}{4 \times 2} - \dfrac{7}{8}$

$\boxed{A = \dfrac{-1}{8}}$

$B = \dfrac{96 \times 10^{-6} \times (-5) \times 10^{-1}}{2^{-6} \times 3 \times 5^{-6} \times 2}$

$= -\dfrac{2^5 \times 3 \times 10^{-6} \times 5 \times 10^{-1}}{10^{-6} \times 3 \times 2}$

$= -\dfrac{2^4 \times 5 \times 10^{-1}}{1}$

$B = -2^3$

$\boxed{B = -8}$

but $A \times B = \left(-\dfrac{1}{8}\right) \times (-8)$

$= 1$

A

Thus, A is the reciprocal of B.

3) In $\triangle$ AOB we have:
I is the mid pt of [AB] (given)
So, [OI) is a median relative to [AB]
but, IO = IA = IB (given)
then, $\triangle$ AOB is right at O (by converse of median relative to hyp)

but, O belongs to [AC] (given)
So, $\hat{AOB} = \hat{BOC} = 90°$ (Supplementary angles)
then $\triangle$ BOC is right at O.
but J is the midpt of [BC] (given)
then [OJ) is a median relative to hyp [BC]
hence, JO = JC = JB.
Thus, $\triangle$ JOC is isosceles at O (having two equal sides) (A)

4) $F = \frac{(3x-5)(-2)}{(5x-2)(-2)} + \frac{16-9x}{4-10x}$ For F to exist: $5x-2 \neq 0$ $\boxed{x \neq \frac{2}{5}}$

$= \frac{-6x+10+16-9x}{-2(5x-2)}$

$F = \frac{26-15x}{-2(5x-2)}$

which is a literal fraction defined for all values of x except for $x = \frac{2}{5}$ since it has a variable in its denominator (A)

5) $A = \frac{\frac{3\times 2}{3\times 1} + \frac{1}{3} + \frac{1}{\frac{3\times 1}{3\times 1} + \frac{1}{3}}}{\frac{2\times 1}{2\times 1} - \frac{1}{2}}$

$= \frac{\frac{7}{3} + \frac{1}{\frac{4}{3}}}{\frac{1}{2}}$

$A = \frac{\frac{4\times 7}{4\times 3} + \frac{3\times 3}{4\times 3}}{\frac{1}{2}}$

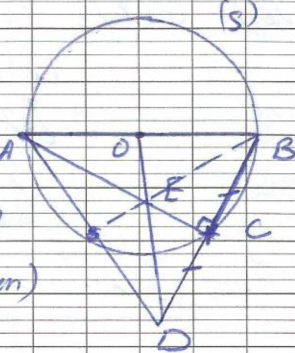
$A = \frac{\left[\frac{37}{12}\right]}{\frac{1}{2}}$

$= \frac{37\times 2 \div 2}{12 \div 2}$

$A = \frac{37}{6} = 6.1666...$

$A = 6.17$ approximated by excess to nearest hundredth. (C)

6) In $\triangle ABD$ we have:
(S) is a circle of diameter [AB]
and center O (given)
then, O is the midpt [AB].
So, [DO) is a median relative to [AB]
And, D is the symm. of B w.r.t C (given)
So, C is the midpt [BD].
So, [AC) is a median relative to [BD]
but (AC) & (DO) in intersect at E (given)
Then E is the centroid of $\triangle ABD$.
Thus, [BE) is a median relative to [AD] (st line issued from vertex & passes through centroid).



Ⓐ

Ex-2: 1a) $H(x) = (x+4)^2 - (x+3)(x+4) + 2x^2 - 32$
$= x^2 + 8x + 16 - [x^2 + 3x + 4x + 12] + 2x^2 - 32$
$= 3x^2 + 8x - 16 - x^2 - 7x - 12$
$\boxed{H(x) = 2x^2 + x - 28}$

b) $H(x) = -28$
$2x^2 + x - 28 = -28$
$2x^2 + x = 0$
$x(2x+1) = 0$
If product of two factors is zero then at least one of them is zero. (1)
So, $x = 0$ or $2x + 1 = 0$
$x = -\frac{1}{2}$.
Solution set of eqn. $H(x) = -28$ is $x = \{0, -\frac{1}{2}\}$.

2a) $G(x) = 4(x-1)^2 - (3x+2)^2$

$= [2(x-1)]^2 - (3x+2)^2$

$= [2(x-1) - (3x+2)][2(x-1) + (3x+2)]$

$= (2x-2-3x-2)(2x-2+3x+2)$

$= (-x-4)(5x)$

Thus, $\boxed{G(x) = -5x(x+4)}$

$H(x) = (x+4)^2 - (x+3)(x+4) + 2x^2 - 32$

$= (x+4)^2 - (x+3)(x+4) + 2(x^2 - 4^2)$

$= (x+4)^2 - (x+3)(x+4) + 2(x-4)(x+4)$

$= (x+4)[x+4 - (x+3) + 2(x-4)]$.

Thus, $\boxed{H(x) = (x+4)(2x-7)}$.

b) To find roots of $G(x)$

we solve factorized form of $G(x) = 0$

So $-5x(x+4) = 0$

by ① so,

$\frac{-5x}{-5} = \frac{0}{-5}$ or $x+4 = 0$

$x = 0$ or $x = -4$

Thus, the roots of the eqn $G(x) = 0$ are 0 & -4

3) a) $SA = G(x)$ (given)

& $x = -4$ is a root of $G(x)$ (proved)

So, $G(-4) = 0$

Thus, the side SA doesn't exist for $x = -4$ since it is reduced to zero

b) $AL = H(x)$ (given)

$H(x) = (x+4)(2x-7)$

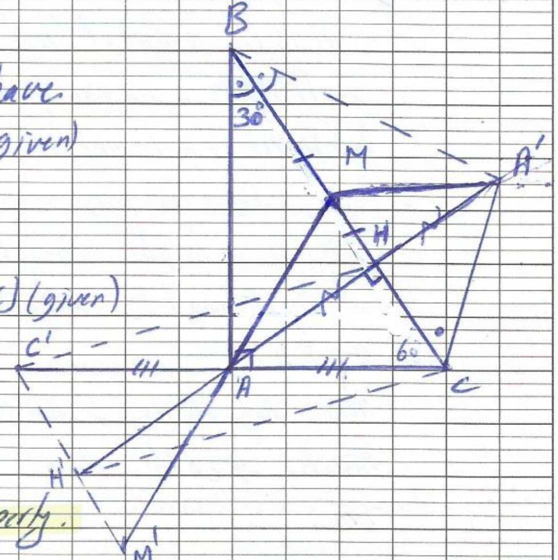
So $H(1) = (5)(2-7)$

$= -25 < 0$

we notice that.
$H(1) = -25 < 0$
which is rejected, since a side exists if it is strictly positive.

c) For SALI to be a rhombus
then $SA = AL$ (consecutive sides in a rhombus are equal)
So, $G(x) = H(x)$
So $(x+4)(2x-7) = -5x(x+4)$
$(x+4)(2x-7) + 5x(x+4) = 0$
$(x+4)[(2x-7) + 5x] = 0$
$(x+4)(7x-7) = 0$
by ① so, $x+4=0$ or $7x-7=0$
then $x=-4$ or $x=1$
which are both rejected
since $x=-4$ deminishes $G(x)$ to zero
& $x=1$ makes $H(x)$ negative
Thus, there is no value of x for which SALI is a rhombus

Ex-3. 1) Drawn. ✓



2) In Δ's AHC & A'HC we have
A' is the symm. of A w.r.t H (given)
So, H is the midpt of [AA']
→ then $AH = A'H$.
[AH) is a height relative to [BC) (given)
→ So, $A\hat{H}C = A'\hat{H}C = 90°$
→ [HC] is a common side.
Thus, Δ's AHC & A'HC
are equal by S.A.S property.

• ΔABC is right at A (given)
& $A\hat{B}C = 30°$ (given)

So, $B\hat{C}A = 60°$ (sum of base angles in a right Δ)
but AHC & A'HC are congruent (proved).
Thus, $A\hat{C}B = B\hat{C}A' = 60°$ (by homologeous elements.

3a) In right triangle ABC of hyp [BC] we have:
M is the midpt of [BC] (given)
So, [AM) is a median relative to hyp. [BC]
Then, $AM = \frac{1}{2} BC$
but $BC = 6cm$ (given)
Thus, $AM = 3cm$.

- In Δ ACM we have
$MA = MC = MB$ (by median relative to hyp)
but $B\hat{C}A = 60°$ (proved)
Thus, Δ ACM is equilateral (having two equal sides & a 60° angle)

b) In quadrilateral CAMA' we have:
H is the midpt of [AA'] (proved)
so, [MH) is a median
but [MH) is a height
- hence, (MH) is the perp. bisector of [AA']
[AH) is a height issued from a vertex of equilateral Δ AMC.
- So [AH) is perp. bisector of [MC].
hence diagonals [MC] & [AA'] are perp. bisector of each other.
Thus, CAMA' is a rhombus.

4) In Δ's ABC & A'BC we have:
[BC] is a common side
$A\hat{C}B = B\hat{C}A' = 60°$ (proved)
CAMA' is a rhombus (proved)
So, $AC = CA'$ (sides of a rhombus)
Thus, Δ's are equal by S.A.S property

- Δ ABC and A'BC are equal (proved)
So, $A\hat{B}C = A'\hat{B}C = 30°$ (by homologeous elements).
Thus, [BC) is the bisector of $A\hat{B}A'$.

5) drawn ✓

a) In quadrilateral CHC'H' we have:
C' & H' are the resp. symmetries of C & H w.r.t A (given)
then A is the mid pt of [CC'] & [HH']
Thus, CHC'H' is a parallelogram (its diagonals bisect each other at the same midpt)

5b) [AH] is a height issued from vertex of equilateral
△AMC (given)
then, [AH] is a median relative to [MC]
hence, H is the midpt of [MC].
but MA = MC = 3cm (proved)
Then, $HC = \frac{MC}{2} = 1.5$ cm

but HCH'C' is a parallelogram (proved)
Thus, HC = H'C' = 1.5cm (opp. sides of a parallelogram)

c) △AMC is equilateral (proved)
So, AM = AC (sides of equilateral △)
but C' & M' are symmetries of C & M
w.r.t A resp. (given)
Thus, CC' = MM' (double of equals are equal)

• In quadrilateral CMC'M' we have:
C' & M' are resp. symmetries of C & M
w.r.t. A resp (given)
& CC' = MM' (proved)
Thus, CMC'M' is a rectangle
(having its diagonals bisect each other at same midpt and equal)

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P-7.