8^{th} Grade

Mathematics
Correction-standards of Mid-term-Exam

2015-2016.

Exercise-I:

1. $x(x-5)=(x-5)^2$

$x(x-5)-(x-5)^2=0$

$(x-5)[x-x+5]=0$

$5(x-5)=0$

Thus, $x=5$ which is a unique soln. (A)

2. $A=\dfrac{2\frac{3}{4}+\frac{1}{5}}{\frac{5\times3}{5\times4}-\frac{1}{2}+\frac{7\times4}{5\times4}}$

$=\dfrac{\frac{5\times11}{5\times4}+\frac{1\times4}{5\times4}}{\frac{15-10+28}{20}}$

$A=\dfrac{\frac{55+4}{20}}{\frac{33}{20}}$

$A=\dfrac{\left[\frac{59}{20}\right]}{\left[\frac{33}{20}\right]}$

$A=\dfrac{59\times\cancel{20}}{33\times\cancel{20}}$

$\boxed{A=\dfrac{59}{33}}$

$B=\dfrac{1}{7}+\dfrac{2}{1}\times\dfrac{2}{5}+\dfrac{26}{35}$

$=\dfrac{1\times5}{7\times5}+\dfrac{4\times7}{5\times7}+\dfrac{26}{35}$

$B=\dfrac{5+28+26}{35}$

$\boxed{B=\dfrac{59}{35}}$

A & B are two fraction of the same neumerator then the one with the greater denominator is the smaller.

but $35>33$

Thus, $\boxed{A>B}$ (B)

3. In ΔABD we have:

$DA=DB$ (given)

then ΔADB is isosceles at D (having two equal sides)

but $A\hat{B}D=35^\circ$ (given)

then $B\hat{A}D=35^\circ$ (base angles in an isosceles Δ).

but, D is the incenter of ΔABC (given)

So, $[AD)$ & $[BD)$ are bisectors of $C\hat{A}B$ & $C\hat{B}A$ resp (incenter is the intersection pt of the bisectors)

then, $C\hat{A}B=C\hat{B}A=2D\hat{A}B=2(35)=70^\circ$.

Thus, $A\hat{C}B=40^\circ$ (sum of angles in a Δ)

(A)

4. $BC = \dfrac{75^2+125^2}{85\times 25}$

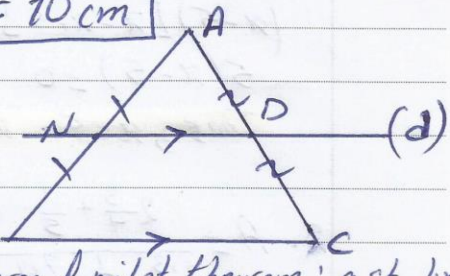
$= \dfrac{(25\times 3)^2 + (25\times 5)^2}{17\times 5\times 25}$

$= \dfrac{25^{\cancel{2}}(3^2+5^2)}{\cancel{25}\times 5\times 17}$

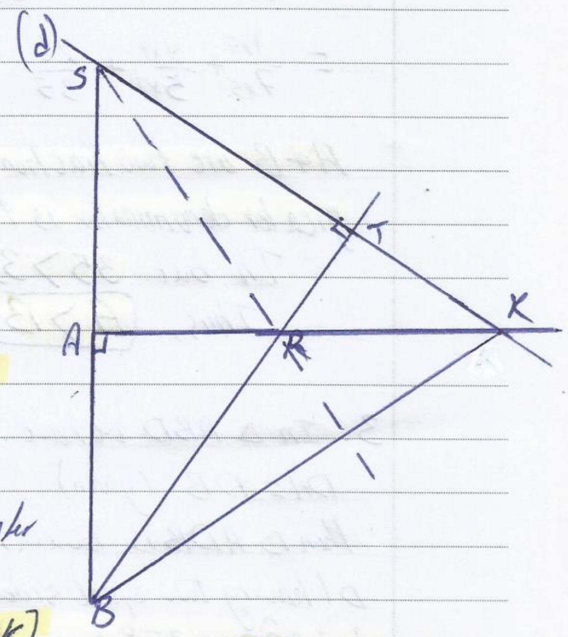
$BC = \dfrac{5(9+25)}{17}$

$= \dfrac{5\times 34 \div 17}{17\div 17}$

$\boxed{BC = 10\text{ cm}}$



In ΔABC we have:
N is the midpt of [AB] (given)
& (ND) is parallel to (BC) (given)
then D is the midpt of [AC] (by converse of midpt theorem: a st. line drawn from midpt of a side & parallel to another side cuts the third side of the Δ at its midpt).
hence $ND = \frac{1}{2} BC$ (by midpt theorem in a Δ: segment joining midpts of two sides of a Δ is half the third side)
but $BC = 10$ cm (proved)
Thus, $\boxed{ND = 5\text{ cm}}$



5. In Δ SBK we have:
ΔBAR is right at A (given)
So, [KA) is a height relative to [SB]
[BT) is perp to [SK] at T (given)
So [BT) is a height relative to [SK]
but [BT) & [KA) intersect at R (given)
hence R is the orthocenter of Δ SBK.
but [SR) is a st. line issued from the vertex of a Δ & passing through orthocenter R.
Then, [SR), is a height relative to [BK]
Thus, (SR) is perpendicular to (BK)

c

6> $A = \left(\frac{-2}{3}\right)^{-2} - \frac{1-\frac{1}{2^2}}{2+\frac{1}{2^2}}$

$= \frac{3^2}{2^2} - \frac{\frac{1\times4}{1\times4} - \frac{1}{4}}{\frac{2\times4}{1\times4} + \frac{1}{4}}$

$A = \frac{9}{4} - \frac{\left[\frac{3}{4}\right]}{\left[\frac{9}{4}\right]}$

$= \frac{9}{4} - \frac{3\times4}{9\times4}$

$= \frac{9\times3}{4\times3} - \frac{1\times4}{3\times4}$

$A = \frac{27-4}{12}$

$\boxed{A = \frac{23}{12}}$

$B = \frac{0.24\times1.8^2}{0.48\times0.36}$

$= \frac{24\times10^{-2}\times(18\times10^{-1})^2}{48\times10^{-2}\times(36\times10^{-2})}$

$= \frac{24\div24}{48\div24} \times \frac{18^2}{36}$

$= \frac{1}{2} \times \frac{18\times18\div18}{36\div18}$

$B = \frac{1}{2}\times\frac{18}{2}$

$\boxed{B = \frac{9}{2}}$

$\frac{A}{B}\times54 = \frac{\frac{23}{12}}{\frac{9}{2}}\times54$

$= \frac{23\times2}{12\times9}\times54$

$\frac{A}{B}\times54 = \frac{23}{6\times9}\times54$

$= \frac{23}{54}\times54$

$\frac{A}{B}\times54 = 23$

$= 2.3\times10^1$

(B)

Exercise-II:

1> $p(x)$ is a 2^{nd} degree polynomial in x since it is formed of sum of monomials having natural exponents.

$p(x)$ is defined for all real values of x, since it has no variables in its denominator.

2> Since, $x=2$ is a root of $p(x)$ (given)

then, $p(2)=0$

So, $p(2) = (2)^2 - m + 3(2-1)(2-2)$

$0 = 4 - m + 3(0)$

Thus, $\boxed{m=4}$

3a>. $p(x) = x^2 - 4 + 3(x-1)(x-2)$

$= x^2 - 4 + 3[x^2 - 2x - x + 2]$

$= x^2 - 4 + 3[x^2 - 3x + 2]$

$p(x) = 4x^2 - 9x + 2$

where $a=4$, $b=-9$ & $C=2$

b) $p(x) = 2$.

$4x^2 - 9x + 2 = 2$

$4x^2 - 9x = 0$

$x(4x-9) = 0$

(*) If product of two factors is zero then at least one of them is zero

so, $x = 0$ or $4x - 9 = 0$

$x = 9/4$.

Thus, the solution set of $p(x) = 2$ is $x = \{0, \frac{9}{4}\}$.

c) $p(x) = x^2 - 4 + 3(x-1)(x-2)$

$= x^2 - 2^2 + 3(x-1)(x-2)$

$= (x-2)(x+2) + 3(x-1)(x-2)$

$p(x) = (x-2)[x+2+3x-3]$

$\boxed{p(x) = (x-2)(4x-1)}$

which are 1st-degree binomials.

4a) $Q(x) = (3x+5)^2 - (x-6)^2$

$= [3x+5-x+6][3x+5+x-6]$

$\boxed{Q(x) = (2x+11)(4x-1)}$ verified.

b) To find roots of $Q(x)$ we solve $Q(x) = 0$

so, $(2x+11)(4x-1) = 0$

by (*) then, $2x+11 = 0$ or $4x-1 = 0$

Thus, the roots of $Q(x)$ are $x = \frac{-11}{2}$ & $x = \frac{1}{4}$.

5) a) $F(x) = \frac{p(x)}{Q(x)}$ is a literal fraction since it has a variable in its denominator.

b) $F(x)$ is defined when its lower part $\neq 0$

Cond: $Q(x) \neq 0$

then $x \neq \frac{-11}{2}$ & $x \neq \frac{1}{4}$.

but neither of the roots of $Q(x)$ are natural no's.

Thus, $F(x)$ is defined for all natural values of x

c) $F(x) = \frac{(x-2)(4x-1)}{(2x+11)(4x-1)}$ so for all x belong to set of natural no's.

$\boxed{F(x) = \frac{x-2}{(2x+11)}}$

d) $F(x) \stackrel{?}{=} 1 - \frac{x+13}{2x+11}$

$\stackrel{?}{=} \frac{2x+11-(x+13)}{2x+11}$

$\boxed{F(x) = \frac{x-2}{2x+11}}$ verified.

e) $F(\frac{1}{2}) = \frac{(\frac{1}{2}) - \frac{2\times 2}{1\times 2}}{2(\frac{1}{2})+11}$

$= \frac{\left[\frac{-3}{2}\right]}{\left[\frac{12}{1}\right]}$

$F = \frac{-3 \div 3}{2\times 12 \div 3}$

$\boxed{F = -\frac{1}{8}}$

which is the inverse of -8. since $(-\frac{1}{8}) \times (-8) = 1$

f) $F(x) = \frac{1}{4}$

$\frac{(x-2)}{(2x+11)} = \frac{1}{4}$

$4(x-2) = 2x+11$

$4x - 8 = 2x+11$

$2x = 19$

$x = \frac{19}{2}$

which is accepted since it doesn't vanish the lower term of $F(x)$.

6) a) Shaded area = Area of ABCD − Area of EFGH

$= Q(x) - P(x)$

$= (2x+11)(4x-1) - (x-2)(4x-1)$

$= (4x-1)[2x+11-x+2]$

$= (4x-1)(x+13)$.

b) for $x=0$; Shaded area $= (4(0)-1)(0+13)$

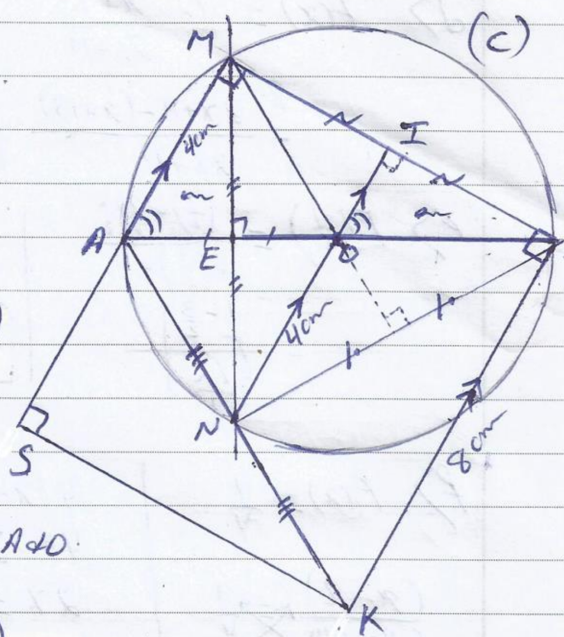
$= -1(13)$

$= -13$

which means that area of shaded region doesn't exist for $x=0$.

Exercise-III:

1) Drawn. ✓



2) a). M is a pt of circle (c) of center O & diameter [AB] (given)
So, OM = OA (radii of same circle)

M is a pt on the perp. bisector of [AB] (given)
So, M is equidistant from extremities A & O.
then, MO = MA.
Thus, MA = OA (by comparison)

b). In triangle AMO we have:
- OM = OA (proved)
- MA = OA (proved)

hence, OM = OA = MA.
Thus, Δ AMO is equilateral (having 3 equal sides)

3 a) [OM] is a radius of (c) (given)
[AB] is a diameter of (c) (given)
So, $OM = \frac{1}{2} AB$ (radius & diameter of same circle)

b) In Δ AMB we have:
O is the center of (c) with diameter [AB] (given)
So, O is the midpt of [AB]
then, [MO) is the median relative to [AB]
but $MO = \frac{1}{2} AB$ (proved)

hence ΔMAB is right at M (by converse of median relative to hypotenuse)

but Δ AMO is equilateral (proved)

then, $O\hat{A}M = 60°$ (angle formed by sides of an equilateral Δ)

Thus, Δ MAB is semi-equilateral at M (right + 60°).

4) In quadrilateral AMON we have:

N is on the perp. bisector of [AO] (given)

So, NA = NO

& MA = MO (proved.)

but OM = ON (radii of (C))

hence, OM = MA = AN = NO (by comparison)

Thus, quadrilateral AMNO is a rhombus (having 4 equal sides)

5) a) In Δ ABM we have:

O is the midpt of [AB] (proved)

I is the midpt of [MB] (given) ①

Thus, (OI) is parallel to (AM) (by midpt theorem: st. line holding the midpts of two sides of a Δ is parallel to the 3rd side)

b) AO = 4cm (given)

OA = AM = 4cm (proved)

& $OI = \frac{1}{2} AM$ (by midpt theorem: segment holding the midpts of two sides of a Δ is half the 3rd side).

Thus, OI = 2cm.

c). (OI) // (AM) (proved)

(ON) // (AM) (opp. sides of rhombus AMON)

Then, (OI) // (ON) (two st. lines parallel to the same st. line are parallel)

but O is a common pt.
Thus, pts O, N & I are collinear.

6 a) In $\triangle$'s OIB & MEO we have:
AM = OA (proved)
but OA = OB (radii of (C))
→ So, AM = OB (by comparison)
(OI) // (AM) (proved)
→ So, $M\hat{A}E = I\hat{O}B = 60°$ (corresponding angles between parallel st. lines)
Again (OI) // (AM) (proved)
but (AM) ⊥ (MB) ($\triangle$ AMB is right at M)
then, (OI) ⊥ (MB) (a st. line perp. to one of two perp. st. lines is perp. to the other) ---- $\triangle$
So, $O\hat{I}B = 90°$.
but, (NM) is the perp. bisector of [AO] at E (given)
So, $A\hat{E}M = 90°$
→ hence, $O\hat{I}B = A\hat{E}M = 90°$ (by comparison)
Thus, $\triangle$'s AME & OIB are equal by R.H.S property.

$\boxed{2}$

b) IB = EM (homologous elements of equal $\triangle$'s AME & OIB)
but E is the midpt of diagonal [MN] (center of a rhombus)
then EN = EM
Thus, IB = EN (by comparison)

7) In $\triangle$ MNB we have:
I is the midpt of [MB] (given)
So [NI) is a median relative to [MB]
E is the midpt of [MN] (proved)
So, [BE) is a median relative to [MN]
but, [NI) & [BE) intersect at O
Thus, O is the centroid of $\triangle$ MNB.

8) [MO) is a st. line Issued from vertex of a triangle & passes through O (the centroid of $\triangle$ MNB) (proved)
then, [MO) is a median relative to [NB]
Thus, [MO) cuts [NB] at its midpt.

9a) In $\triangle$ ABK we have:
O is the midpt of [AB] (proved)
K is the symmetric of A w.r.t N (given)
So, N is the midpt of [AK].
hence (NO) // (BK) (by midpt theorem: 1)
In quadrilateral NOBK we have:
(NO) // (BK) (proved)
So, NOBK is a trapezoid (having a pair of parallel sides)
but, OB = AM (by $\boxed{2}$)
& AN = AM (sides of rhombus AMON)
(i)... So, OB = AN. (by comparison)
but N is the midpt of [AK] (proved)
(ii)... So, AN = NK.
hence, NK = OB (by comparing (i) & (ii))
Thus, trapezoid BKNO is isosceles (having equal legs)

b) In quadrilateral SMBK we have:
→ $S\hat{M}B = 90^\circ$ (proved)
(AM) // (NO) (opp. sides of rhombus AMON.)
(NO) // (KB) (proved)
So, (AM) // (KB) (two st. lines parallel to same st. line are parallel)
but (AM) ⊥ (MB) (proved)
Then, (KB) ⊥ (MB) (by $\boxed{A}$)
→ So, $M\hat{B}K = 90^\circ$
but, S is the orth. proj. of K on (AM) (given)

So, $K\hat{S}M = 90°$

Thus, quad. SMBK is a rectangle (having 3 right angles)

c) $ON = 4cm$ (radius)

$KB = 2\,NO$ (proved)

so, $KB = 8cm$

let $MB = x$

Then, perimeter of $MBKS = 2\,(\text{length} + \text{width})$

$= 2\,(MB + BK)$

$28 = 2\,(x + 8)$

$14 = x + 8$

$x = 6\,cm$ which is accepted since (+)ve.

Rabih Khater