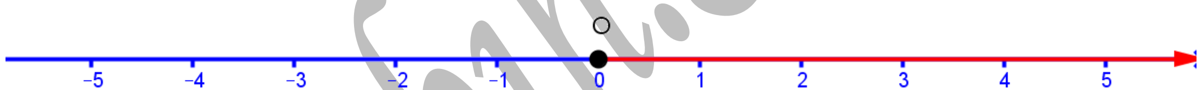
Sets of numbers: numbers are classified into sets (groups) according to their purpose.

1. ***Natural numbers:***
When you are counting the number of pages of a book or any other thing we use:
$0, 1, 2, 3, 4, 5, \dots$ etc.
The set of natural numbers is denoted by: $\mathbb{N} = \{0, 1, 2, 3, 4, 5, \dots.\}$

Ex$_1$: Solve the following equations in $\mathbb{N}$ and what do you notice: $x + 3 = 5$
$2x + 5 = 3.$

2. ***Integers***:
As you noticed, from the above example that some equations of the form $x + a = b$, where $a \,\&\, b$ belong to $\mathbb{N}$ cannot be solved in the set $\mathbb{N}$. So we will extend the set $\mathbb{N}$ to the set of integers $\mathbb{Z}$, which stands for ***Zahlen*** and consists of the set $\mathbb{N}$ and its opposite.
In other words: $\mathbb{Z} = \{\dots -3, -2, -1, 0, 1, 2, 3 \dots\}$.



➢ ***Notice that:*** $\mathbb{N} \subset \mathbb{Z}$

Ex$_2$: Solve the following equations in $\mathbb{Z}$: $x^2 - 4 = 0$, $2x + 3 = 0$.

3. ***Rational numbers:***
$\mathbb{Z}$, is insufficient to solve some equations of the form $ax + b = 0$, where $a \,\&\, b$ belong to $\mathbb{Z}$.
So, we will extend the set $\mathbb{Z}$ into the set of ***rational*** numbers.
The word rational is derived from ratio. So, we can deduce that any number that can be written in the ratio form, $\frac{a}{b}$, where $a \,\&\, b$ are integers such that $b \neq 0$ is said to be a rational number.
The set of rational numbers includes numbers of the form:

$$\mathbb{Q} = \left\{ \dots, -\frac{1}{3}, -\frac{1}{2}, -\frac{1}{1}, 0, \frac{1}{3}, \frac{1}{2}, \frac{1}{1}, \dots \right\}$$

$$\mathbb{Q} = \{x \mid x = \frac{a}{b}, \forall a \in Z \,\&\, b \in Z^*\} \text{ In comprehension.}$$

➢ ***Notice that: :*** $\mathbb{Z} \subset \mathbb{Q}$

4. ***Irrational numbers:***

Are numbers that ***can't*** be written in the form of a ratio, $\frac{a}{b}$ where a & b belong to $\mathbb{Z}$ and $b \neq 0$.

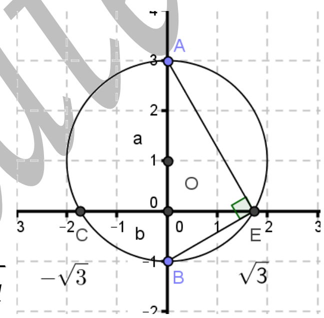
Common forms of irrational numbers:

- $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{6}...$
- $\pi \approx 3.1415...$
- Numbers, in decimal form with infinite non periodic decimal part. E.g: 7.9425721...

How to represent an irrational number on a number line?

To represent a number of the form: $\sqrt{a}$

1- Square the given number to get a.
2- Place on $y-axis$ a point A so that $OA = a\,cm$.
3- Always place on $y-axis$ from below a point B so that $OB = 1cm$.
4- Trace a circle whose diameter is $[AB]$.
5- The intersection of the circle with the $\oplus ve\ x-axis$ is the position of $\sqrt{a}$



5. ***Real numbers:***

Is the set of all numbers, it includes all of the above sets, $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$ and the set of irrational numbers.

This set includes numbers of the form: $\Re = \left\{..., -2, -\frac{1}{2}, 0, 1, \sqrt{2}, \pi, 4.5...\right\}$

Conclusion: $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \Re$

Note that:

I - If a set of numbers is raised to the symbol "$*$" "***star***" then this means that it is the same set ***except*** we exclude from it ***zero***.

$$N^* = \{1, 2, 3, 4, ...\}$$

$$N^* = N - \{0\}$$

II - If a set of numbers is raised to the symbol "$+$" "***plus***" then this means that it includes only the ***positive*** part of the set.

$$Z^+ = \{0, 1, 2, 3, ...\}$$

$$Z^+ = \{x \mid x \in N, \forall x \geq 0\}$$

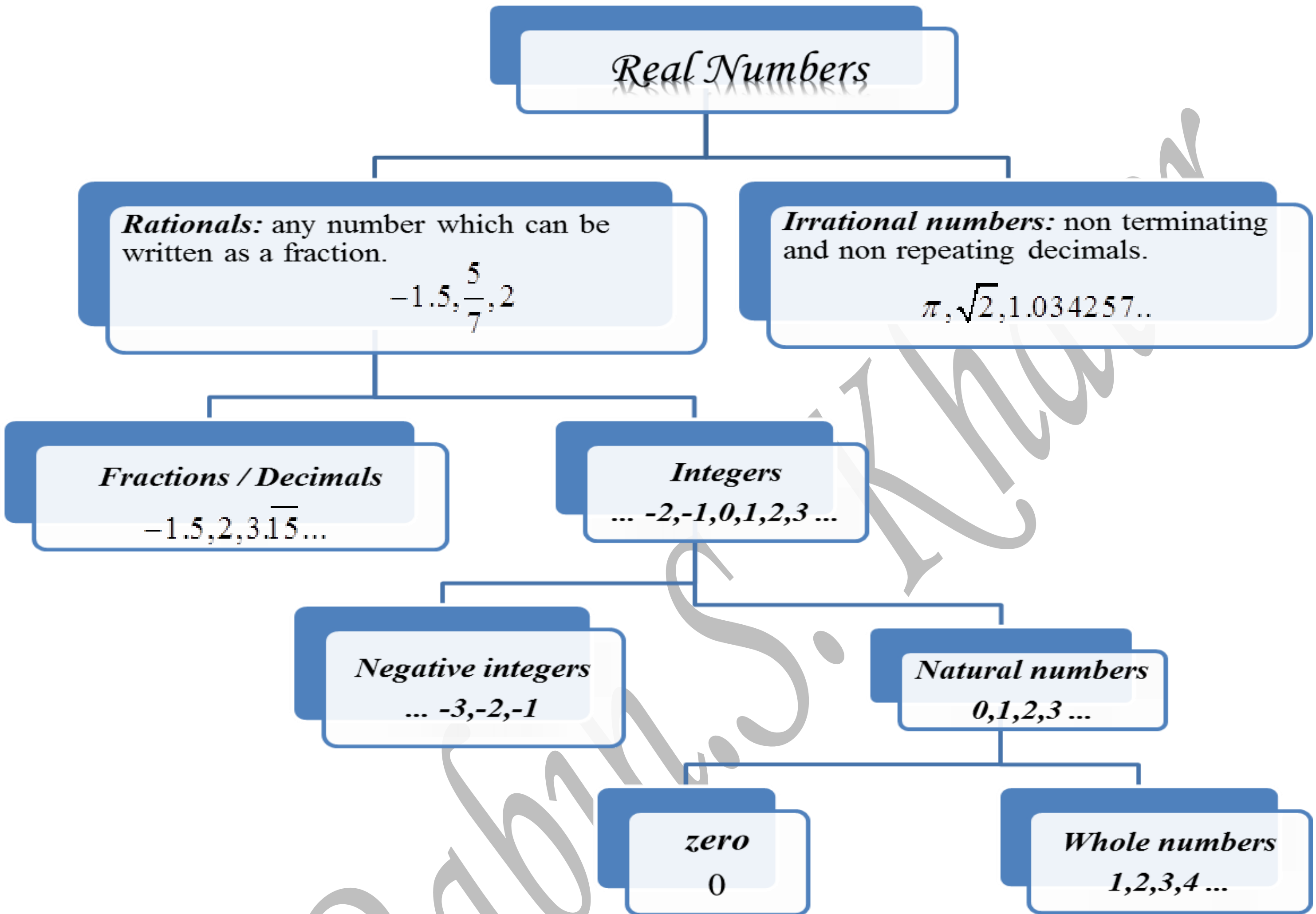
III - If a set of numbers is raised to the symbol "$-$" "***minus***" then this means that it includes only the ***negative*** part of the set.

$$Z^- = \{..., -3, -2, -1, 0\}$$

$$Z^- = \{x \mid x \in Z, \forall x \leq 0\}$$

Summary of the lesson using:

Flow Chart:



Venn Diagram: