9 Lycée Des Arts $\quad$ Mathematics $\quad 8^{\text {th_G_Grade. }}$

Consider the two distinct circles $C(O ; R) \& C^{\prime}\left(O ; R^{\prime}\right)$, where $R \& R^{\prime}$ are positive non-zero numbers.

| $\mathcal{N}$ O. | Relative the tw | osition of circles | Graphical representation | Mathematical relation |
| :---: | :---: | :---: | :---: | :---: |
| 1. | Two Circles are Disjoint: | $\begin{array}{\|c} \text { Internally } \\ \text { If } \end{array}$ |  | $O O^{\prime}<R-R^{\prime}, \text { where }\left(R>R^{\prime}\right)$ |
|  |  | Externally If |  | $O O^{\prime}>R+R^{\prime} .$ |
| 2. | Two Circles are Tangent: | $\begin{array}{\|c} \text { Externally } \\ \text { If } \end{array}$ |  | $O O^{\prime}=R+R^{\prime}$. |
|  |  | Internally If |  | $O O^{\prime}=R-R^{\prime}$.where $\left(R>R^{\prime}\right)$ |
| 3. | Two Ci <br> Interse | cles are cting if |  | $R-R^{\prime}<O O^{\prime}<R+R^{\prime} .\left(R>R^{\prime}\right)$ |


| Summalry: | Externally | Internally |  |
| :---: | :---: | :---: | :---: |
|  | Tangent | $O O^{\prime}=r+r^{\prime}$ | $O O^{\prime}=r-r^{\prime}$ |
|  | Disjoint | $O O^{\prime}>r+r^{\prime}$ | $O O^{\prime}<r-r^{\prime}$ |

If none of the above applies, then circles are secant

## Application

Whenever you want to work an exercise that askes you about relative position, try this way, it works all the time.
Determine relative positions of circles $C(O, r=3 \mathrm{~cm}) \& n\left(P, r^{\prime}=5 \mathrm{~cm}\right)$, where $O P=7 \mathrm{~cm}$. To study relative positions of two circles:

| Explanation | Calculation |
| :---: | :---: |
| $1^{\text {st }}$ : calculate the sum and the difference between the radii | $\begin{aligned} & r+r^{\prime}=3+5=8 \mathrm{~cm} \\ & r^{\prime}-r=5-3=2 \mathrm{~cm} \end{aligned}$ |
| $2^{\text {nd }}$ Find the distance between the centers | $O P=7 \mathrm{~cm}$ |
| $3{ }^{\text {rd }}$ Compare the distance between the centers to the sum and difference between the radii. | 1) Is $O P=r+r^{\prime}(7 \neq 8$ false $)$, then circles are not tangent externally. <br> 2) Is $O P=r-r^{\prime}(7 \neq 2$ false $)$, then circles are not tangent internally. <br> 3) Is $O P>r+r^{\prime}$ (false), then circles are not disjoint externally. <br> 4) Is $O P<r-r^{\prime}$ (false), then circles are not disjoint internally. <br> 5) Thus, circles are secant $\left(r-r^{\prime}<O P<r+r^{\prime}\right)$ |

$\boldsymbol{E x}$-1: Consider the circles $\lambda(O, 5 \mathrm{~cm}) \& \Delta\left(O^{\prime}, 3 \mathrm{~cm}\right)$ where $O O^{\prime}=\frac{3^{3}+243}{135} \mathrm{~cm}$
a) Prove that $O O^{\prime}$ is a natural number to be determined.
b) Find the difference between the two radii.
c) Draw on the adjacent $\operatorname{grid}(\lambda) \&(\Delta)$.
d) At how many points do $(\lambda) \&(\Delta)$ intersect?
e) Deduce the relative positions of $(\lambda) \&(\Delta)$.

$\boldsymbol{E x}$-2: In the adjacent figure $(C)$ is a circle of center $O$ and radius $r=5 \mathrm{~cm}$.

1) Trace a circle $C^{\prime}\left(O^{\prime}, r^{\prime}=3 \mathrm{~cm}\right)$, so that $O O^{\prime}=5.6-2 \times 0.23 \times 10 \mathrm{~cm}$
$\qquad$
2) Compare $r-r^{\prime}$ with $O O^{\prime}$ :
3) Deduce the relative position of $(C) \&\left(C^{\prime}\right)$ ?

