Al Mahdi High — Mathematics — 10^{th}-Grade.

Name: . . . . — "Factorizing Techniques" — E.S-1

There are a variety of techniques to write an expression in a product form (factorized)

A. *Taking a **common factor:***

It works when **all terms** of an expression have a common:

1. ***Coefficient*: ** that is if the GCD of the coefficients of the expression is different than **1**.

Expanded form	Factorized form
$3x^2 - 9$	Since, $GCD(3;9) = 3$ so $3(x^2 - 3)$
$21z^5 + 28z - 7$	Since, $GCD(21;28;7) = 7$ so $7(3z^5 + 4z - 1)$
$-\pi y^2 - 2x\pi$	Since, $GCD(\pi;2\pi) = \pi$ so $-\pi(y^2 + 2x)$

2. ***Common Term:*** that is if terms of expression have a common *variable or factor*.

Expanded form	Factorized form
$x^2 - 12x$	$x(x-12)$
$3x^2z^3 + 5xz^2 - 11x^2z^2$	$xz^2(3xz + 5 - 11x)$
$(x-2)(x^2+2) - (x-2)(3x^2-5)$	$(x-2)[(x^2+2) - (3x^2-5)]$

B. ***By grouping:***

If a polynomial has ***FOUR*** terms, the expression may be factorable by grouping, This technique works if there is no common factor to all terms. However, a common factor can be found between the two terms and another common factor between the second two terms, then we ***GROUP*** the terms and factor out the common factor.

Expanded form	Factorized form
$ac + ad + bc + bd$	1^{st}-step: Put in groups $\underbrace{ac + ad}_{1^{st}-Group} + \underbrace{bc + bd}_{2^{nd}-Group}$ 2^{nd}-Step:Take common $a(c+d) + b(c+d)$ 3^{rd}-step:Factor out again $(c+d)(a+b)$

C. *Using **remarkable identities**:*

Frequently used identities

Description	Expanded form	Factorized form
Difference of two squares:	$a^2 - b^2$	$(a-b)(a+b)$
Square of sum:	$a^2 + 2ab + b^2$	$(a+b)^2$
Square of difference:	$a^2 - 2ab + b^2$	$(a-b)^2$
Difference of two cubes:	$a^3 - b^3$	$(a-b)(a^2 + ab + b^2)$
Sum of two cubes:	$a^3 + b^3$	$(a+b)(a^2 - ab + b^2)$
Cube of sum:	$a^3 + 3a^2b + 3ab^2 + b^3$	$(a+b)^3$
Cube of difference:	$a^3 - 3a^2b + 3ab^2 - b^3$	$(a-b)^3$

D. *Using **trial and error** technique:*

It is used only in case if the expression is of the form: $1x^2+bx+c$.

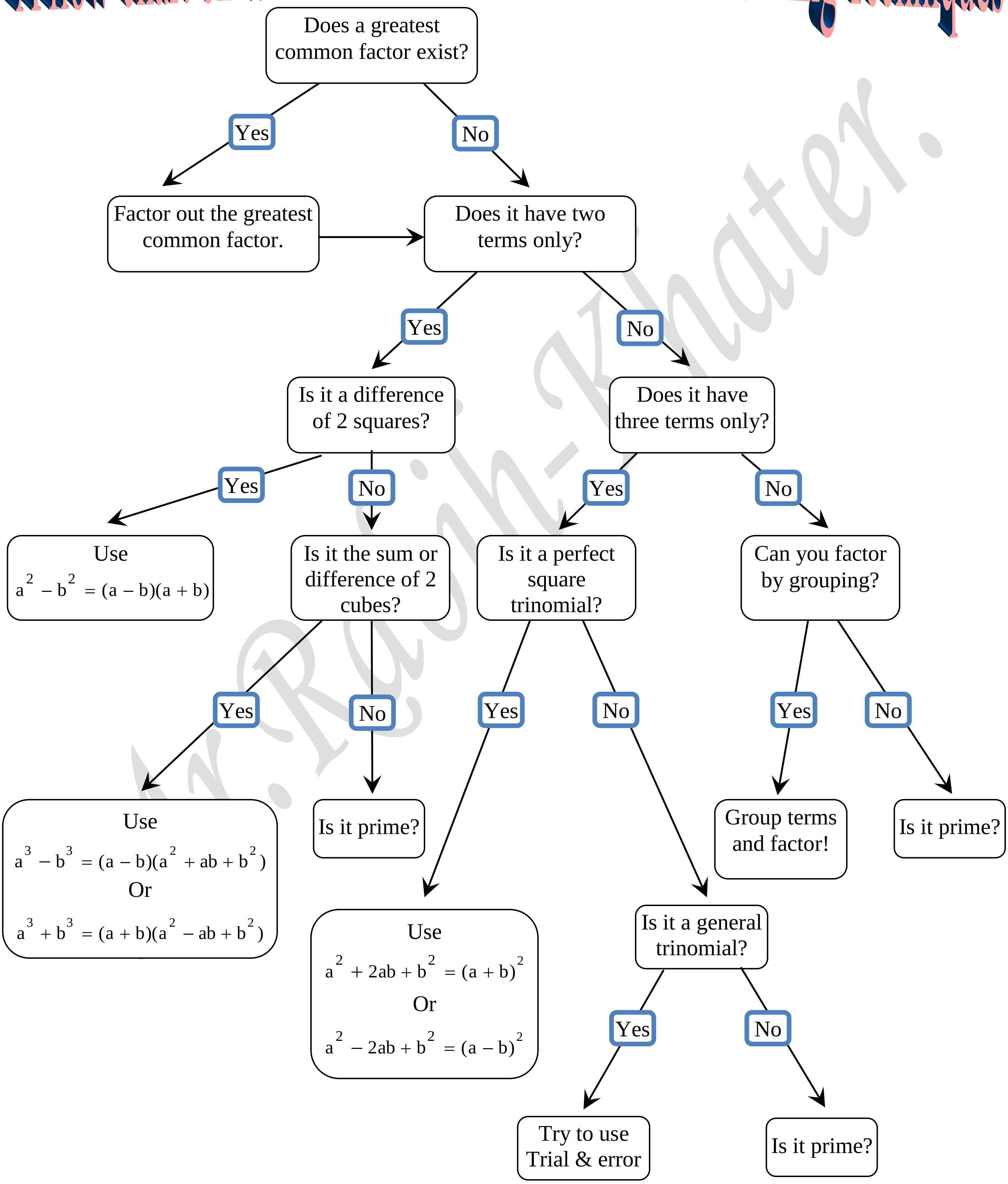
Expanded form	*Factorized form*	*Application*
$1x^2+bx+c$.	***1st-step:*** Put in product form: $\underbrace{(x\quad)}_{1^{st}-factor}\underbrace{(x\quad)}_{2^{nd}-factor}$ ***2nd-Step:*** Check the sign of c: ✍ If the sign of c is $\oplus ve$, then: ↳ Find a pair of numbers in which their: - Product is equal to c. That is, $c=r\times n$. - And sum is equal to b. That is, $b=r+n$. • If the sign of b is $\oplus ve$, then write $(x+r)(x+n)$. • If sign of b is $-ve$, then write $(x-r)(x-n)$. ✍ If the sign of c is $-ve$, then: ↳ Find a pair of numbers in which their: - Product is equal to c. That is, $c=r\times n$. - And difference is equal to b. That is, $b=r-n$. • If the sign of b is $-ve$, then greater is $-ve$. • If the sign of b is $\oplus ve$, then greater is $\oplus ve$. ***3rd-step:*** Insert numbers and signs in factors. $(x\quad r)(x\quad n)$	x^2-5x+6 1) $(x\quad)(x\quad)$ 2) $c=+6\,is\oplus ve$ $\Rightarrow 6=\begin{cases}6\times1\\3\times2\end{cases}$ $so,use\ 3+2=5$ 3) $(x\quad 3)(x\quad 2)$ $but,we\ need\ -5$ $so,-3+(-2)=-5$ $\therefore (x-3)(x-2)$

E. ***Perfect squaring** technique:*

It is used only in case if the expression is of the form: ax^2+bx+c.

Expanded form	*Factorized form*	*Application*
ax^2+bx+c.	***1st-step:*** If $a>1$ then take it as a common factor if not, do the other steps: $a\left(x^2+\frac{b}{a}x+\frac{c}{a}\right)$. ***2nd-Step:*** Write expression: $a\left[(x\qquad)^2+\frac{c}{a}\right]$. ***3rd-step:*** Always insert sign of b next to x. ***4th-step:*** Divide the term $\frac{b}{a}$ by 2 to get $\frac{b}{2a}$ ***5th-step:*** Put this term next to x: $a\left[\left(x+\frac{b}{2a}\right)^2+\frac{c}{a}\right]$. ***6th-step:*** Always subtract the square of, $\frac{b}{2a}$ from $\frac{c}{a}$: $a\left[\left(x+\frac{b}{2a}\right)^2+\frac{c}{a}-\left(\frac{b}{2a}\right)^2\right]$.	$3x^2-12x-4$ $=3\left[x^2-4x-\frac{4}{3}\right]$ $=3\left[x^2-4x..........-\frac{4}{3}\right]$ $=3\left[x^2-4x+...-...-\frac{4}{3}\right]$ $=3\left[\underbrace{x^2-4x+\left(\frac{4}{2}\right)^2}-\left(\frac{4}{2}\right)^2-\frac{4}{3}\right]$ $=3\left[\left(x-\frac{4}{2}\right)^2-4-\frac{4}{3}\right]$

A flow chart on when and how to use different Factorizing Techniques



Expanding binomials of the form $(x+y)^n$

Pascal's Triangle is a technique that serves best in determining the coefficients of the expanded form of a binomial.
1st 8 rows of ***Pascal's Triangle*** (which was known to ***Omar Khayyam*** before 500 years)
A usage of ***Pascal's Triangle*** (Out of many other important usages)

							1							
						1		1						
					1		2		1					
				1		3		3		1				
			1		4		6		4		1			
		1		5		10		10		5		1		
	1		6		15		20		15		6		1	
1		7		21		35		35		21		7		1

Application

$(x+y)^0 = 1$

$(x+y)^1 = 1x + 1y$

$(x+y)^2 = 1x^2 + 2xy + 1y^2$

$(x+y)^3 = 1x^3 + 3x^2y + 3xy^2 + 1y^3$

$(x+y)^4 = 1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$

$(x+y)^5 = 1x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + 1y^5$