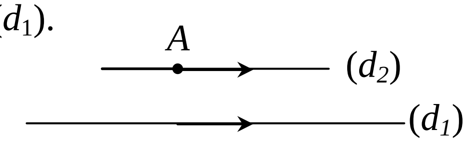
Lycée Des Arts | Mathematics | 9^{th}-Grade

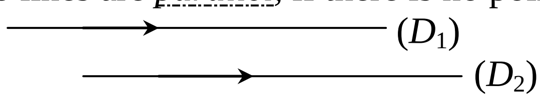
Name: . . . . | "General Flash Back" | E.S-2

I- *Euclidian postulate*: From any point "A" distinct (outside) from any line (d_1), we can draw *one and only one* line (d_2) parallel to (d_1).



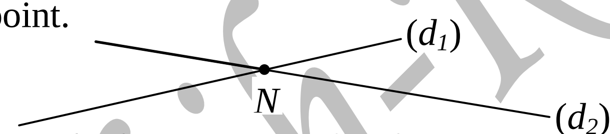
II- *Relative positions of straight lines*: Any set of straight lines in a plane can be

a. ***Parallel***: Two lines are *parallel*, if there is no point of intersection between them.



b. ***Confounded***: Two or more lines are *confounded*, if they have more than one point of intersection.

c. ***Intersecting***: Two or more lines are *intersecting*, if they have one common point, called the intersection point.

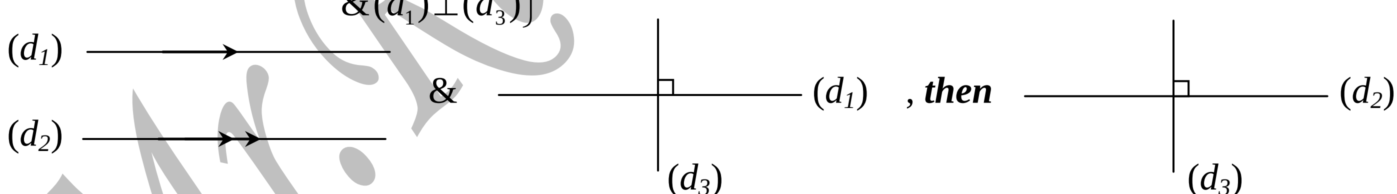


III- *How to prove two straight lines Perpendicular* ?

i- If (d_1) & (d_2) are any two parallel lines, then every line (d_3) perpendicular to (d_1) is perpendicular to (d_2).

Thus, a st. line perpendicular to one of two parallel lines is perpendicular to the other.

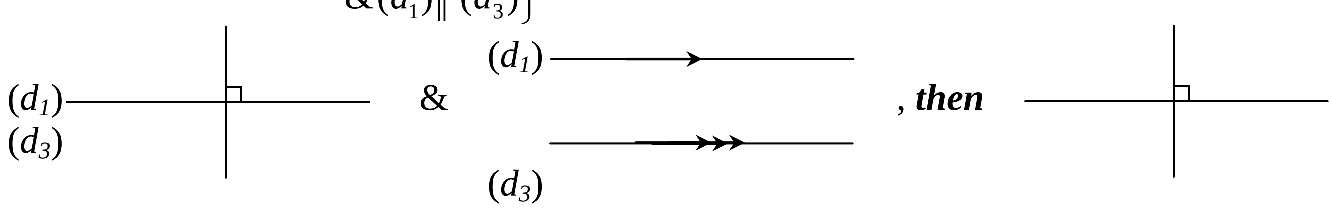
$$\text{i.e. if, } \left.\begin{matrix}(d_1)\parallel(d_2)\\ \&(d_1)\perp(d_3)\end{matrix}\right\}, \text{then } (d_2)\perp(d_3).$$



ii- If (d_1) & (d_2) are any two perpendicular lines, then every line (d_3) parallel to (d_1), is perpendicular to (d_2).

Thus, a st. line parallel to one of two perpendicular lines is perpendicular to the other.

$$\text{i.e. if, } \left.\begin{matrix}(d_1)\perp(d_2)\\ \&(d_1)\parallel(d_3)\end{matrix}\right\}, \text{then } (d_2)\perp(d_3).$$



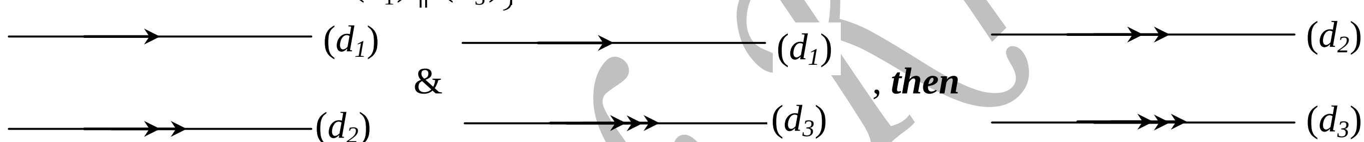
IV- *How to prove two straight lines parallel* ?

a- If (d_1) & (d_2) are perpendicular to a third line (d_3), then (d_1) and (d_2) are parallel.
<u>Thus</u>, two st. lines perpendicular to a third line are parallel.

i.e. if, $\left.\begin{matrix}(d_1)\perp(d_3)\\ \&(d_2)\perp(d_3)\end{matrix}\right\}, then\ (d_1)\parallel(d_2).$

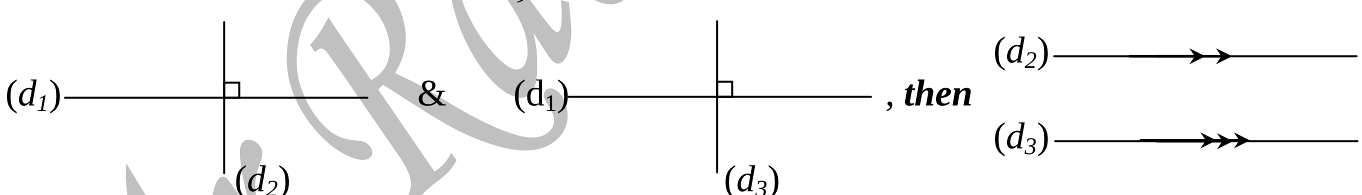
b- If (d_1) & (d_2) are any two parallel lines, then every line (d_3) parallel (d_1), is parallel to (d_2).
<u>Thus</u>, a st. line parallel to one of two parallel lines is parallel to the other.

i.e if, $\left.\begin{matrix}(d_1)\parallel(d_2)\\ \&(d_1)\parallel(d_3)\end{matrix}\right\}, then\ (d_2)\parallel(d_3).$



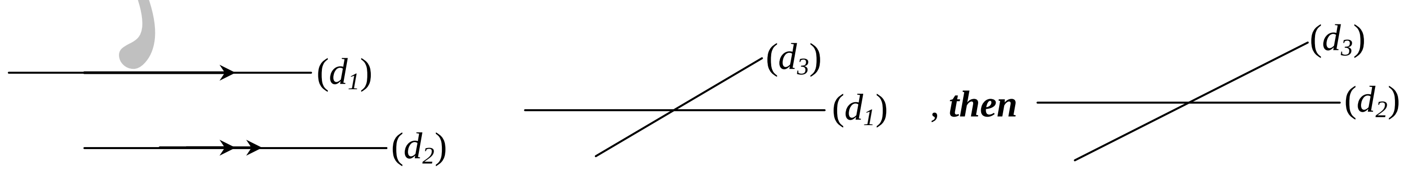
c- If (d_1) & (d_2) are any two perpendicular lines, then every line (d_3) perpendicular (d_1) is parallel to (d_2).
<u>Thus</u>, a st. line perpendicular to one of two perpendicular lines is parallel to the other.

i.e. if, $\left.\begin{matrix}(d_1)\perp(d_2)\\ \&(d_1)\perp(d_3)\end{matrix}\right\}, then\ (d_2)\parallel(d_3).$



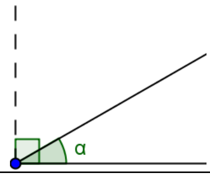
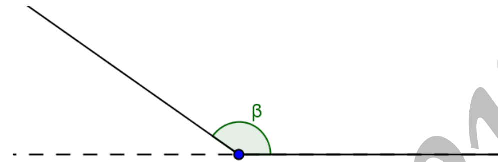
V- *How to prove two straight lines intersecting* ?

If (d_1) & (d_2) are any two parallel lines, then every line (d_3) intersects (d_1), must intersect (d_2).
<u>Thus</u>, a st. line intersecting one of two parallel lines intersects the other.

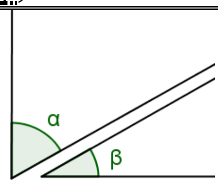
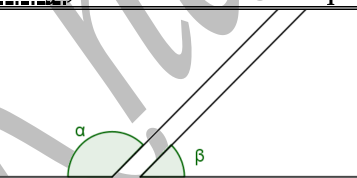


VI- Notion about Angles :

A. Acute and obtuse angles:

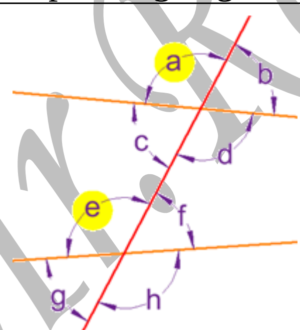
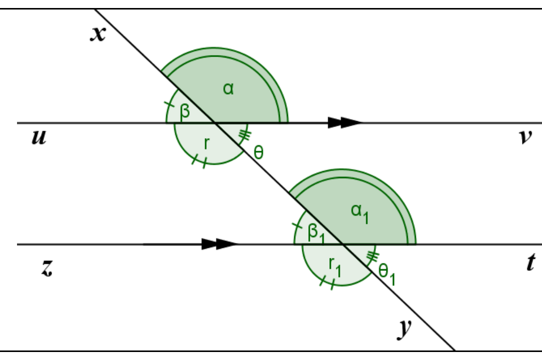
An angle α is said to be ***acute***, if its measure is between 0^o and 90^o.	An angle β is said to be ***obtuse***, if its measure is between 90^o and 180^o.
	
i.e. $0^\circ \langle \alpha \langle 90^\circ$	i.e. $90^\circ \langle \beta \langle 180^\circ$

B. Complementary and supplementary angles:

Two or more angles are said to be ***complementary***, if their sum is equal to 90^o.	Two or more angles are said to be ***supplementary***, if their sum is equal to 180^o.
	
i.e. $\alpha + \beta = 90^\circ$	i.e. $\alpha + \beta = 180^\circ$

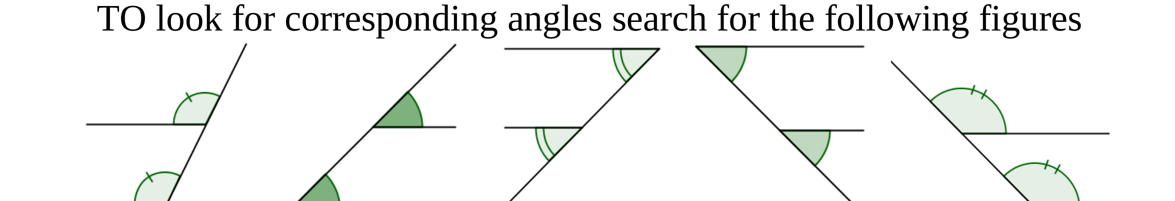
C. Corresponding, alternating, and vertically opposite angles:

a. Corresponding angles:

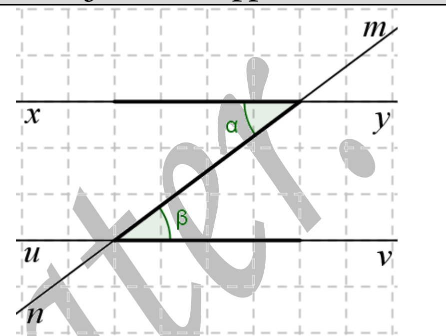
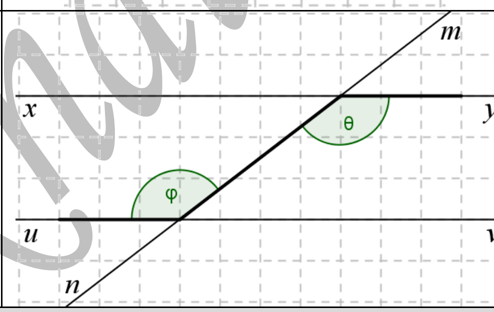
Corresponding Angles	***Equal Corresponding Angles***
If two lines are crossed by a ***transversal***, ***then*** the angles formed in the ***matching corners*** are ***non-equal*** corresponding angles	Corresponding angles are ***equal if and only if*** they are enclosed between ***parallel*** lines.
	

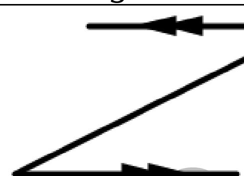
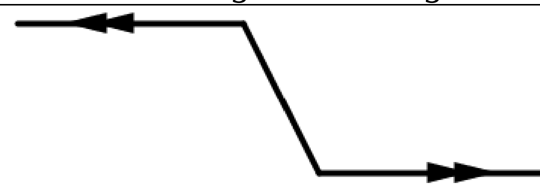
$$IF \left. \begin{array}{l} (uv) \parallel (zt), \& \\ (xy)\, is\, a\, transversal \end{array} \right\}, then\, we\, have \left\{ \begin{array}{l} \alpha;\ \alpha_1 \\ \beta;\ \beta_1 \end{array} \right\} \& \left\{ \begin{array}{l} r;\ r_1 \\ \theta;\ \theta_1 \end{array} \right\} (are\ equal\ corresponding\ angles)$$

TO look for corresponding angles search for the following figures

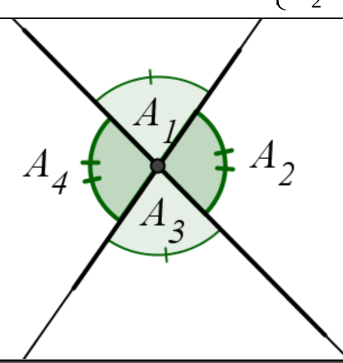


b. *Alternating angles*: We distinguish two types of ***equal*** alternating angles:
Alternating angles are ***equal if and only if*** they are enclosed between ***parallel*** lines.

	Analytic approach	*Geometric approach*
Alternating interior angles	$IF \left.\begin{matrix}(xy)\parallel(uv),\&(mn)\\ is\ a\ transversal\end{matrix}\right\}, then \left\{\begin{matrix}\alpha\ \&\ \beta\ are\ equal\\ \text{alter.interior angles}\end{matrix}\right.$	
Alternating exterior angles	$IF \left.\begin{matrix}(xy)\parallel(uv),\&(mn)\\ is\ a\ transversal\end{matrix}\right\}, then \left\{\begin{matrix}\varphi\ \&\ \theta\ are\ equal\\ \text{alter.exterior angles}\end{matrix}\right.$	

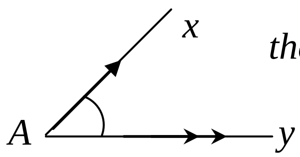
TO look for alternating angles search for the following figures	
Alternating interior angles:	*Alternating exterior angles*
 Z- SHAPE	 Stair-SHAPE

c. *Vertically opposite angles:* are formed by two intersecting lines.

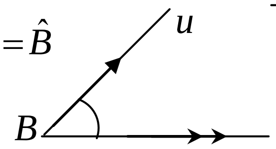
Analytic approach	$IF\ (xy)\ \&\ (uv)\ \text{are two intersecting lines , then} \left\{\begin{matrix}\hat{A}_1=\hat{A}_3\\ \hat{A}_2=\hat{A}_4\end{matrix}\right\}\ \text{(vertically opp.angles)}$
Geometric approach	

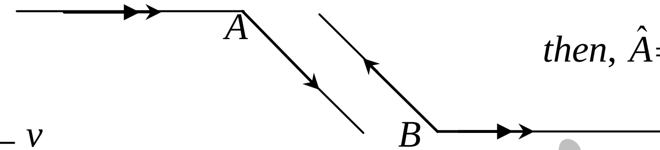
D. *Angles with their sides respectively parallel*:

Two angles (acute or obtuse) with their sides respectively parallel, are ***equal***.



then, $\hat{A} = \hat{B}$



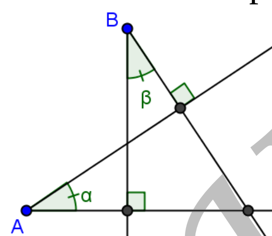


then, $\hat{A} = \hat{B}$

Conclusion: ***Angles enclosed between parallel lines are equal.***

E. *Angles with their sides respectively* perpendicular:

Two angles (acute or obtuse) with their sides respectively perpendicular are ***equal***.

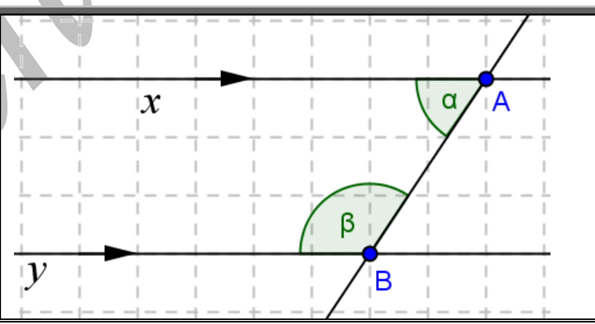


then, $\hat{A} = \hat{B}$

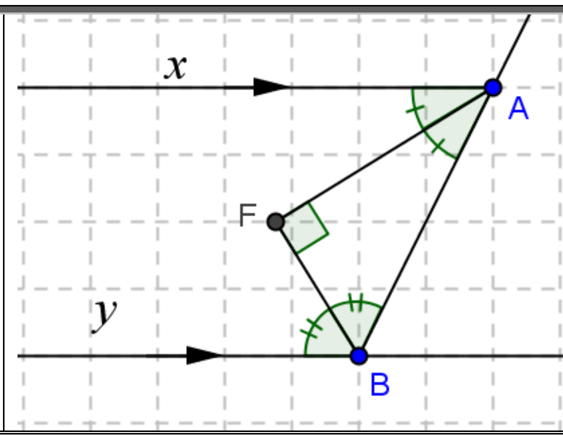
Conclusion: ***Angles with their sides (arms) mutually perpendicular are equal.***

VII- *Relative positions of lines and angles*:

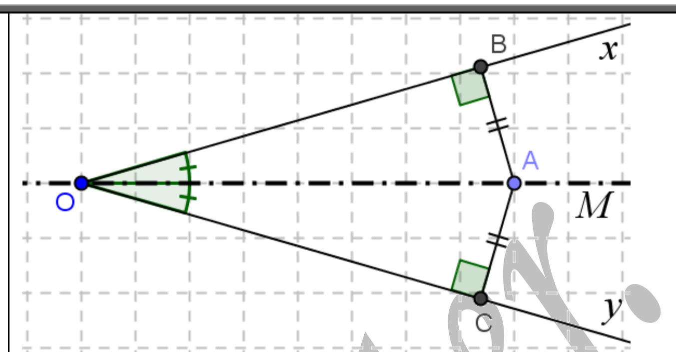
a. Co-interior angles

If $[Ax) \parallel [By)$, then angles formed between them are called co-interior angles So that, $\alpha + \beta = 180^\circ$	
Note that: ***The sum of two co-interior angles is 180°.***	

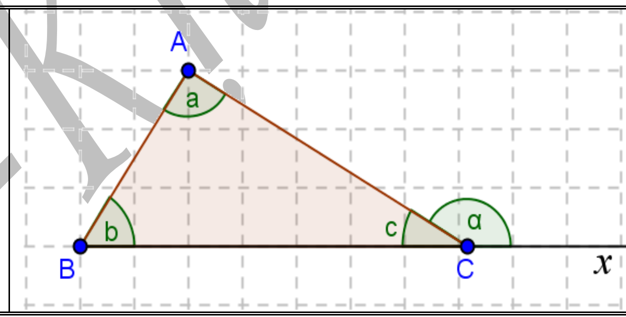
b. Bisectors of two co-interior angles

$IF \begin{cases} [Ax) \parallel [By), \\ [AF) \text{ bisector of } x\hat{A}B, \\ \&[BF) \text{ bisector of } y\hat{B}A \end{cases}$ *then,* $A\hat{F}B = 90^\circ$. $$i.e [AF) \perp [BF)$$	
Conclusion: ***Bisectors of two co-interior angles form a right angle.***	

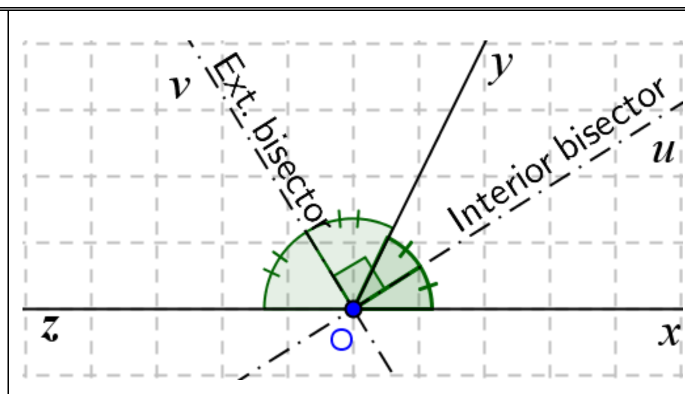
c. *Point on a bisector of an angle:*

IF $\begin{cases} [OM) \text{ bisector of } x\hat{O}y, \\ A \text{ belongs } [OM), \\ [AB) \perp [Ox), \\ \& [AC) \perp [Oy) \end{cases}$ *then,* $AB = AC$.	
Conclusion: ***Any point on the bisector of an angle is equidistant from its arms.***	

d. *Exterior angle in a triangle:*

Rule:	*If* α is an exterior angle, then $\alpha = a + b$	
Proof:	$\alpha + c = 180^\circ$ (Supplementary angles) *so,* $\alpha = 180^\circ - c$ but, $(a + b) = 180^\circ - c$ (Sum of angle in a Δ) *Thus,* $\alpha = a + b$ (By comparison)	
Conclusion: ***The exterior angle is equal to the sum of the two opposite interior angles.***		

e. *Bisectors of two adjacent supplementary angles:*

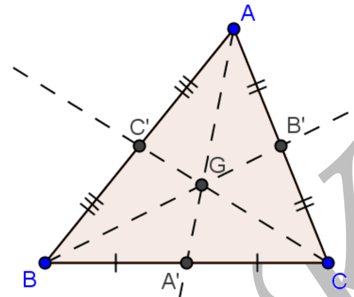
Rule:	*IF* $\begin{cases} x\hat{o}y + y\hat{o}z = 180^\circ \ (Supplementary) \\ [ou) \text{ bisector of } x\hat{o}y, \& \\ [ov) \text{ bisector of } y\hat{o}z \end{cases}$ *then,* $u\hat{o}v = 90^\circ$.	
Proof:	$x\hat{o}y + y\hat{o}z = 180^\circ$ *(Given)* $[Ou)$ *bisector of* $x\hat{o}y$ *(Given)* *so,* $x\hat{o}y = 2u\hat{o}y$ *(property of bisector)* $[Ov)$ *bisector of* $y\hat{o}z$ *(Given)* *so,* $y\hat{o}z = 2y\hat{o}v$ *(property of bisector)* *hence,* $2u\hat{o}y + 2y\hat{o}v = 180^\circ$ *(By substitution)* *Thus,* $u\hat{o}y + y\hat{o}v = 90^\circ$	
Conclusion: ***The bisectors of two adjacent supplementary angles form a right angle.***		

VIII- Remarkable lines:

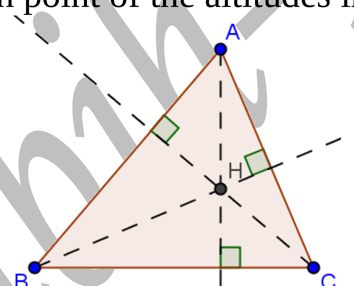
1. **Median:** is a line that divides a segment into two equal parts.
2. **Altitude (*height*):** or foot of perpendicular is a line that falls perpendicularly on a segment.
3. **Perpendicular bisector:** is a line that both bisects & falls perpendicularly on a segment.

IX- Remarkable points in a triangle:

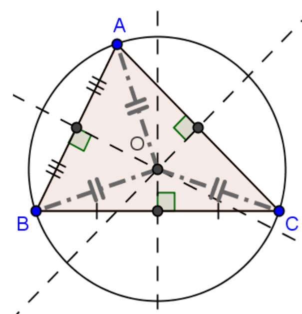
a- Center of gravity or centroid: is the intersection of ***medians*** in a triangle.

Properties	Geometric figure	Uses
The centroid divides each median in the ratio $\frac{2}{3}$ starting from the vertex ***That is***; $AG = (2/3)\,AA'$ $A'G = (1/3)\,AA'$ $AG = 2\,A'G$		***Center of mass*** is a point at which the mass of an object is concentrated.

b- Orthocenter: is the intersection point of the altitudes in a triangle.

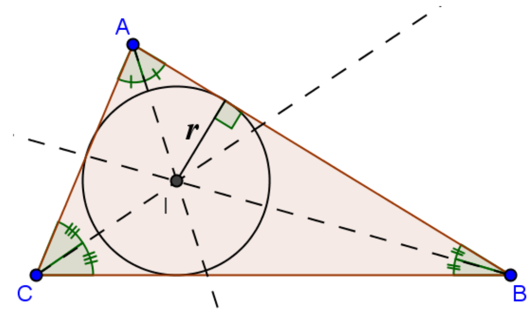


c- Circumcenter or center of the circumscribed circle: is the intersection point of the perpendicular bisector.



- ✓ *Uses:* The ***circumcenter*** is a point at which we can plot the center of the circle that passes through the three vertices of a given triangle.

d- Incenter or the center of the inscribed circle: is the intersection point of the bisectors of the angles in a triangle.



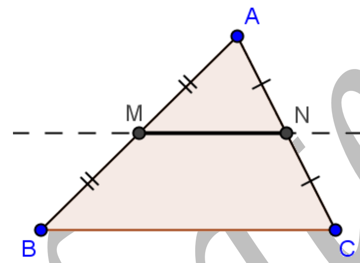
- ✓ *Uses:* The ***incenter*** is a point at which we can plot the center of the circle that remains tangent to all three sides of the given triangle.

X- *Lines joining the midpoints of the sides* :

1- Midpoint theorem in a triangle:

Discussion: *If* $\begin{cases} \text{1- } M \text{ is the midpoint of } [AB] \text{ ,and} \\ \text{2- } N \text{ is the midpoint of } [AC] \end{cases}$, *then* $\begin{cases} \text{1- } (MN) \parallel (BC) \\ 2 - MN = \frac{1}{2} BC \end{cases}$.

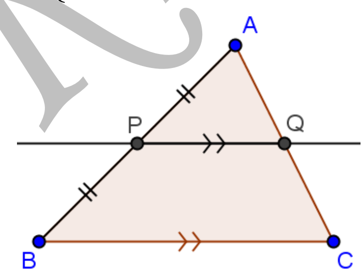
Usage: *To* $\begin{cases} 1 - \textit{Prove two st.lines parallel.} \\ 2 - \textit{Find relation between sides.} \end{cases}$



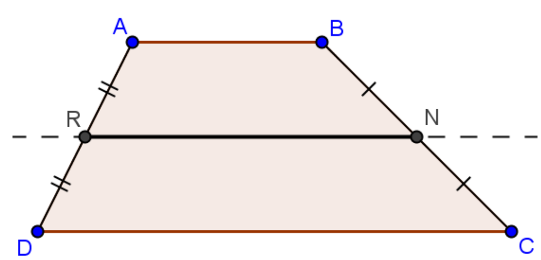
2- Converse of midpoint theorem in a triangle:

Discussion: *If* $\begin{cases} \text{1- } P \text{ is midpoint of } [AB] \text{ ,} \\ \text{2- } (PQ) \parallel (BC) \end{cases}$, *then* $\begin{cases} \text{1- } Q \text{ is the midpoint of } [AC] \\ 2 - PQ = \frac{1}{2} BC \end{cases}$.

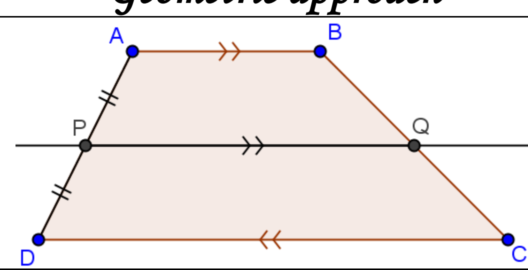
Usage: *To* $\begin{cases} 1 - \textit{Prove } \text{midpoint.} \\ 2 - \textit{Find relation between sides.} \end{cases}$



3- Midpoint theorem in a trapezoid:

Analytic approach	*Geometric approach*
If $\begin{cases} R \text{ is the midpoint of } [AD] \text{ ,} \\ \& N \text{ is the midpoint of } [BC] \end{cases}$, *then,* $\begin{cases} \text{1- } (RN) \parallel (AB) \text{ \& } (CD) \\ \text{2- } RN = \frac{(AB + CD)}{2} \end{cases}$.	

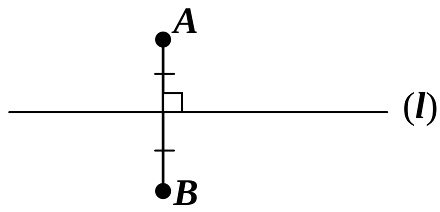
4- Converse of midpoint theorem in a trapezoid:

Analytic approach	*Geometric approach*
If $\begin{cases} P \text{ is the midpoint of} [AD] \text{ ,} \\ \& PQ \parallel (AB) \textit{ or } (CD) \end{cases}$, *then,* Q is the midpoint of $[BC]$.	

XI- "Symmetry":

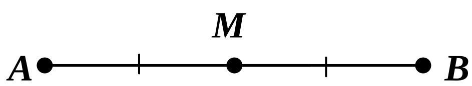
a. ***Symmetry with respect to a line***:

A point ***A*** is said to be symmetric to another point ***B*** *w.r.t* a line (***l***), *if* (***l***) is the perpendicular bisector of [***AB***].



b. ***Symmetry with respect to a point***:

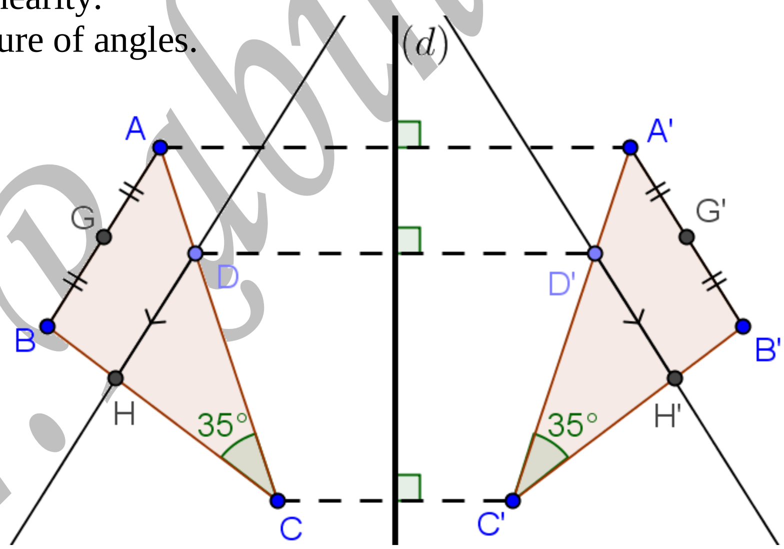
A is the *symm*. of ***B*** *w.r.t* point ***M***, *if* ***M*** is the midpoint of [***AB***].



c. ***Properties of symmetry:***

Symmetry preserves:

1- Midpoint.
2- Parallelism.
3- Length.
4- Collinearity.
5- Measure of angles.
6- Area.



✐ ***Note that:*** Since the left figure is symmetric to the right figure with respect to st. line (***d***), ***Then*** ***A*** is *symmetric* to ***A'*** *w.r.t* (***d***) and similarly for all the above points,

1 - G is mipt of $[AB]$	, then G' is mipt of $[A'B']$.
2 - $(AB) \parallel (DH)$	, then $(A'B') \parallel (D'H')$.
3 - $BC = 2\,cm$	, then $B'C' = 2\,cm$.
4 - $A, C \& D$ are collinear	, then $A', C' \& D'$ are collinear.
5 - $B\hat{C}A = 35^\circ$.	, then $B'\hat{C}'A' = 35^\circ$.