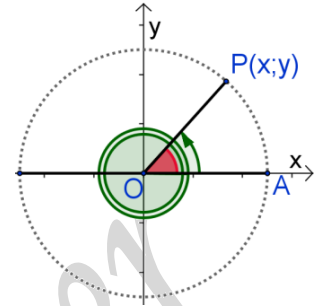


**A. Principal determination:**

Principal angle is the measure of the oriented arc  $AP$  that belongs to the interval:

$] -\pi, \pi ]$	$] -180^\circ, 180^\circ ]$
If angle is in radian	If angle is in degrees



**B. Range of trigonometric ratios:**

Bounding trigonometric lines			
$ \cos \alpha  \leq 1$	means	$-1 \leq \cos \alpha \leq 1$	For all $\alpha \in \mathbb{R}$
$ \sin \alpha  \leq 1$		$-1 \leq \sin \alpha \leq 1$	
$-\infty < \tan \alpha < +\infty$		For all $\alpha \in \mathbb{R}$	
$-\infty < \cot \alpha < +\infty$			

**C. Remarkable angles:**

Degrees versus radian										
$\alpha$ : in degrees	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$135^\circ$	$150^\circ$	$180^\circ$	$270^\circ$
$\beta$ : in radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{3\pi}{2}$

From the table

If  $\alpha$  and  $\beta$

Quadrant	1 <sup>st</sup> - quadrant				
Complements					
Angle ( $\alpha$ )	0 rad	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
Ratios					
$\sin \alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos \alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan \alpha$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\infty$
$\cot \alpha$	$\infty$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0

We notice that

Complementary

$$\alpha + \beta = \frac{\pi}{2}$$

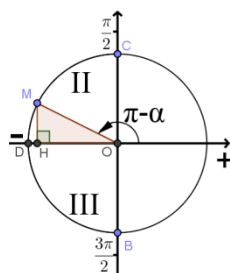
$$\cot \alpha \times \cot \beta = 1$$

$$\tan \alpha \times \tan \beta = 1$$

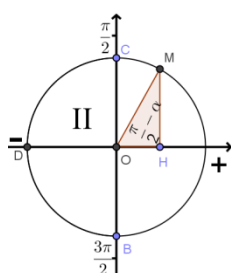
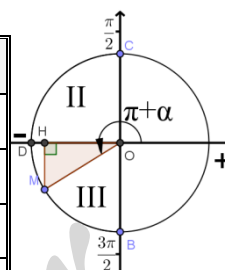
$$\tan \alpha = \cot \beta$$

$$\cos \alpha = \sin \beta$$

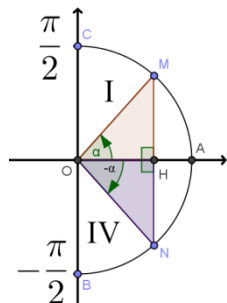
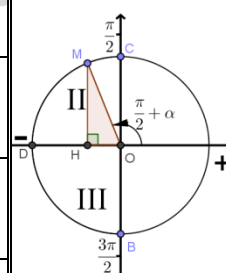
### D. Associated arcs for:



Supplementary arcs: $(\alpha \ \& \ \pi - \alpha)$	Arcs differing by $\pi$ : $(\alpha \ \& \ \pi + \alpha)$
$\cos(\pi - \alpha) = -\cos \alpha$	$\cos(\pi + \alpha) = -\cos \alpha$
$\sin(\pi - \alpha) = \sin \alpha$	$\sin(\pi + \alpha) = -\sin \alpha$
$\tan(\pi - \alpha) = -\tan \alpha$	$\tan(\pi + \alpha) = \tan \alpha$
$\cot(\pi - \alpha) = -\cot \alpha$	$\cot(\pi + \alpha) = \cot \alpha$



Complementary arcs: $(\alpha \ \& \ \frac{\pi}{2} - \alpha)$	Arcs differing by $\frac{\pi}{2}$ : $(\alpha \ \& \ \frac{\pi}{2} + \alpha)$
$\cos(\frac{\pi}{2} - \alpha) = \sin \alpha$	$\cos(\frac{\pi}{2} + \alpha) = -\sin \alpha$
$\sin(\frac{\pi}{2} - \alpha) = \cos \alpha$	$\sin(\frac{\pi}{2} + \alpha) = \cos \alpha$
$\tan(\frac{\pi}{2} - \alpha) = \cot \alpha$	$\tan(\frac{\pi}{2} + \alpha) = -\cot \alpha$
$\cot(\frac{\pi}{2} - \alpha) = \tan \alpha$	$\cot(\frac{\pi}{2} + \alpha) = -\tan \alpha$



Opposite arcs: $(\alpha \ \& \ -\alpha)$	Arcs differing by $2k\pi$ : $(\alpha \ \& \ \alpha \pm 2k\pi) \text{ s.t. } (k \in \mathbb{Z})$
$\cos(-\alpha) = \cos \alpha$	$\cos(\alpha \pm 2k\pi) = \cos \alpha$
$\sin(-\alpha) = -\sin \alpha$	$\sin(\alpha \pm 2k\pi) = \sin \alpha$
$\tan(-\alpha) = -\tan \alpha$	$\tan(\alpha \pm 2k\pi) = \tan \alpha$
$\cot(-\alpha) = -\cot \alpha$	$\cot(\alpha \pm 2k\pi) = \cot \alpha$

### E. Equations:

$\cos x = \cos \alpha \overset{\text{means}}{\Leftrightarrow} \begin{cases} x = \alpha + 2k\pi \\ \text{or} \quad \text{where } k \ \& \ k' \in \mathbb{Z} \\ x = -\alpha + 2k'\pi \end{cases}$	$\sin x = \sin \alpha \overset{\text{means}}{\Leftrightarrow} \begin{cases} x = \alpha + 2k\pi \\ \text{or} \quad \text{where } k \ \& \ k' \in \mathbb{Z} \\ x = \pi - \alpha + 2k'\pi \end{cases}$
$\cos x = 0 \overset{\text{means}}{\Leftrightarrow} x = \frac{\pi}{2} + k\pi$	$\sin x = 0 \overset{\text{means}}{\Leftrightarrow} x = k\pi$
$\tan x = \tan \alpha \overset{\text{means}}{\Leftrightarrow} \{x = \alpha + k\pi$	
$\tan x = 0 \overset{\text{means}}{\Leftrightarrow} x = k\pi$	

## Transformation formulas

### Basic formulas:

$\sin^2 \alpha + \cos^2 \alpha = 1$		
$1 + \cot^2 \alpha = \frac{1}{\sin^2 \alpha}$	$\tan \alpha \times \cot \alpha = 1$	$1 + \tan^2 \alpha = \frac{1}{\cos^2 \alpha}$
$\sin^2 \alpha = \frac{1}{1 + \cot^2 \alpha} = \frac{\tan^2 \alpha}{1 + \tan^2 \alpha}$	$\tan \alpha = \frac{\sin \alpha}{\cos \alpha}$	$\cos^2 \alpha = \frac{1}{1 + \tan^2 \alpha} = \frac{\cot^2 \alpha}{1 + \cot^2 \alpha}$

### Formulas for:

Sum and difference of two angles	Double the angle (If $a = b$ , then)
$\cos(a + b) = \cos a \cdot \cos b - \sin a \cdot \sin b$ $\cos(a - b) = \cos a \cdot \cos b + \sin a \cdot \sin b$	$\cos(2a) = \cos^2 a - \sin^2 a$ Or $\begin{cases} \cos(2a) = 2\cos^2 a - 1 \\ \cos(2a) = 1 - 2\sin^2 a \end{cases}$
$\sin(a + b) = \sin a \cdot \cos b + \cos a \cdot \sin b$ $\sin(a - b) = \sin a \cdot \cos b - \cos a \cdot \sin b$	$\sin(2a) = 2\sin a \cdot \cos a$
$\tan(a + b) = \frac{\tan a + \tan b}{1 - \tan a \cdot \tan b}$ $\tan(a - b) = \frac{\tan a - \tan b}{1 + \tan a \cdot \tan b}$	$\tan(2a) = \frac{2 \tan a}{1 - \tan^2 a}$

Half the angle (If $\alpha = 2a$ , then)	
$\cos \alpha = \cos^2 \left( \frac{\alpha}{2} \right) - \sin^2 \left( \frac{\alpha}{2} \right)$ Or $\begin{cases} \cos(\alpha) = 2\cos^2 \left( \frac{\alpha}{2} \right) - 1 \\ \cos(\alpha) = 1 - 2\sin^2 \left( \frac{\alpha}{2} \right) \end{cases}$	$\cos^2 a = \frac{1 + \cos(2a)}{2}$ $\sin^2 a = \frac{1 - \cos(2a)}{2}$
$\sin \alpha = 2 \sin \frac{\alpha}{2} \cdot \cos \frac{\alpha}{2}$	
$\tan(\alpha) = \frac{2 \tan \frac{\alpha}{2}}{1 - \tan^2 \frac{\alpha}{2}}$	

<i>Linearization</i>
$\cos a \cdot \cos b = \frac{1}{2} [\cos(a + b) + \cos(a - b)]$
$\sin a \cdot \sin b = \frac{1}{2} [\cos(a - b) - \cos(a + b)]$
$\sin a \cdot \cos b = \frac{1}{2} [\sin(a + b) + \sin(a - b)]$

<i>Triple the angle</i>
$\cos 3a = 4\cos^3 a - 3\cos a$
$\sin 3a = 3\sin a - 4\sin^3 a$

<i>Product form</i>
$\cos a + \cos b = 2\cos\left(\frac{a+b}{2}\right) \cdot \cos\left(\frac{a-b}{2}\right)$
$\cos a - \cos b = -2\sin\left(\frac{a+b}{2}\right) \cdot \sin\left(\frac{a-b}{2}\right)$
$\sin a + \sin b = 2\sin\left(\frac{a+b}{2}\right) \cdot \cos\left(\frac{a-b}{2}\right)$
$\sin a - \sin b = 2\cos\left(\frac{a+b}{2}\right) \cdot \sin\left(\frac{a-b}{2}\right)$
$\tan a + \tan b = \frac{\sin(a+b)}{\cos a \cdot \cos b}$
$\tan a - \tan b = \frac{\sin(a-b)}{\cos a \cdot \cos b}$

