| AlMahdifigh School | Mathematics | 11 th_Grade |
| :--- | :---: | :---: |
| Name:........ | "Study of Symmetry" | E.S-3 |

## Center of symmetry of a polygon:

I- Consider the following polygons:
a. Determine the center and axis of symmetry for each of the following figures if it exists:



Square


Kite


Regular hyptagon

Center:
Axis:
b. Define a center of symmetry of a polygon:

## Center of symmetry of a rational function:

How to prove that a point is a center of symmetry for the graph of a function?
II- On the adjacent figure $A(x ; f(x))$ is any point on the curve of a numerical function $f$.
$a$. Determine the values of $x$ for which $f$ is defined.
b. If $C(a ; b)$, is the center of symmetry of $C_{f}$, then locate the point $B$ the symmetric of $A$ with respect to a point $C$.
c. As $A$ varies on $C_{f}$, where would the point $B$ move? $\qquad$
d. What is the relative position of $C$ w.r.t $[A B]$ ?
$e$. Determine the coordinates of $B$ :
$f$. Deduce the relation: $f(x)+y_{B}=2 b$
$g$. When is the relation $f(x)+f(2 a-x)=2 b$ valid?


Fig-1.
h. List the steps to prove that a point is a center of symmetry for the graph of a function

How to determine center of symmetry of rational functions?

## III- Consider the following table:


a) Complete the above table.
b) What do you notice in the above table?
c) What do you conclude?
d) Does your conclusion work for all degrees for rational functions?

## Center of symmetry of a quadratic function:

IV- Consider the function $h$ defined by $h(x)=x^{2}-4 x+1$
$a$. Is the domain of $h$ centered at origin? Justify.
b. Determine $h^{\prime}(x)$ :
c. Find coordinates of $S$ the absolute minimum of $C_{h}$.
d. Complete the following table of variation:

| Values of $x$ | $-\infty$ | $+\infty$ |
| :---: | :--- | :--- |
| $h^{\prime}(x)$ |  |  |
| $h(x)$ |  |  |

$e$. Trace on the adjacent grid the curve of $h$.
$f$. Does the $C_{h}$, admit a center of symmetry? Justify.


Fig-2.

## Axis of symmetry of a quadratic function:

How to prove that a straight line is an axis of symmetry for the graph of a function?
$\boldsymbol{V}$ - Let $R(x, r(x))$ be a point on the function $r$ defined by its curve $C_{r}$ and the straight line, $(d): x=2$, to be its axis of symmetry.
$a$. Is the domain of $r$ centered at origin?
b. Use $C_{r}$ to determine $r(x)$.
$c$. Justify the placement of $N$ the symmetric of $R$ w.r.t $(d)$.
d. Compare the ordinates of $N \& R$.
$e$. Let $I(a, b)$ be the midpoint of $[R N]$.

1. Locate I.
2. Determine the abscissa of $N$ in terms of $a$.

$f$. When is the relation, $r(x)=r(2 a-x)$ valid?
$g$. How can we prove that the straight line $(d): x=a$ is an axis of symmetry for a function $f$ ? $\qquad$ . $\qquad$
How to find the equation of the axis of symmetry for the graph of a function?
VI- In the table below $(d),(l) \&(\Delta)$ are the respective axes of symmetries of $C_{f}, C_{g} \& C_{h}$.

| Graphs |  |  |  |
| :---: | :---: | :---: | :---: |
| Image: | $f(x)=$ | $g(x)=$ | $h(x)=$ |
| Derivative: | $f^{\prime}(x)=$ | $g^{\prime}(x)=$ | $h^{\prime}(x)=$ |
| Equate the derivatives to zero. |  |  |  |
| Equation of axis of symmetry | (d): | (l) : | $(\Delta)$ : |

a) Complete the above table.
b) What do you notice in the above table?
c) What do you conclude?

VII- Consider the function $p$ defined by its image $p(x)=-x^{2}-2 x+3$ and its curve $C_{p}$.


Fig-3.
a. Prove that: $p(x)=-(x+1)^{2}+4$.
b. Deduce the equation of the axis of symmetry of $C_{p}$
c. Can you use the same technique to find the axis of symmetry of higher polynomial functions? Justify.
d. Determine $p^{\prime}(x)$ : $\qquad$
$e$. Use the derivative of $p(x)$ to determine the equation of the axis of symmetry of $C_{p}$ :

