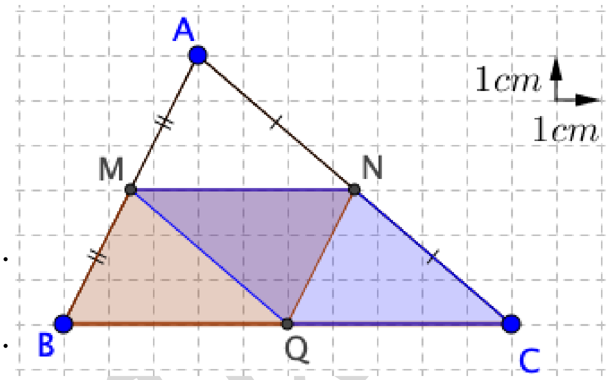
Lycée Des Arts | Mathematics | 8^{th}-Grade

Name: . . . . "Midpoint theorem in a triangle" E.S-4

Focusing event:

Consider M, N & Q to be the respectve midponts of sides $[AB], [AC]$ & $[BC]$ of a tiangle ABC:
Where $MNQB$ and $MNCQ$ are two parallelograms



1) Is it true that $\frac{AM}{AB} = \frac{1}{2}$?

   ……………………………………………………………

2) Find a ratio of AN to AC

   ……………………………………………………………

3) Compare MN, BQ & QC:

   ……………………………………………………………

   ……………………………………………………………

4) Deduce the ratio of MN to BC:……………………………………………………………

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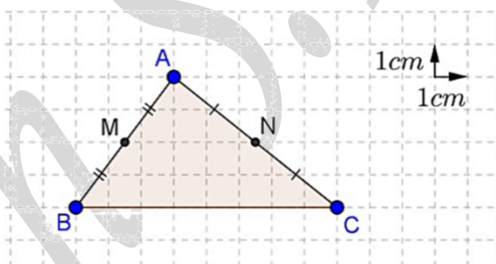
Midpoint theorem in any triangle

If ABC is a triangle where M & $N$are respective midpoints of sides $[AB]$ & $[AC]$, then



Conclusion:

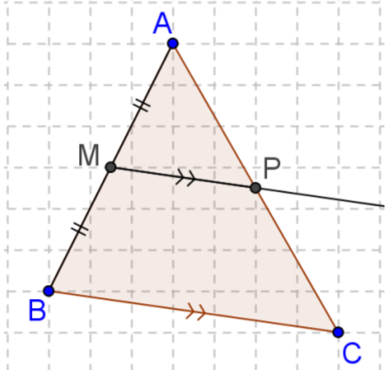
- ✓ The segment joining the midpoints of two sides of a triangle is half the third side $MN = \frac{1}{2}BC$ or $BC = 2MN$.
- ✓ The straight line joining the midpoints of two sides of a triangle is parallel to third side $(MN)//(BC)$

Conditions and usage:

- ✓ Conditions: To use midpoint theorem we should have:
  - ***Two midpoints***
- ✓ Usage: We use the midpoint theorem to:
  - ***Find length of segment joining midpoints***
  - ***Side opposite to the midsegment***
  - ***Prove parallel***

Converse of midpoint theorem

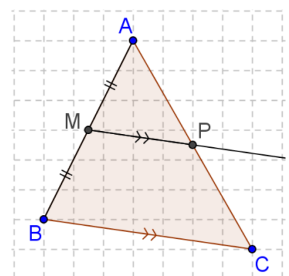
In the adjacent figure M is the midpoint of $[AB]$ and $[MP)$ is parallel to (BC).



Conclusion:
- ✓ If a line is issued from the ***midpoint*** of a side of a triangle and ***parallel*** to the second side, then it must cut the third side at its midpoint.

Conditions and usage:
- ✓ Conditions: To use converse of midpoint theorem we should have:
  - ***One midpoint***
  - ***Parallel form midpoint to third side***
- ✓ Usage: We use the converse of midpoint theorem to:
  - ***Find midpoint***

I- In the adjacent figure M is the midpoint of $[AB]$ and $[MP)$ is parallel to (BC).
Prove that P is the midpoint of $[AC]$.



Solutions:

Focusing event

1. Yes, since M & N are respective midpoints of sides $[AB]$ & $[AC]$ (given)
2. Since, N is midpoint of sides $[AC]$ (given)
   Then, $\frac{AN}{AB} = \frac{1}{2}$
3. $MNQB$ is a parallelogram (given)
   So, $MN = BQ$ (opposite sides of a parm are equal)
   $MNCQ$ is a parallelogram (given)
   So, $MN = QC$ (opposite sides of a parm are equal)
   Thus, $MN = BQ = QC$ (by comparison)
4. Since, $MN = BQ = QC$ (Proved)
   Thus, $\frac{MN}{BC} = \frac{1}{2}$

I- In ΔABC we have:

M is the midpoint of $[AB]$ $(given)$
$[MP)//(BC)$ (given)
Thus, P is the midpoint of $[AC]$ (By converse of midpoint theorem in a triangle: line issued from ***midpoint*** of a side of a triangle and ***parallel*** to the second side, cuts third side at its midpoint)