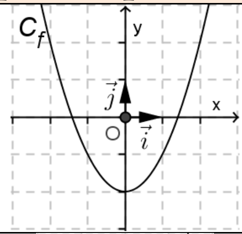
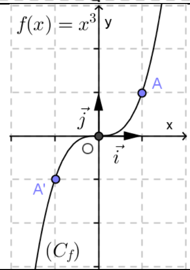
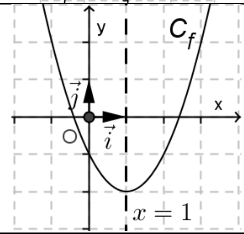
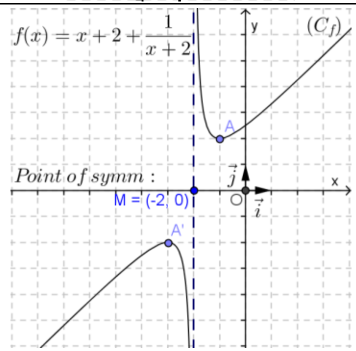
Al Mahdi High School	Mathematics	11th-Grade
Name:	"Study of Symmetry"	E.S-4

Symmetry of a function

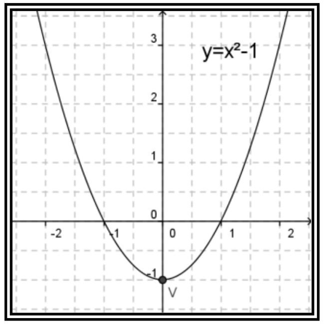
Parity of a function	Element of symmetry	Relation & condition	Graphical representation
Even	Symmetric about: $y-axis$	1- ***Condition:*** D_f is centered at origin. 2- ***Relation:*** $f(-x)=f(x)$.	
Odd	Symmetric *w.r.t* origin: $O(0;0)$	1- ***Condition:*** D_f is centered at origin. 2- ***Relation:*** $f(-x)=-f(x)$.	
Neither even nor odd	Symmetric *w.r.t* a vertical st. line of eqn: $x=a$	1- ***Condition:*** D_f is centered at $x=a$. 2- ***Relation:*** $f(2a-x)=f(x)$	
Neither even nor odd	Symmetric *w.r.t* a point $M(x,y)$	1- ***Condition:*** D_f is centered at $x=a$. 2- ***Relation:*** $f(x)+f(2a-x)=2b$ ***Proof***: Symmetric *w.r.t* $x=a$ then, $2a=x+x' \text{ or } x'=2a-x$ $So,\ f(x')=f(2a-x)$ $Also, 2b=y+y' \text{ or } f(x')=2b-y$ $Thus,\ f(2a-x)+f(x)=2b$	

To graph any absolute valued functions of the form $g(x)=|f(x)|$

1) Graph the given function ***without absolute value***.
2) ***Find*** the interval for which $f(x)<0$ (that is curve is below $x-axis$)
3) ***Reflect*** this part with respect to the $x-axis$ (Find symmetry $w.r.t\ x-axis$)

Ex1: Consider the two functions h & k so that $h(x)=x^2-1$ and $k(x)=|h(x)|$, graph h then deduce the graph of k.

Soln: 1st - Step



Therefore, we can say that

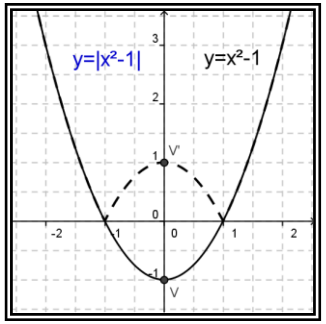
$$k(x)=\begin{cases} h(x) & if\ x\in]-\infty;-1]\cup[1;\infty[\\ -h(x) & if\ -1\le x\le 1 \end{cases}$$

Comparing graphs of h & k we say:

a) C_h & C_k are ***confounded*** if $x\in]-\infty;-1]\cup[1;\infty[$

b) C_h & C_k are ***symmetric*** w. r.t $x-axis$ $-1\le x\le 1$

2nd - Step

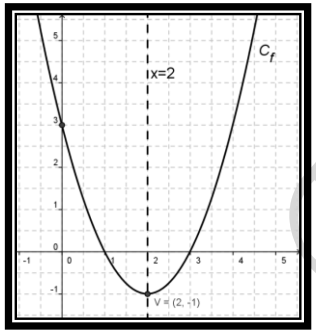


To graph any absolute valued functions of the form $g(x)=f(|x|)$

1) Graph the given function ***without absolute value***.
2) **Find** the interval for which $x>0$
3) **Reflect** this part with respect to the $y-axis$ (find symmetry $w.r.t\ y-axis$)

Ex: Consider the functions f defined by its curve C_f & $g(x)=f(|x|)$, deduce the graph of g using C_f.

0 Soln:



Therefore, we can say that

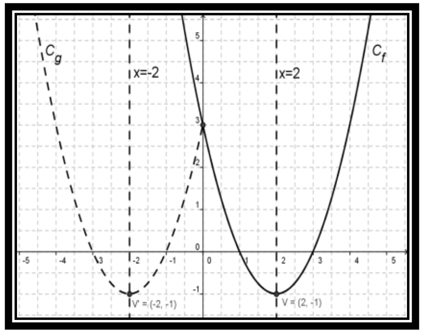
$$g(x)=\begin{cases} f(x) & for\ x\ge 0 \\ f(-x) & for\ x\le 0 \end{cases}$$

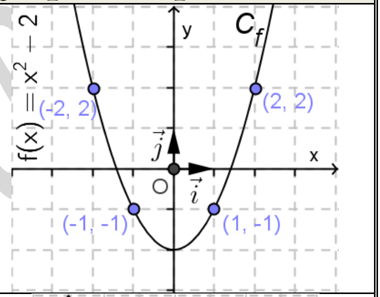
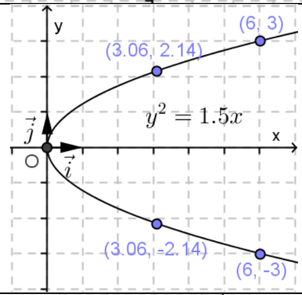
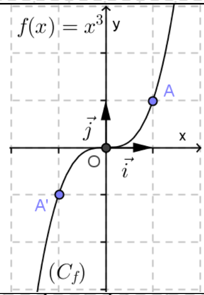
Comparing graphs of f & g we say:

i) C_f & C_g are ***confounded*** for all $x\ge 0$

ii) C_f & C_g are ***symmetric*** with respect to $y-axis$ for all $x\le 0$

Notice that: $g(x)=f(|x|)$ ***is an even function.***



Symmetry of a Curve	*With respect to*	*Testing symmetry by coordinates*	*Graphical representation*
	The, $y-axis$	For all points $(x;y)\in C_f$, the points $(-x;y)\in C_f$ *as well*.	
	The, $x-axis$	For all points $(x;y)\in C_f$, the points $(x;-y)\in C_f$ *as well*.	
	The, origin $O(0;0)$	For all points $(x;y)\in C_f$, the points $(-x;-y)\in C_f$ *as well*.	
	The, 1st -bisector $y=x$	For all points $(x;y)\in C_f$, the points $(y;x)\in C_f$ *as well*.