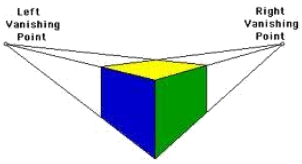
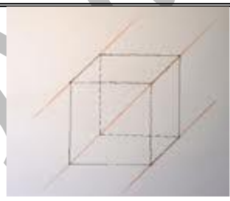
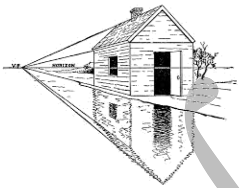
Draw on your copy book the following solids:

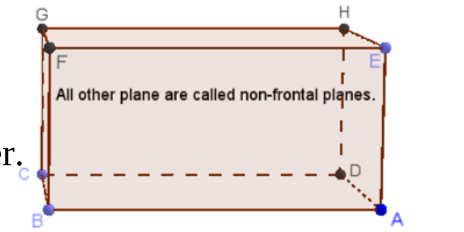


It is a bit difficult to represent solid objects of space in a plane.
To facilitate such a task we will use a drawing technique known as cavalier perspective.

- **Def:** Perspective (view point) is a drawing technique, where a three dimensional object is represented in a plane.
- **Types:** There are two main types of perspective:

True perspective, where is an escape point	*Cavalier perspective*
	
	

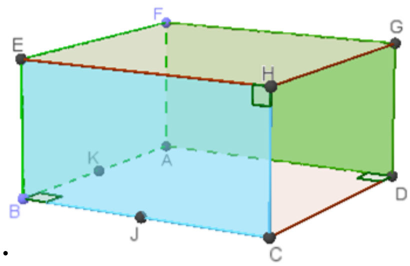
- Terminologies:



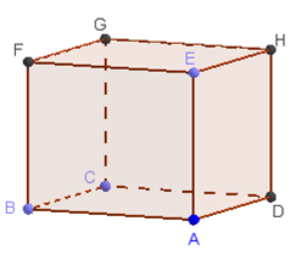
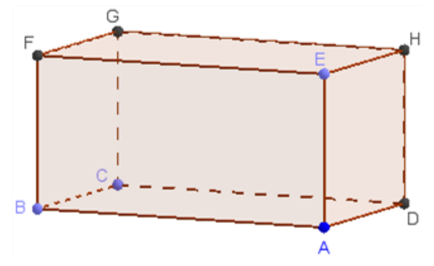
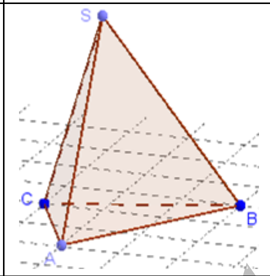
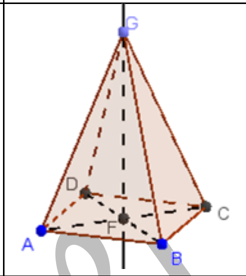
Plane (ABFE) is called the frontal plane.

- A plane is an unlimited surface.
- Frontal plane: is the plane that is directly in front of observer.
- Face: is a subset of a plane.
- Perspective:
  a) Line: is a line that is perpendicular to frontal plane. Ex: (BC) & (AD).
  b) Angle: angle formed by a horizontal line & a perspective line.

- Principle rules of Cavalier's perspective:

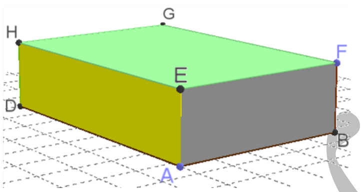


1- The full lines are seen by the observer.
2- The dotted lines are hidden with respect to the observer
3- Parallelism is conserved, that is parallel lines are presented by parallel straight lines.
4- Midpoints of segments are conserved.
5- Right angles are represented by right angles only in ***frontal*** plane.
6- Segments subset, of frontal plane are presented in true dimensions.
7- The ratio of segments having the same direction is preserved.

Basic solids in space				
Number of:	Cube	Parallelepiped (rectangular prism)	Tetrahedron	Pyramid
				
Faces				
Edges				
Vertices				

App-1:

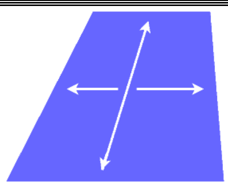
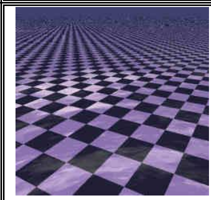
1- Redraw the solid rectangular prism *ABCDEFGH* in Cavalier perspective:



2- Complete the following table:

Non-hidden	Edges	
	Faces	
Hidden	Edges	
	Faces	

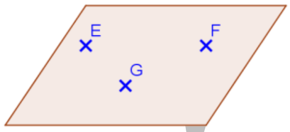
Notion about planes

Definition		***Properties***	
In solid geometry, a plane is a two-dimensional "surface" (similar to the surface of still water, or a sheet of paper, but with no thickness)			A plane is unlimited, that is to say it stretches infinitely in all directions
Representation	A plane is generally presented by a parallelogram 		

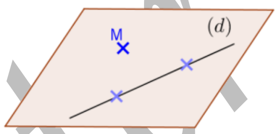
Determination of a plane

A unique plane can be determined by: *Denoted by*

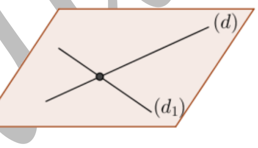
1- Three distinct non-collinear points. — (EFG)



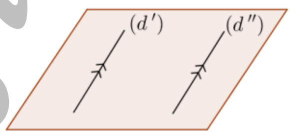
2- A straight line and any point that is not on this straight line. — pl$(M;(d'))$



3- Two intersecting straight lines — pl$((d);(d_1))$



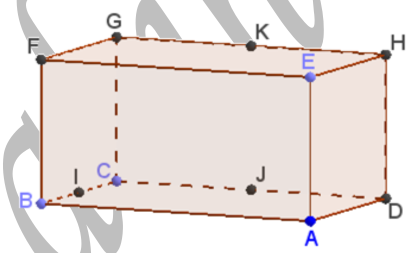
4- Two parallel straight lines. — pl$((d');(d''))$



Note that: Any geometric figure, such as (triangle, circle, quadrilateral...) determines a plane.

App-2:

For the given rectangular prism *ABCDEFGH* list (more than 2):



1- Straight lines	a) Parallel to (CG)	
	b) Subset of plane (ABC)	
2- Planes formed of	- Three non collinear points	
	-Two intersecting lines.	
	- (IK) and a point.	
	- Any two parallel st. lines.	

How many planes can be formed of the straight lines (KJ) & (BF)?

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What can you say about the planes formed in- 2)? ……………………………………..

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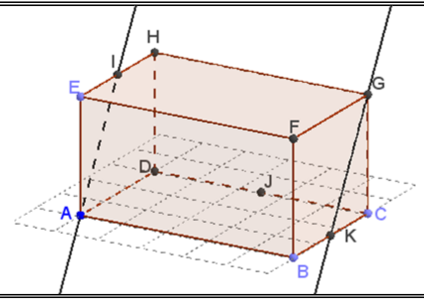
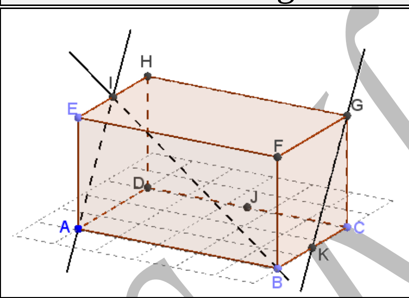
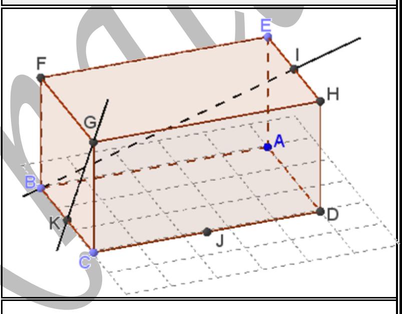
Relative positions of two straight lines in space

In space two straight lines can be:

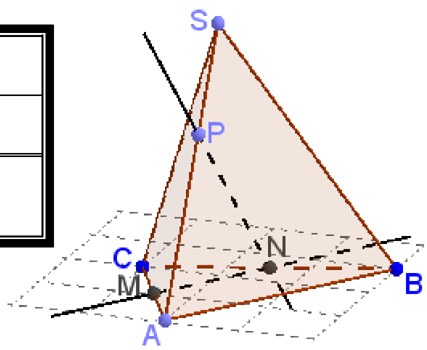
	Coplanar		Non - Coplanar
Definition	Two straight lines are coplanar *iff* they are subset of the same plane		Two straight lines are non-coplanar (Skew) *iff* they are subset of two different planes
Types	Parallel	Intersecting	Skew
Graphs			
Analytical study	$\left.\begin{matrix}(AI)\subset(AKI)\\(KG)\subset(AKI)\end{matrix}\right\}$ $So, (AI)\ \&\ (KG)\ are\ coplanar$ $(AI)\cap(KG)=\varnothing$ Thus, $(AI)\parallel(KG)$	$\left.\begin{matrix}(AI)\subset(ABI)\\(BI)\subset(ABI)\end{matrix}\right\}$ $So, (AI)\ \&\ (BI)\ are\ coplanar$ $(AI)\cap(BI)=\{I\}$ Thus, $(AI)\ \&\ (BI)$ are intersecting	$\left.\begin{matrix}(GK)\subset(BCG)\\(BI)\subset(BDI)\end{matrix}\right\}$ Thus, $(GK)\ \&\ (BI)\ are$ $non-coplanar$ or skew.

App-3:

Consider the tetrahedron $SABC$ where M & N are respective midpoints of $[AC]\ \&\ [BC]$ $\&P\in(AS)$.

1) Indicate the straight lines that are:

Coplanar	Parallel to (MN)	
	Intersecting with (PN)	
Non-coplanar	(MP)	

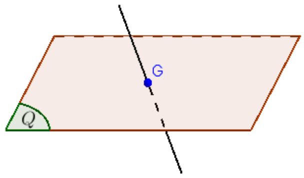
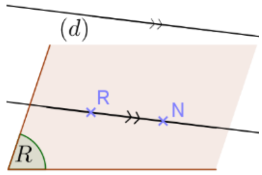
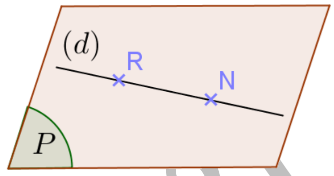


2) Are the straight lines $(PN)\ \&\ (CS)$ intersecting? Justify.

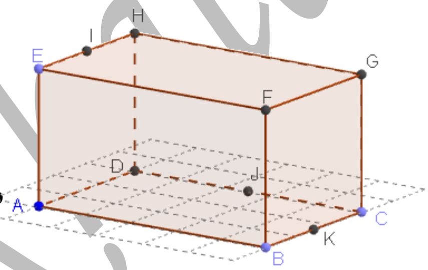
3) Find $\begin{cases}(MNP)\cap(ABC)\\(MNP)\cap(ACS)\\(MNP)\cap(ABS)\end{cases}$.

Relative positions of a straight line and a plane in space

In space a straight line can be:

Position	Intersecting with a plane	Parallel to plane	Subset of a plane
Geometric approach			
Analytic approach	$(Q)\cap(d)=\{G\}$	$(R)\cap(d)=\varnothing$	$(P)\cap(d)=(RN)$

App-4:



In the adjacent figure I, J & K are the respective midpoints of $[EH],[DC]\&[BC]$:

A- How to prove that a line is parallel to a plane?

1) Starting from the definition, why are $(EF)\&(ABC)$ parallel?

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2) Prove that the following are parallel:

$(DH)\&(ABF)$	
$(JK)\&(FGH)$	
$(KI)\&(ABF)$	

Conclusion: How do we prove that any straight line is parallel to a plane?

To prove that a straight line (d) is parallel to a plane (p):

1- Look for a straight line $(d')\subset(p)$
2- Prove that (d') is parallel to (d)

Outcome: $(d)\&(p)$ are parallel.

B- How to prove that a line is perpendicular to a plane?

3) Prove that the following are perpendicular:

$(DH)\&(ABC)$	
$(JK)\&(FGH)$	

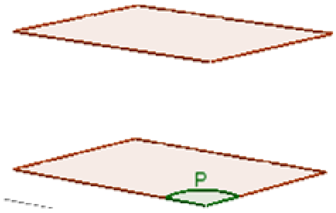
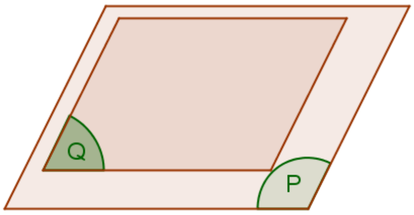
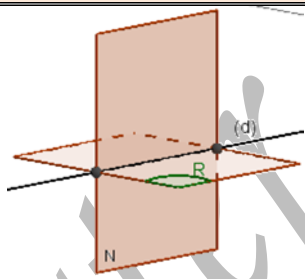
Conclusion: How do we prove that any straight line is parallel to a plane?

To prove that a straight line (d) is perpendicular to a plane (p):

- Look for two ***intersecting*** straight lines $(l)\&(\Delta)\subset(p)$
- Prove that (d) is perpendicular to both $(l)\&(\Delta)$

Outcome: $(d)\&(p)$ are perpendicular.

Relative positions between two planes in space

Position	*Parallel*	*Confounded*	*Secant(intersecting)*
Geometric approach			

How to prove that two planes are perpendicular?