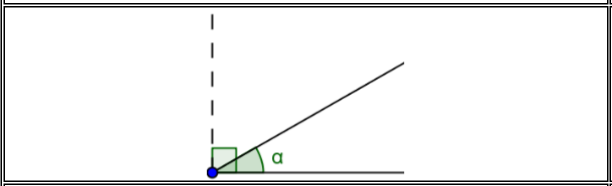
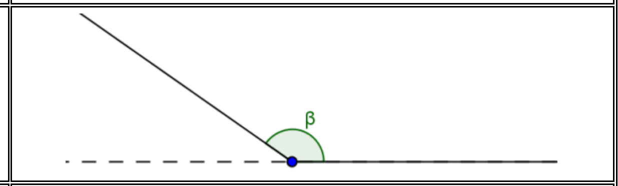
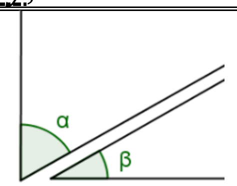
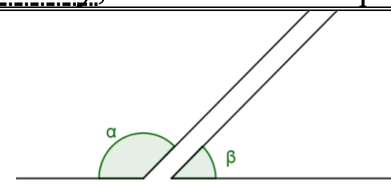
Lycée Des Arts | Mathematics | 8^{th}-Grade

Name: . . . . | "All about Angles" | E.S-6

A. *Acute and obtuse angles:*

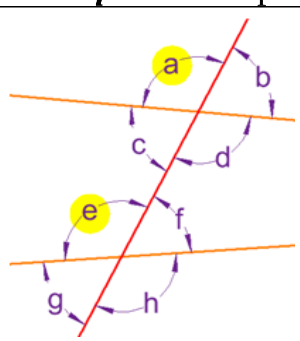
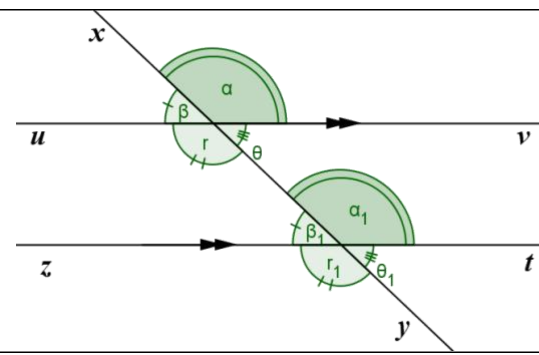
An angle α is said to be ***acute***, if its measure is between 0° and 90°.	An angle β is said to be ***obtuse***, if its measure is between 90° and 180°.
	
$i.e.\, 0°\langle\alpha\langle 90°$	$i.e.\, 90°\langle\beta\langle 180°$

B. *Complementary and supplementary angles:*

Two or more angles are said to be ***complementary***, if their sum is equal to 90°.	Two or more angles are said to be ***supplementary***, if their sum is equal to 180°.
	
$i.e.\, \alpha + \beta = 90°$	$i.e.\, \alpha + \beta = 180°$

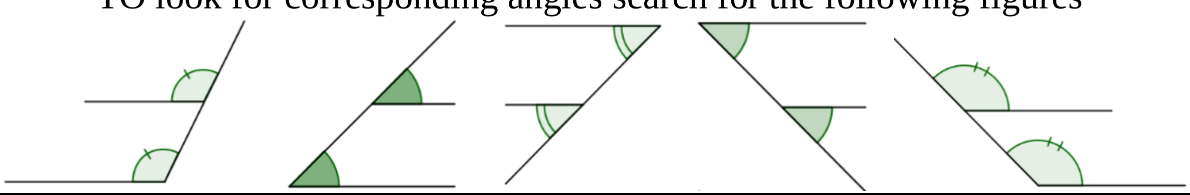
C. *Corresponding, alternating, and vertically opposite angles:*

a. ***Corresponding angles:***

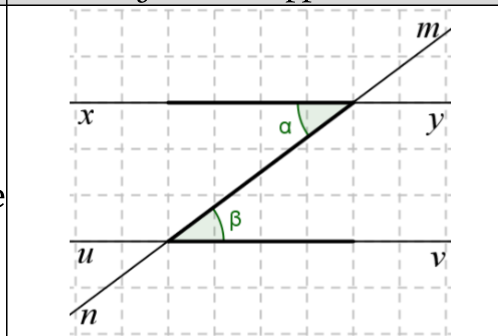
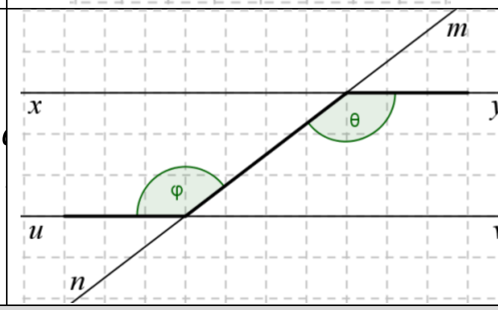
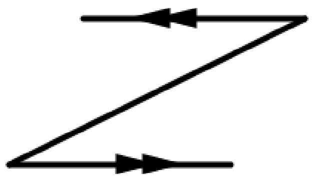
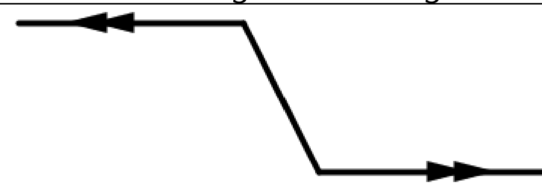
Corresponding Angles	***Equal Corresponding Angles***
If two lines are crossed by a ***transversal***, ***then*** the angles formed in the ***matching corners*** are ***non-equal*** corresponding angles	Corresponding angles are ***equal if and only if*** they are enclosed between ***parallel*** lines.
	

$$IF \left.\begin{matrix}(uv)\|(zt),\\ (xy) is\ a\ transversal\end{matrix}\right\}, then\ we\ have \begin{Bmatrix}\alpha;\alpha_1\\ \beta;\beta_1\end{Bmatrix} \& \begin{Bmatrix}r;r_1\\ \theta;\theta_1\end{Bmatrix} (are\ equal\ corresponding\ angles)$$

TO look for corresponding angles search for the following figures



b. ***Alternating angles***: We distinguish two types of ***equal*** alternating angles:
Alternating angles are ***equal if and only if*** they are enclosed between ***parallel*** lines.

<table>
<tr><th></th><th>Analytic approach</th><th>Geometric approach</th></tr>
<tr><td>Alternating interior angles</td><td>$IF \left.\begin{matrix} (xy)\parallel(uv), \\ and\ (mn) \\ is\ a\ transversal \end{matrix}\right\}, then \left\{\begin{matrix} \alpha\ and\ \beta are\ equal \\ \text{alter.interior angle} \end{matrix}\right.$</td><td>
</td></tr>
<tr><td>Alternating exterior angles</td><td>$IF \left.\begin{matrix} (xy)\parallel(uv), and\ (mn) \\ is\ a\ transversal \end{matrix}\right\}, then \left\{\begin{matrix} \varphi\ and\ \theta\ are\ e \\ \text{alter. exterior} \end{matrix}\right.$</td><td>
</td></tr>
<tr><td colspan="3">TO look for alternating angles search for the following figures</td></tr>
<tr><td colspan="2">Alternating interior angles:</td><td>Alternating exterior angles</td></tr>
<tr><td colspan="2">
Z- SHAPE</td><td>
Stair-SHAPE</td></tr>
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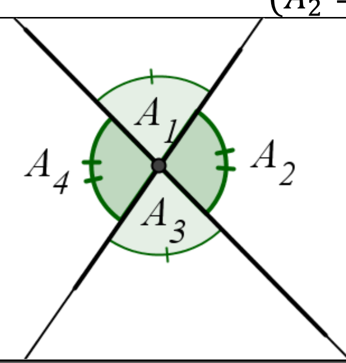
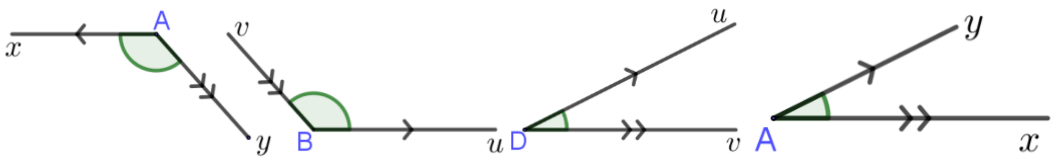
c. ***Vertically opposite angles:*** are formed by two intersecting lines.

Analytic approach	$IF\,(xy)\&(uv)$are two intersecting lines,then $\left\{\begin{matrix} \hat{A}_1 = \hat{A}_3 \\ \hat{A}_2 = \hat{A}_4 \end{matrix}\right\}$ (vertically opp. angles)
Geometric approach	

D. *Angles with their sides respectively parallel*:

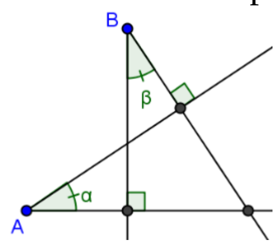
Two angles (acute or obtuse) with their sides respectively parallel, are ***equal***.



Conclusion: ***Angles enclosed between parallel lines are equal.***

E. *Angles with their sides respectively* perpendicular:

Two angles (acute or obtuse) with their sides respectively perpendicular are ***equal***.



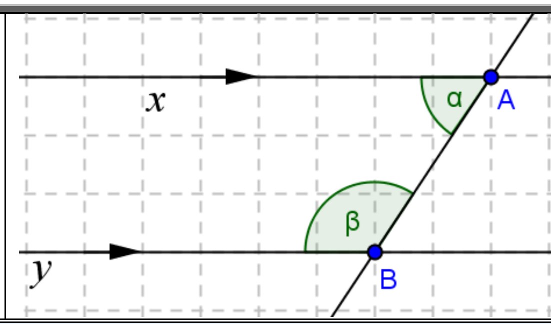
then, $\hat{A} = \hat{B}$

Conclusion: ***Angles with their sides (arms) mutually perpendicular are equal.***

I- *Relative positions of lines and angles*:

a. Co-interior angles

If [*Ax*) $\parallel$ [*By*), then angles formed between them are called co-interior angles
So that, $\alpha + \beta = 180°$

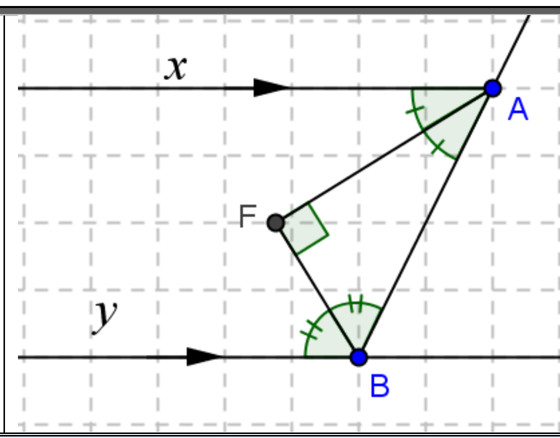


Note that: ***The sum of two co-interior angles is 180º.***

b. Bisectors of two co-interior angles

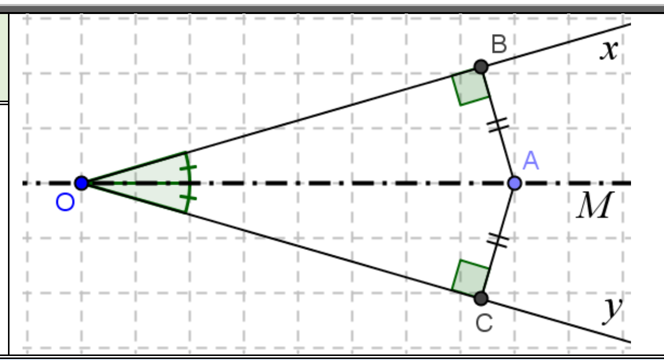
$$IF \begin{cases} [Ax) \parallel [By)\ , \\ [AF) \text{ bisector of } x\hat{A}B, \\ [BF) \text{ bisector of } y\hat{B}A \end{cases} then, A\hat{F}B = 90°.$$

$$i.e [AF) \perp [BF)$$

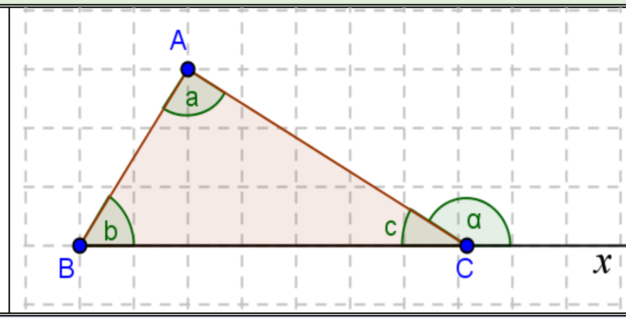


Conclusion: ***Bisectors of two co-interior angles form a right angle.***

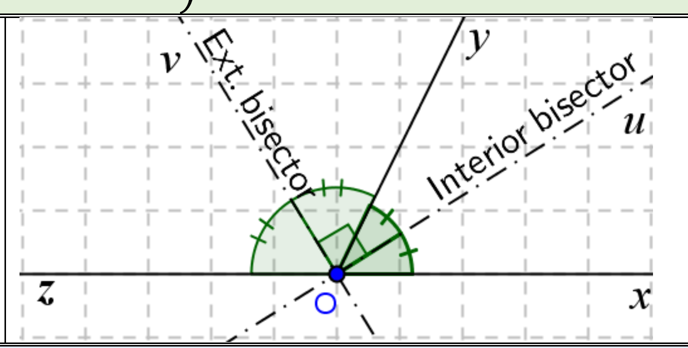
c. *Point on a bisector of an angle:*

IF	*then*	
$\begin{cases} [OM) \text{ bisector of } x\hat{O}y, \\ A \text{ belongs } [OM), \\ [AB) \perp [Ox), \\ [AC) \perp [Oy) \end{cases}$	$AB = AC$. *And* *OA=OC*	
Conclusion: **Any point on the bisector of an angle is equidistant from its arms.**		

d. *Exterior angle in a triangle:*

Rule:	*If* α is an exterior angle, then $\alpha = a + b$	
Proof:	$\alpha + c = 180°$ (Supplementary angles) $so, \alpha = 180° - c$ but, $(a + b) = 180° - c$ (Sum of angle in a Δ) $Thus, \alpha = a + b$ (By comparison)	
Conclusion: ***The exterior angle is equal to the sum of the two opposite interior angles.***		

e. *Bisectors of two adjacent supplementary angles:*

Rule:	$IF \begin{cases} x\hat{o}y + y\hat{o}z = 180° (Supplementary) \\ [ou) bisector\ of\ x\hat{o}y, \\ [ov)\ bisector\ of\ y\hat{o}z \end{cases}$ $then, u\hat{o}v = 90°$.	
Proof:	$x\hat{o}y + y\hat{o}z = 180° (Given)$ $[Ou) bisector\ of\ x\hat{o}y (Given)$ $so, x\hat{o}y = 2u\hat{o}y (property\ of\ bisector)$ $[Ov) bisector\ of\ y\hat{o}z (Given)$ $so, y\hat{o}z = 2y\hat{o}v\ (property\ of\ bisector)$ $hence, 2u\hat{o}y + 2y\hat{o}v = 180° (By\ substitution)$ $Thus, u\hat{o}y + y\hat{o}v = 90°$	
Conclusion: ***The bisectors of two adjacent supplementary angles form a right angle.***		