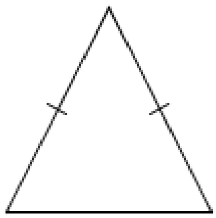
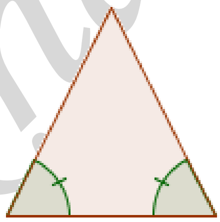
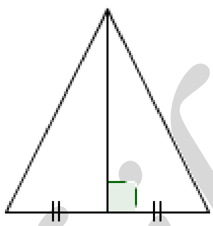
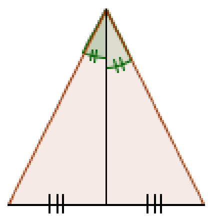
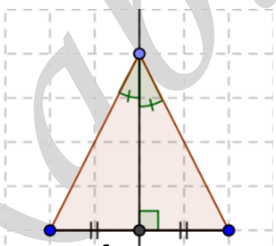
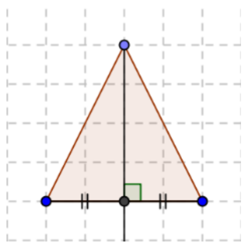
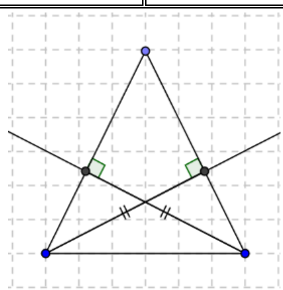
To prove a triangle **isosceles** look for one of the following:

- Two equal sides.
- Two equal angles.
- Two equal altitudes.
- A height to be a median at the same time or vice versa.
- A bisector to be a median at the same time or vice versa.
- An axis of symmetry passing through one vertex of the triangle.
- A perpendicular bisector issued from one vertex of the triangle.

In other words a triangle is said to be isosceles if:

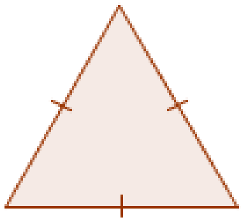
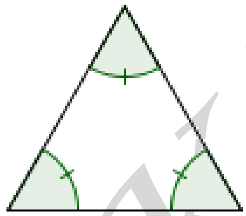
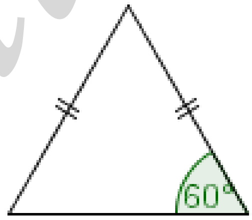
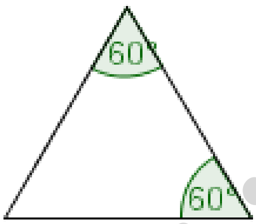
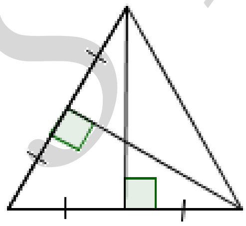
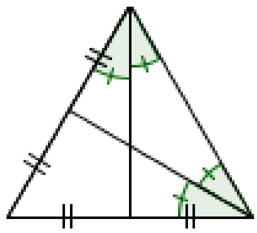
a.  It has two equal sides.	*b.*  It has two equal angles.
c.  It has a median as a height or vice versa.	*d.*  It has a median as a bisector of the main vertex or vice versa.
e.  It has an axis of symmetry passing through one of its vertices	*f.*  It has a perpendicular bisector passing through one of its vertices
g. It has two equal altitudes 	

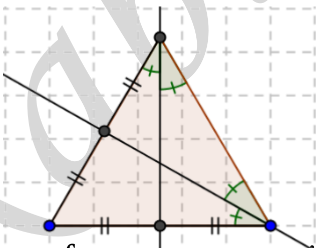
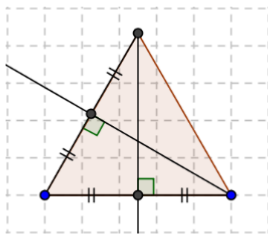
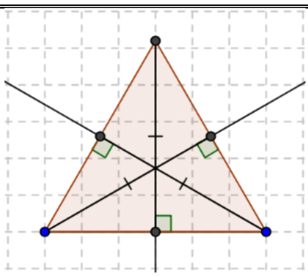
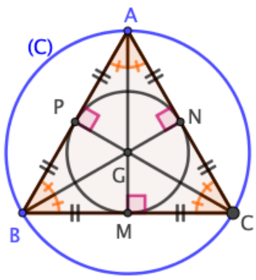
Rabih S Khater.

To prove a triangle **equilateral** look for one of the following:

- Three equal sides.
- Three equal angles.
- Three equal altitudes.
- Two equal sides and 60° angle (Isosceles triangle and a 60° angle).
- Two 60° angles.
- Two heights to be as medians at the same time or vice versa.
- Two bisectors to be as medians at the same time or vice versa.
- Two axes of symmetry passing through two vertices of the triangle.
- Two perpendicular bisectors issued from two vertices of the triangle.

In other words a triangle is said to be equilateral if:

1.  It has three equal sides	2.  It has three equal angles	3.  It has two equal sides and one 60° angle
4.  It has two equal 60° angles	5.  It has two medians as heights	6.  It has two medians as bisectors

7.  It has two axes of symmetry passing through two of its vertices	8.  It has two perpendicular bisectors passing through two of its vertices
 9. It has three equal altitudes.	10. All centers are confounded. (incenter, circumceter, orthocenter, centroid) 

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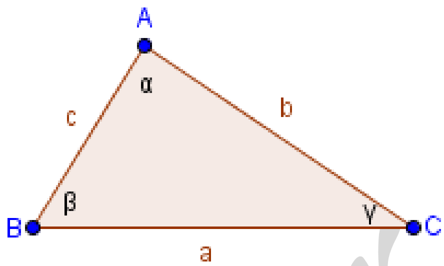
How to prove a triangle right?

1- *By sum of angles in a triangle:*



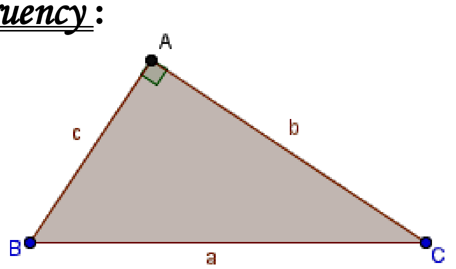
In triangle ABC we have: $\beta + \gamma = 90^\circ$.(Given)

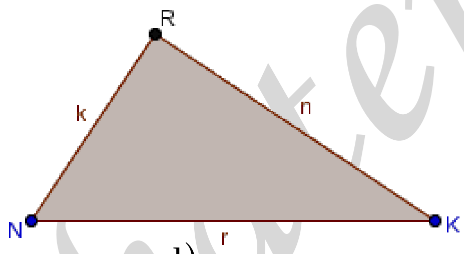
But $\alpha + \beta + \gamma = 180^\circ$.(Sum of angles in a triangle)

$So, \alpha = 90^\circ$.(By substitution)

Thus, ΔABC *is right at A.*

2- *By congruency* :

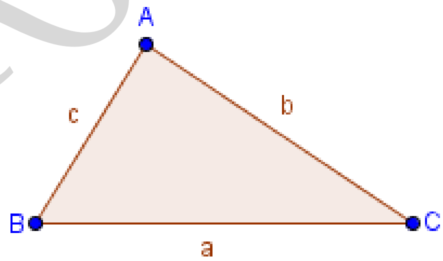




$IF \begin{cases} \text{Triangles } RNK \text{ and } ABC \text{ are congruent } (\text{given or proved}) \\ \text{Triangle } RNK \text{ is right at } R (\text{given}) \end{cases}$

Then, ABC is right at A (by homologous elements)

3- *By converse of Pythagoras' Theorem:*



Given length of all sides

Then, if $a^2 = b^2 + c^2$

Thus, ΔABC *is right at A.*

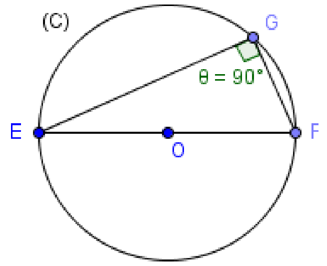
4- *By central and inscribed angles:*

If G is a point on (C) of diameter $[EF]$

Then, $E\hat{G}F = \frac{1}{2} E\hat{O}F$.(mes of inscribed angle = $\frac{1}{2}$ mes of intercepted arc.)

But $E\hat{O}F = 180^\circ$.

$\Rightarrow E\hat{G}F = \dfrac{1 \times 180^\circ}{2} = 90^\circ$.

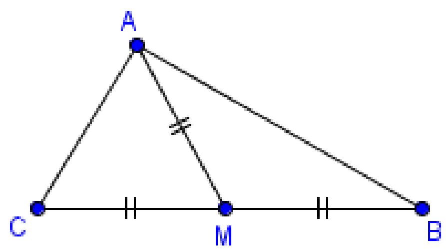


Thus, ΔEGF *is right at G.*

5- *By Converse of median relative to hypotenuse:*

If $AM = \frac{1}{2} BC$.(given or proved)

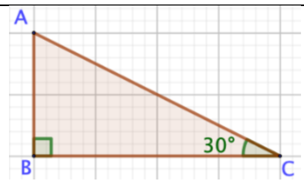
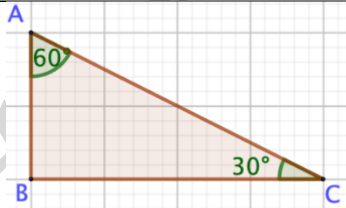
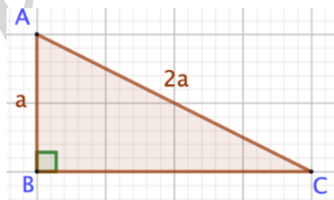
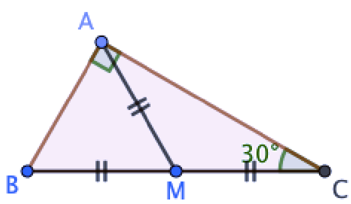
Or $MA = MB = MC$.



Thus, ΔABC *is right at A.*(By converse of median relative to hypotenuse)

How to prove a triangle semi-equilateral?

Rabih S Khater.

Using angles only	1) $90° + 60°$	
	2) $90° + 30°$	
	3) $30° + 60°$	
Using sides	4) $90°$ & $smallest\ side = \frac{1}{2} longest\ side$	
	5) $90°$ & $longest\ side = 2\ smallest\ side$	
	6) Converse of pythagorean theorem + $60°\ or\ 30°$	
	7) Converse of median relative to hypotenuse + $60°\ or\ 30°$	

Rabih S Kha

Special lines in special right triangles

In a right isosceles triangle:

☆ The side facing 45° angle is expressed as:

$c = \frac{\sqrt{2}}{2} \times hyp.$ (Side facing 45° of a right isosceles Δ)

Proof: According to Pythagoras' theorem:

$hyp^2 = leg^2 + leg^2$

$hyp^2 = c^2 + c^2$

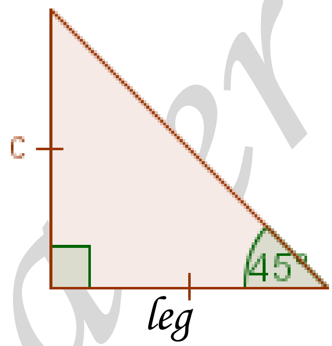
$hyp^2 = 2c^2$

$so, 2c^2 = hyp^2$

$c^2 = \frac{hyp^2}{2}$

$c = \frac{hyp}{\sqrt{2}}$

$Thus,\ c = \frac{\sqrt{2}}{2} \times hyp.$



☆ Hypotenuse is expressed as:

$hyp = \sqrt{2} \times c.$ (Hypotenuse of a right isosceles Δ)

In a semi equilateral triangle:

☆ Side facing **30°** angle is equal to ***half*** the hypotenuse

$Then,\ a = \frac{1}{2} \times hyp.$ (Side facing 30° of a semi equilateral Δ)

☆ Side facing 60° angle is expressed as:

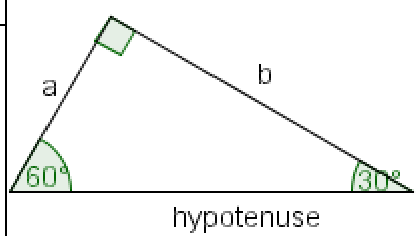
$b = \frac{\sqrt{3}}{2} \times hyp.$ (Side facing 60° of a semi equilateral Δ)

OR

$b = \sqrt{3} \times a.$ (Sides facing 60° & 30° of a semi equilateral Δ)

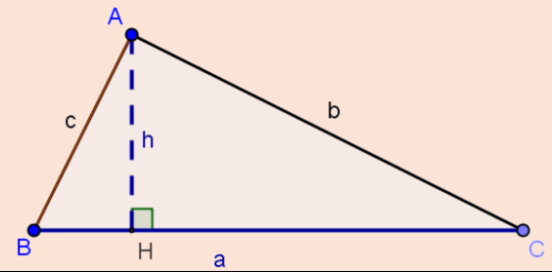
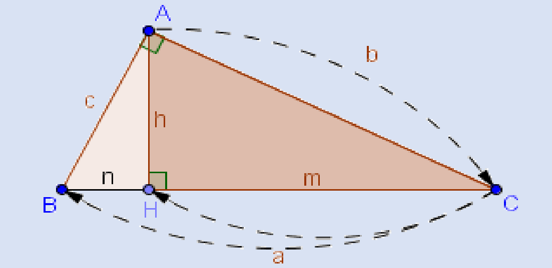
☆ Hypotenuse is expressed as:

$hyp = 2 \times a.$ (Hypotenuse of a semi equilateral Δ)



Metric Relations of a Right triangle

Rabih S Khader.

<table>
<tr><th>No.</th><th>Relations</th><th>Name of relation</th><th>Geometric Figures</th></tr>
<tr><td>1.</td><td>$a \times h = b \times c.$</td><td>Height- Hypotenuse relation</td><td>
</td></tr>
<tr><td>2.</td><td>$b^2 = m \times a.$</td><td rowspan="3">Geometric means</td><td>
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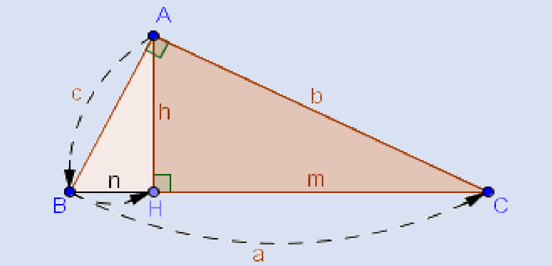
<tr><td>3.</td><td>$c^2 = n \times a.$</td><td>
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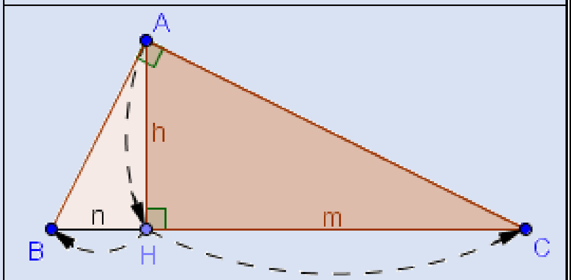
<tr><td>4.</td><td>$h^2 = m \times n.$</td><td>
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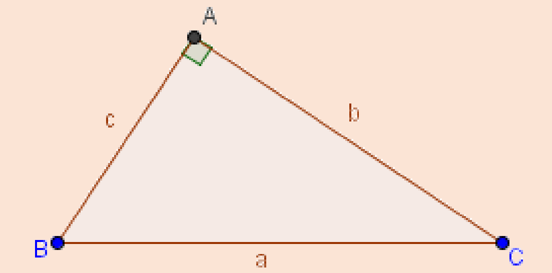
<tr><td>5.</td><td>$a^2 = b^2 + c^2$</td><td>Pythagoras theorem</td><td>
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