

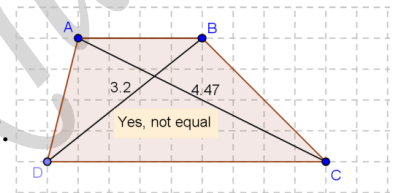
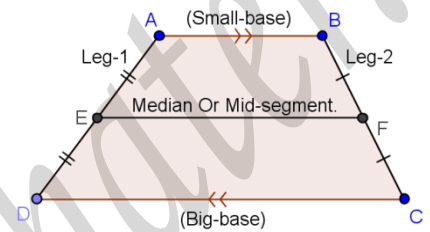
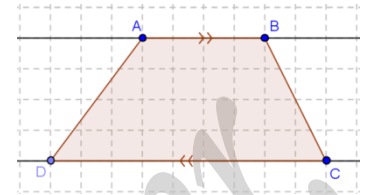
**Definition:** A *trapezoid* is a quadrilateral with exactly **one pair** of its sides parallel.

✓ **Vocabulary:**

- ↪ **Bases of a trapezoid:** are the parallel sides.
- ↪ **Legs of a trapezoid:** are the non-parallel sides.
- ↪ **Median or mid-segment of a trapezoid:** is the segment joining the midpoints of the legs.

👁👁 **Note that:** In a trapezoid diagonals

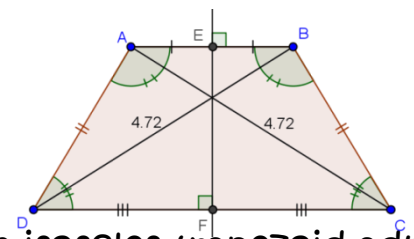
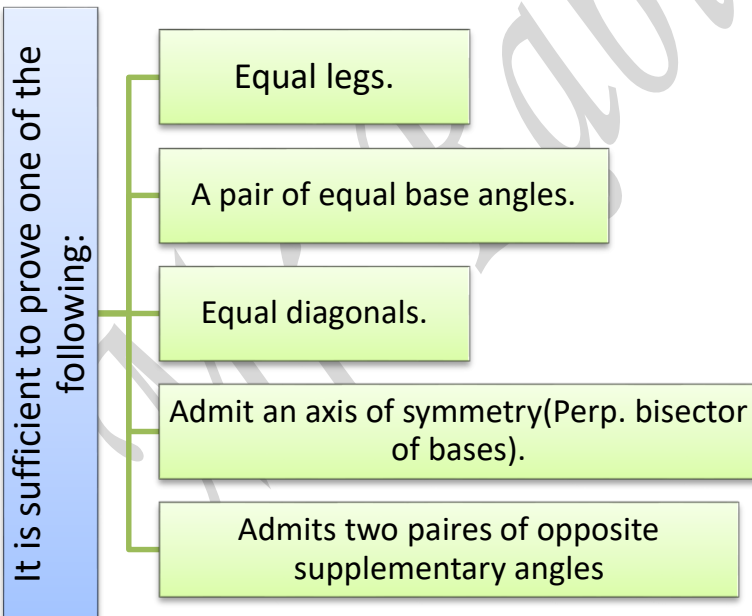
- Are **not Equal**.
- Do **not** bisect each other **they only intersect each other**.




✓ **Types of a trapezoid:**

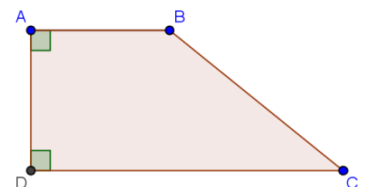
 Isosceles trapezoid:

**How to prove a trapezoid Isosceles?**



Notice that, an isosceles trapezoid admits one **axis of symmetry**, the perpendicular bisector of the bases.

 Right trapezoid: I'm a trapezoid in which one of my legs is perpendicular to my bases.



# Proving a quadrilateral to be a parallelogram

Starting from

## Sides

## Diagonals

## Angles

Definition

Two pairs of sides

A pair of sides

A quadrilateral with **two pairs of opposite sides parallel** is a parm

A quadrilateral with **two pairs of opposite sides equal** is a parm

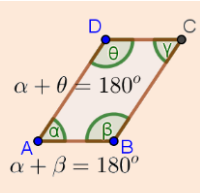
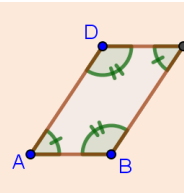
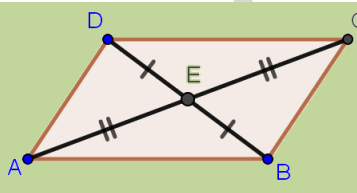
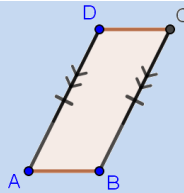
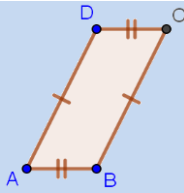
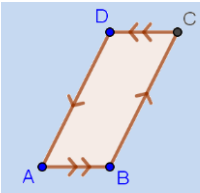
A quadrilateral with **a pair of opposite sides equal & parallel** is a parm

A quadrilateral with its **diagonals bisect each other at same midpoint** is a parm

A quadrilateral that admits a **center of symmetry** is a parm

A quadrilateral with **two pairs of opposite angles equal** is a parm

A quadrilateral with **two pairs of adjacent angles supplementary** is a parm



How to prove a quadrilateral a rectangle?

Definition

• A quadrilateral with four equal angles is a rectangle.

Angles

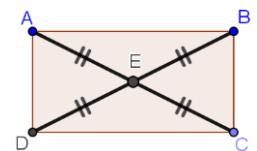
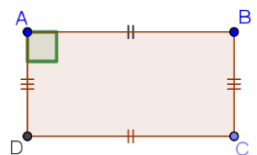
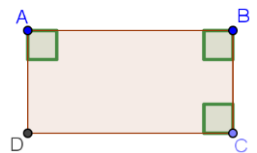
• A quadrilateral with three right angles is a rectangle.

Sides & angles

• A quadrilateral with its opposite sides equal or (parallel) and have one right angle is a rectangle.

Diagonals

• A quadrilateral in which its diagonals are equal and bisect each other is a rectangle.



Starting from the:

How to prove a parallelogram to be a rectangle?

## Starting from the

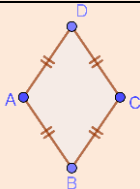
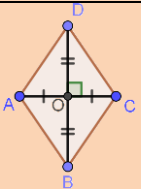
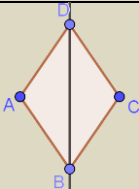
*Angles*

A parallelogram with one right angle is a rectangle

*Diagonals*

A parallelogram with equal diagonals is a rectangle.

Proving a quadrilateral to be a rhombus starting from

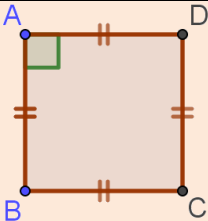
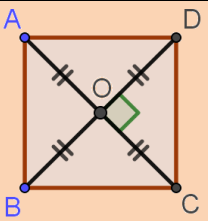
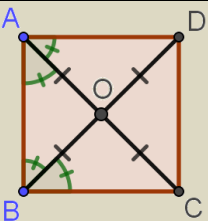
	Definition	Diagonals	Axes of symmetry
<i>In words</i>	A quadrilateral with four equal sides is a rhombus	A quadrilateral whose diagonals are perpendicular and bisect each other is a rhombus	A quadrilateral whose diagonals are axes of symmetry is a rhombus.
<i>Graphically</i>			



How to prove a parallelogram a rhombus?

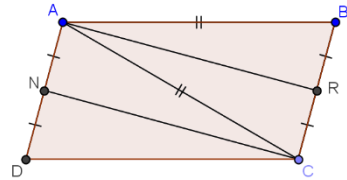
- i- Starting from sides: A parallelogram with two equal consecutive sides is a rhombus.
- ii- Starting from diagonals: A parallelogram with perpendicular diagonals is a rhombus.
- iii- Starting from diagonals: A parallelogram with one diagonal is a bisector of its one angles is a rhombus.

Proving a quadrilateral to be a square starting from

	Definition	Diagonals	Diagonals & angles
<i>In words</i>	Four equal sides and one right angle	Diagonals are perpendicular, equal and bisect each other	Diagonals are equal and bisect the angles of the quadrilateral
<i>Graphically</i>			

## Applications

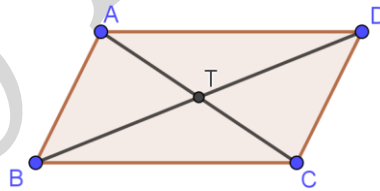
- I-** Consider the parallelogram  $ABCD$ .
- Indicate the properties included in the adjacent figure.
  - Prove that  $ARCN$  is a rectangle.



- II-** Let  $C(O; R)$  and  $C'(O'; R)$  be two intersecting circles.
- Draw figure.
  - $(C)$  &  $(C')$  intersect at the points  $R$  &  $N$ . What is the nature of the quadrilateral  $ORON'$ ?

- III-**  $ROME$  is a square of center  $N$ .
- Construct sketch.
  - Let  $J$  be any point of  $[RM]$ . Locate  $K$  the symmetric of  $J$  with respect to  $O$ .
  - What is the nature of quadrilateral  $JOKE$ ?

- IV-**  $CORE$  is a parallelogram such that  $CO = 2OR$ .
- Sketch the figure.
  - Let  $N$  &  $K$  be the respective midpoints of sides  $CO$  and  $RE$ .
    - Prove that  $NORK$  and  $CNKE$  are two rhombuses.
    - Show that triangle  $COK$  is right at  $K$ .



- V-** Let  $ABCD$  be a parallelogram of center  $T$ .  
The following parts are independent:
- If  $\hat{ABC} = 135^\circ$ , then find the measure of  $\hat{BAD}$ .
  - If  $AC = 5x - 12$  and  $AT = 14$ , then find the value of  $x$ . (check existence).
  - If  $AB = 12, BC = 9$  and  $\hat{ABC} = 90^\circ$ , then calculate the length of  $BD$ .
  - If  $BT = 3x + 1$  and  $BD = 4x + 8$ , then determine the value of  $x$ . (check existence).
  - If  $BC = 4x - 7$  and  $AD = 8x - 5$ , then compute the value of  $x$ . (check existence).
  - If  $\hat{BCD} = 3x + 14$  and  $\hat{ADC} = x + 10$ , then work out the exact value of  $\hat{ADC}$ .