There are a variety of techniques to write an expression in a product form (factorized)

☆ In what follows we will list some common techniques:

A. Taking a **common factor:**

It works when **all terms** of an expression have a common:

1. Coefficient: that is if the GCD of the coefficients of the expression is different than **1**.

Expanded form	Factorized form
$3x^2-9$	Since, $GCD(3;9)=3$ so $3(x^2-3)$
$21z^5+28z-7$	Since, $GCD(21;28;7)=7$ so $7(3z^5+4z-1)$
$-\pi y^2-2x\pi$	Since, $GCD(\pi;2\pi)=\pi$ so $-\pi(y^2+2x)$

2. Common Term: that is if terms of expression have a common *variable or factor*.

Expanded form	Factorized form
x^2-12x	$x(x-12)$
$3x^2z^3+5xz^2-11x^2z^2$	$xz^2(3xz+5-11x)$
$(x-2)(x^2+2)-(x-2)(3x^2-5)$	$(x-2)[(x^2+2)-(3x^2-5)]$

B. By grouping:

If a polynomial has ***FOUR*** terms, the expression may be factorable by grouping, This technique works if there is no common factor to all terms. However, a common factor can be found between the two terms and another common factor between the second two terms, then we ***GROUP*** the terms and factor out the common factor.

Expanded form	Factorized form
$ac+ad+bc+bd$	1st-step: Put in groups $\underbrace{ac+ad}_{1^{st}-Group}+\underbrace{bc+bd}_{2^{nd}-Group}$ 2nd-Step:Take common $a(c+d)+b(c+d)$ 3rd-step:Factor out again $(c+d)(a+b)$

C. Using **remarkable identities**:

Frequently used identities

Description	***Expanded form***	***Factorized form***
Difference of two squares	a^2-b^2	$(a-b)(a+b)$
Square of sum (binomial)	$a^2+2ab+b^2$	$(a+b)^2$
Square of difference	$a^2-2ab+b^2$	$(a-b)^2$
Difference of two cubes	a^3-b^3	$(a-b)(a^2+ab+b^2)$
Sum of two cubes	a^3+b^3	$(a+b)(a^2-ab+b^2)$
Cube of sum	$a^3+3a^2b+3ab^2+b^3$	$(a+b)^3$
Cube of difference	$a^3-3a^2b+3ab^2-b^3$	$(a-b)^3$

D. *Using **trial and error** technique:*

It is used only in case if the expression is of the form: $1x^2 + bx + c$.

Expanded form	*Factorized form*	*Application*
$1x^2 + bx + c.$	***1st-step:*** Put in product form: $\underbrace{(x \quad)}_{1^{st}-factor}\underbrace{(x \quad)}_{2^{nd}-factor}$ ***2nd-Step:*** Check the sign of c: If the sign of c is $\oplus ve$, then: Find a pair of numbers in which their: - ***Product*** is equal to c. That is, $c = r \times n$. - ***Sum*** is equal to b. That is, $b = r + n$. • If the sign of b is $\oplus ve$, then write $(x+r)(x+n)$. • If sign of b is $-ve$, then write $(x-r)(x-n)$. If the sign of c is $-ve$, then: Find a pair of numbers in which their: - ***Product*** is equal to c. That is, $c = r \times n$. - ***Difference*** is equal to b. That is, $b = r - n$. • If the sign of b is $-ve$, then greater is $-ve$. • If the sign of b is $\oplus ve$, then greater is $\oplus ve$. ***3rd-step:*** Insert numbers and signs in factors. $(x \quad r)(x \quad n)$	$x^2 - 5x + 6$ 1) $(x \quad)(x \quad)$ 2) $c = +6 \, is \oplus ve$ $but,\ 6 = \begin{cases} 6 \times 1 \\ 3 \times 2 \end{cases}$ $so, use\ 3 + 2 = 5$ 3) $(x \quad 3)(x \quad 2)$ $but, we\ need\ -5$ $so, -3 + (-2) = -5$ $Thus, (x-3)(x-2)$

E. ***Perfect squaring** technique:*

It is used only in case if the expression is of the form: $ax^2 + bx + c$.

Expanded form	*Factorized form*	*Application*
$ax^2 + bx + c.$	***1st-step:*** If $a > 1$ then take it as a common factor if not, do the other steps: $a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right)$. ***2nd-Step:*** Write expression: $a\left[(x \quad)^2 + \frac{c}{a}\right]$. ***3rd-step:*** Always insert sign of b next to x. ***4th-step:*** Divide the term $\frac{b}{a}$ by 2 to get $\frac{b}{2a}$ ***5th-step:*** Put this term next to x: $a\left[\left(x + \frac{b}{2a}\right)^2 + \frac{c}{a}\right]$. ***6th-step:*** Always subtract the square of, $\frac{b}{2a}$ from $\frac{c}{a}$: $a\left[\left(x + \frac{b}{2a}\right)^2 + \frac{c}{a} - \left(\frac{b}{2a}\right)^2\right]$.	$3x^2 - 12x - 4$ $= 3\left[x^2 - 4x - \frac{4}{3}\right]$ $= 3\left[x^2 - 4x .... - \frac{4}{3}\right]$ $= 3\left[x^2 - 4x + ... - ... - \frac{4}{3}\right]$ $= 3\left[\underbrace{x^2 - 4x + \left(\frac{4}{2}\right)^2} - \left(\frac{4}{2}\right)^2 - \frac{4}{3}\right]$ $= 3\left[\left(x - \frac{4}{2}\right)^2 - 4 - \frac{4}{3}\right]$

When and how to use different Factorizing Techniques