Remark:
the line of equation
(d): y= ax is an oblique
line passing through the
ORIGIN

Reduced equation

(d): y= 2x + 3

When I see this equation I should know that

1) 2 is the slope/director coefficient of (d)

2) +3 is the y-intercept/ ordinate at the origin of (d) (it means that (d) cuts (y'y) at the point (0,+3))

3) The coo of all points of (d) verify the relation y=2x+3

4) (d) is oblique

(d): y = -5

When I see this equation I should know that

1) 0 is the slope/director coefficient of (d)

2) All points of (d) are of ordinate -5.

3) (d) // (x'x) at y = -5 (horizontal)

4) -5 is the y-intercept/ ordinate at the origin of (d) (it means that (d) cuts (y'y) at the point (0 ,-5))

(d): x = 1

When I see this equation I should know that

1) (d) has NO SLOPE

2) All points of (d) are of abscissa +1.

3) (d) // (y'y) at x = 1 (vertical)

4) NO y-intercept

Rule 1: All points on (x'x) have 0 as ordinate

Rule 2: All points of (y'y) have 0 as abscissa

Rule 3 : 2 points having same abscissa form a line // (y'y)

Rule 4: 2 points having same ordinate form a line // (x'x)

Rule 5: If $(d) // (x'x)$

and $(d') // (y'y)$

then $(d) \perp (d')$

(since $(x'x) \perp (y'y)$)

Length Rule

Given the coordinates of A and B we can calculate the length AB by applying

$$AB = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2}$$

REMARKS:

1) We usually use the rule to prove the nature of a triangle (by calculating the lengths of its 3 sides) or a quadrilateral or to calculate the diameter or radius of a circle.

2) Remember that the center of the circle circumscribed about a right triangle is the midpoint of the hypotenuse.

3) Remember that we can always use the coverse of the Pythagorean theorem to prove a right triangle

Slope Rules

<u>Rk</u>
- (d): $y=b \rightarrow a_d=0$
- (d): $x=m \rightarrow$ no slope

SLOPE when exists

Given the reduced equation of (d)
It is the COEFFICIENT of x

Given 2 points A and B of (d)
It is calculated through this rule

$$a_{AB}=\frac{y_A-y_B}{x_A-x_B}$$

<u>Remember</u>: If you got 0 in the denominator it means that the slope does not exist and the line is parallel to (y'y) and of equation x= nb

If we DON'T HAVE neither the reduced equationof (d) nor 2 points of it then its shoud be // or $\perp$ to another line (d') whose slope can be calculated through 2 points of it or its reduced equation in this case we find $a_{d'}$ first then we use these rules to find a_d

(d) // (d') so
$$a_d=a_{d'}$$

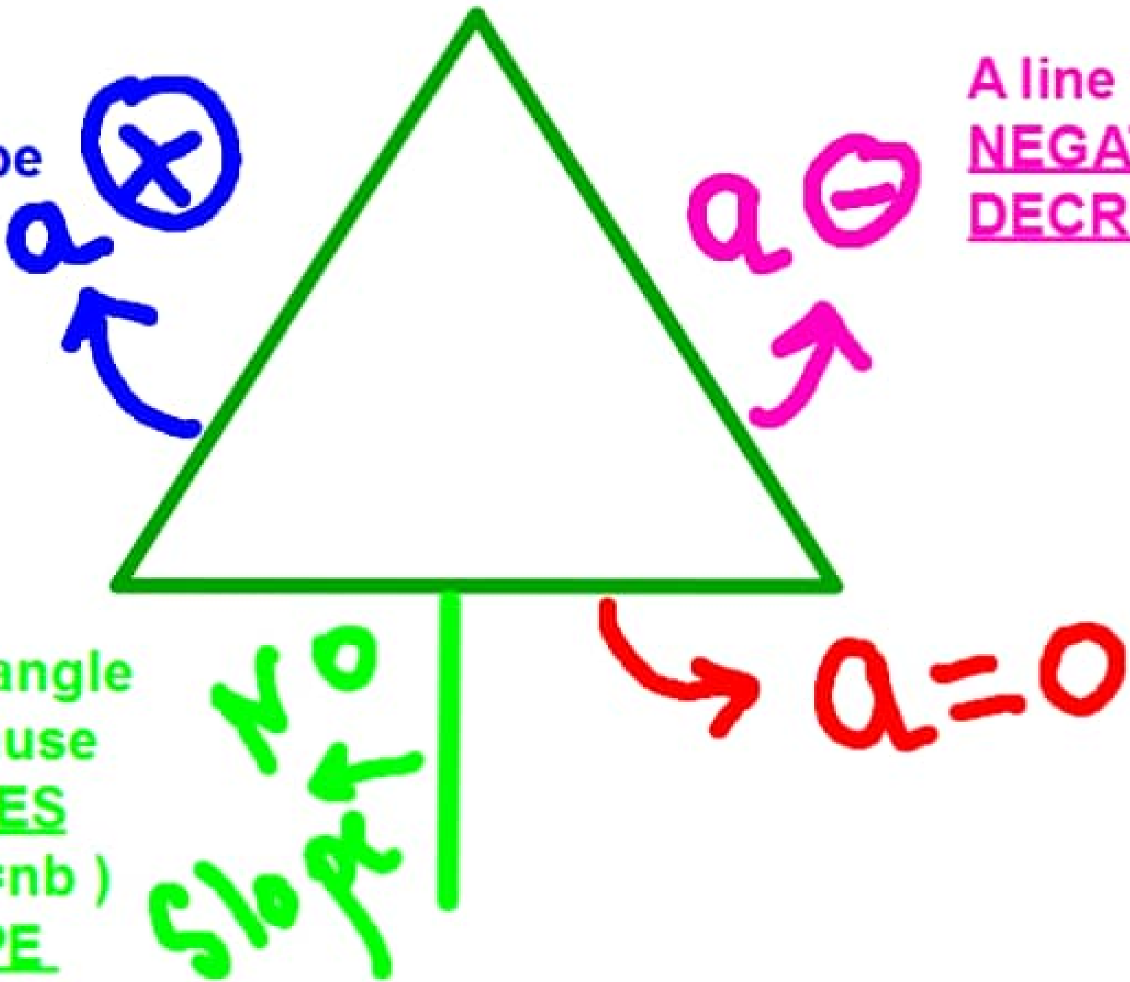
(d) $\perp$ (d') so
$$a_d \times a_{d'}=-1$$

Triangle of slopes

In order to help yourselves verify your figure, you need to remember of this triangle

A line with a <u>POSITIVE</u> slope is an <u>INCREASING</u> line

A line with a <u>NEGATIVE</u> slope is a <u>DECREASING</u> line



This line is outside the triangle of slopes because <u>VERTICAL LINES</u> (//(y'y) of eq x=nb) have <u>NO SLOPE</u>

The slope of a <u>HORIZONTAL</u> line (//(x'x) of equations y=b) is <u>NULL</u>

Midpoint Rule

Calculate

If we were given the coo of A and B and we need to calculate the coo of M midpoint of [AB] we use the rule

$$x_M = \frac{x_A + x_B}{2}$$

$$y_M = \frac{y_A + y_B}{2}$$

Prove / Verify

If we were given the coo of A and B and M and we need to verify or prove whether M is the midpoint of [AB] or not we use the same rule, we replace all the coordinates using the question mark then we get at the end a true equation if M were the midpoint of [AB] and a false one if it weren't

Symmetry Rules

Direct symmetry

If S is the sym of L(-2,1)

1) W.R to the origin

S(+2,-1) change the signs of **both** x_L and y_L to get the coo of S

2) W.R to (x'x)

S(-2,-1) change the sign of **only** y_L to get the coo of S

3) W.R to (y'y)

S(+2,1) change the sign of **only** x_L to get the coo of S

General symmetry

Sym of a pt w.r to another point

To calculate the coo of S symmetric of L(-2,1) w.r to H(0,1) we use the midpoint rule on H.

(Rk:through calculation there'll be "criss cross")

solution

H is the midpoint of [LS] by symmetry so

$$x_H = \frac{x_L + x_S}{2} \qquad\qquad y_H = \frac{y_L + y_S}{2}$$

$$\frac{0}{1} = \frac{-2 + x_S}{2} \qquad\qquad \frac{1}{1} = \frac{1 + y_S}{2}$$

$$-2 + x_S = 0 \qquad\qquad 1 + y_S = 2 \; ; \; y_S = 1$$

$$x_S = 2$$

S(2,1)

How to ??

1) Prove three collinear points A, B and C?

Check their abscissas if $x_A = x_B = x_C$
then A, B and C are collinear

Check their ordinates if $y_A = y_B = y_C$
then A, B and C are collinear

If A, B and C have different abscissas and ordinates then check if this rule is verified

$$\frac{y_A - y_B}{x_A - x_B} = \frac{y_C - y_B}{x_C - x_B}$$

Remark : this rule consists on proving $a_{BA} = a_{BC}$ which means proving that (AB) //(BC) with a common point

How to ??

2) Prove a point A belongs / is on a straight line (d)? or (d) passes through A

We replace the coo of A in the equation of (d)

- True eq → A is on (d)
- False eq → A is not on d

exp: prove that $(d): y = -x + 2$ passes through $F(-2, 4)$

sol: $y_F = -x_F + 2$?

$4 = -(-2) + 2$?

$4 = 2 + 2$?

$4 = 4$ True

So F is on (d) since the coo of F verify the equation of (d)

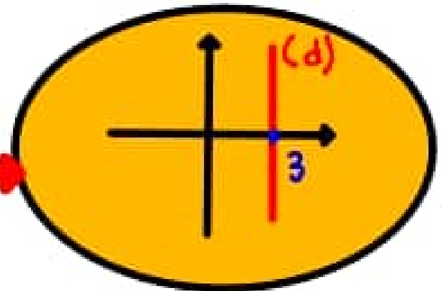
KEEP IN MIND THAT

When they tell you that A is on (d) they MEAN THAT THE COO OF A VERIFY THE EQUATION OF (d) and you can replace them in it once needed

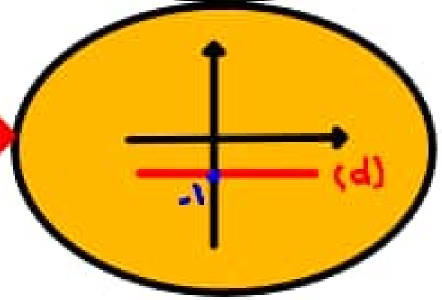
How to ??

3) Draw a straight line given its equation

The equation contains only x as (d):x=3 → **We draw the parallel to (y'y) at x=3**



The equation contains only y as (d):y=-1 → **We draw the parallel to (x'x) at y=-1**



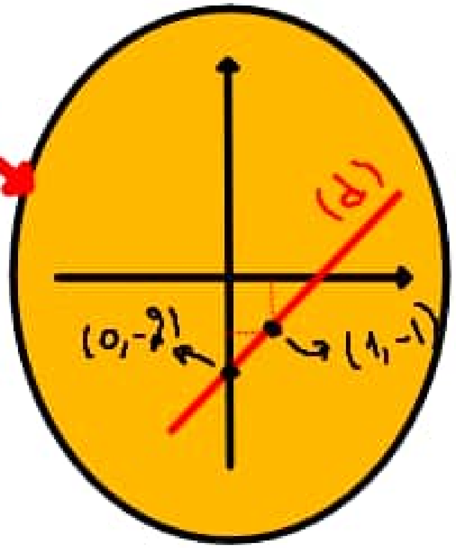
The equation contains y and x as (d):y=x-2 → We need to find 2 points of (d)
We can set the table of points(to obtain pointswe give x a value and we replace it in the eq of (d) to get y)

created

calculated

x	0	1
y	-2	-1
point	(0,-2)	(1,-1)

Now we plot these 2 points and we join them to draw (d)



How to ??

4) Prove (d) $\perp$ (d')?

→ Calculate a_d

→ Calculate $a_{d'}$

→ Calculate $a_d \times a_{d'} = \cdots = -1$

5) Prove (d) // (d')?

→ Calculate a_d

→ Calculate $a_{d'}$

→ $\boxed{a_d = a_{d'}}$

How to ??

6) Cacluate the coordinates of S where (d): y=3x-2 cuts (x'x) in S?

(d) cuts (x'x) in S $\rightarrow$ S is on (x'x) $\rightarrow$ $\boxed{y_S = 0}$

(d) cuts (x'x) in S $\rightarrow$ S is on (d) $\rightarrow$ the co of S verify the eq of (d)

So $y_S = 3x_S - 2$

$0 = 3x_S - 2$

$2 = 3x_S$; $\boxed{x_S = \frac{2}{3}}$

So $S\left(\frac{2}{3} ; 0\right)$

How to ??

7) Cacluate the coordinates of S where (d): y=3x-2 cuts (y'y) in S?

(d) cuts (y'y) in S

S is on (y'y) → $\boxed{x_S = 0}$

S is on (d) → the coo of S verify the eq of (d) So

$y_S = 3x_S - 2$

$y_S = 3(0) - 2$

$y_S = -2$

So $S(0;-2)$

Rk: We could have simply said that the y-intercept of (d) is -2 so (d) cuts (y'y) in S(0,-2)

How to ??

8) Cacluate the coordinates of S where

(d): $y=3x-2$ cuts (d'): $y=x+1$ in S?

(the coo of the intersection point of (d) and (d'))

(d) cuts (d') in S → S is on (d) → $y_S = 3x_S - 2$

→ S is on (d') → $y_S = x_S + 1$

Equate $y_S^{\text{in (d)}} = y_S^{\text{in (d')}}$ → $3x_S - 2 = x_S + 1$; $2x_S = 3$; $\boxed{x_S = \frac{3}{2}}$

Substitute $x_S = \frac{3}{2}$ in the equation of (d') → $y_S = x_S + 1$; $y_S = \frac{3}{2} + 1$

$\boxed{y_S = \frac{5}{2}}$

$S\left(\frac{3}{2} ; \frac{5}{2}\right)$

Remark: we can substitute x_S in the eq of (d) instead of (d')

How to ??

9) Cacluate the coordinates of S where

S is a point of (d): $y = -x-4$ of abscissa -1

s is on (d) so its coo verify the eq of (d) so $y_s = -x_s - 4$

$x_s = -1$

sol • $\boxed{x_s = -1}$ given

• S is a point of (d) so its coo verify the eq of (d) so $y_s = -x_s - 4$

$y_s = -(-1) - 4$

$\boxed{y_s = -3}$

Thus $S(-1, -3)$

How to ??

10) Prove that $(\Delta): 3y - 5x = 30$ is the perpendicular bisector of [AB] where A(-7,4) and B(-2,1)

→ **Prove that (AB) ⊥ (Δ) (we caclulate** a_{AB} **and** a_{Δ} **then** $a_{\Delta} \times a_{AB} = \dots = -1$**)**

→ **We calculate the coo of N the midpoint of [AB] if we didn't have it (the midpoint rule is** $x_N = \frac{x_A + x_B}{2}$ **and** $y_N = \frac{y_A + y_B}{2}$ **)**

we skip this step if we have the midpt

→ **We prove that (Δ) passes through N (we replace the coo of N is the eq of (Δ) and we should get a true eq)**

sol: $(\Delta): 3y - 5x = 30$; $3y = 5x + 30$; $y = \frac{5x + 30}{3}$; $y = \frac{5}{3}x + 10$

So $\boxed{(\Delta): y = \frac{5}{3}x + 10}$ reduced form.

→ $a_{\Delta} = \frac{5}{3}$ (since $(\Delta): y = \frac{5}{3}x + 10$) ; $a_{AB} = \frac{y_A - y_B}{x_A - x_B}$ (we have the coo of A + B)

$= \frac{4 - 1}{-7 - (-2)}$

$\boxed{a_{AB} = \frac{3}{-5}}$; $a_\Delta \times a_{AB} = \frac{5}{3} \times \frac{3}{-5} = -1$

A(-7, 4)
B(-2,1)
(Δ): $y = \frac{5}{3}x + 10$

So $(\Delta) \perp (AB)$

→ Let N be the midpoint of [AB].

$x_N = \frac{x_A + x_B}{2} = \frac{-7-2}{2} = \frac{-9}{2}$; $y_N = \frac{y_A + y_B}{2}$; $y_N = \frac{4+1}{2} = \frac{5}{2}$

So $\boxed{N\left(-\frac{9}{2}, \frac{5}{2}\right)}$

→ (Δ) passes through N ? coo of N verify the eq of (Δ) ?

$\frac{5}{2} = \frac{5}{\cancel{3}}\left(-\frac{\overset{3}{\cancel{9}}}{2}\right) + 10$?

$\frac{5}{2} = -\frac{15}{2} + \frac{10^{\times 2}}{1_{\times 2}}$?

$\frac{5}{2} = \frac{-15}{2} + \frac{20}{2}$?

$\frac{5}{2} = \frac{5}{2}$ ✓

So (Δ) passes through N

Therefore ( ) is the perpendicular biscetor of [AB] since we proved it is the perpendicular at its midpoint

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EQUATION of a straight line

Given 2 points A and B of (d)

If $x_A = x_B$

The equation of the line is

$$\boxed{(AB): x = x_A}$$

Exemple

Given A(-3,1) and B(-3, 5). Write an equation of the line (AB).

sol $x_A = x_B = -3$ so the eq of (AB) is

$$\boxed{(AB): x = -3}$$

If $x_A \neq x_B$

The equation of the line is obtained through the **GOLDEN RULE:**

$$\boxed{\frac{y - y_A}{x - x_A} = \frac{y_B - y_A}{x_B - x_A}}$$

Exp: Given A(-1,3) and B(2,0). Write an equation of (AB).

$x_A \neq x_B$ so the eq of (AB):

$$\frac{y - y_A}{x - x_A} = \frac{y_B - y_A}{x_B - x_A}$$

$$\frac{y-3}{x+1} = \frac{0-3}{2+1}$$

$$3y - 9 = -3x - 3$$

$$3y = -3x + 6$$

$$\boxed{(AB): y = -x + 2}$$

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EQUATION of a straight line

Given 1 point A and the slope of (d)

Rk: if the slope does not exist then the eq of (d) is in the form (d):x= nb, where the number is the abscissa of any point on (d)

Apply the rule of slope-point equation

$$\boxed{y - y_A = a_d (x - x_A)}$$

Exp: Given A(-3,2) and B(0,1) Write an equation of the line (d) perpendicular to (AB) at $\underbrace{S(3,5)}_{\text{pt of (d)}}$

① a_d? (d) ⊥ (AB) so $a_d \times a_{AB} = -1$

↳ $a_{AB} \times a_d = -1$

But $a_{AB} = \frac{y_B - y_A}{x_B - x_A} = \frac{1-2}{0+3} = \frac{-1}{3}$

So $a_d \times \frac{-1}{3} = -1$; $a_d = \frac{-1}{\frac{-1}{3}}$

$\boxed{a_d = 3}$. ② S(3,5) a pt of (d)

So pt-slope eq of (d) is

(d): $y - y_s = a_d(x - x_s)$; $y - 5 = 3(x-3)$

$\boxed{(d): y = 3x - 4}$

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EQUATION of vertical and horizontal lines

- The equation of $(x'x)$ is $(x'x): y=0$
  (since all points on $(x'x)$ have 0 as ordinate)

- The equation of $(y'y)$ is $(y'y): x=0$
  (since all points on $(y'y)$ have 0 as abscissa)

- The equation of a line (d) // $(x'x)$ is $(d): y=b$ where
  b is the y-intercept of (d) or the ordinate of any point on (d)

- The equation of a line (d) // $(y'y)$ is $(d): x=m$ where
  m is the abscissa of of any point on (d)