CH-1: Foundations of Sets

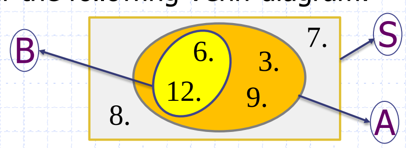
10^{th} Grade

Objectives of session 2:

- Subsets.
  - ✓Introduction & Definition.
  - ✓Properties.
- Equal sets.
- Complement of a set.
- Operations on sets.

Introductive - activity:

Consider the following Venn-diagram:



1. List elements that belong to set:
   1) A = $\{3, 12, 6, 9\}$
   2) B = $\{12, 6\}$
2. Can we say that all elements of B belong to A?
3. Is the converse true?

Subset: - Definition. - Properties.

A Set is said to be a **subset** (part or contained) of another if all of its elements belong to another.

In other words: If for every $x \in A$, we get $x \in B$ then we say that A is Subset of B.

✓In Symbols: $A \subset B$ means A is a subset of B.

✓ Properties:

- ❑ Every set is a subset of itself: $A \subset A$
- ❑ A void set is subset of every set: $\varphi \subset A$
- ❑ If $A \subset B$ and $B \subset D$, then $A \subset D$

Equal sets:

Consider the following sets:

$A = \{x / x \in N \ \& \ x^2 - 4 = 0\}$

$B = \{x / x \in N \ \& \ x \text{ is an even prime}\}$

1) Write the sets A & B in extension.

$A = \{2\}$ & $B = \{2\}$

2) Is $A \subset B$? Justify. Yes, since $\forall x \in A, x \in B$ as well

3) Is $B \subset A$? Justify. Yes, since $\forall x \in B, x \in A$ as well

Conclusion: Two sets are equal if and only if (iff): $A \subset B$ and $B \subset A$

Complement of a set:
- In words.
- Applications.

Let E be the reference, such that $A \subset E$.

The **complement** of the set *A* in *E* is the set of all elements that are in *E* but not in *A* .

The complement of a set is denoted by: $\complement_E^A$ or $\overline{A}$

Eg: Find the complement of:

$A = \{x \mid x \text{ plays football in your class}\}$

$B = \{x \mid x \text{ is strictly taller than } 175cm\}$

$\bar{A} = \{x \mid x \text{ does'nt play football in your class}\}$

$\bar{B} = \{x \mid x \text{ is shorter than or equal to } 175cm\}$

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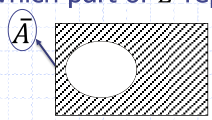
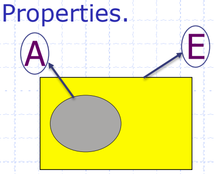
Complement of a set:
- Venn-diagram.
- In comprehension.
- Properties.

Take a set E, so that $A \subset E$.

Which part of E represents $\bar{A}$





In comprehension: $\overline{A} = \{ x \mid x \notin A \}$

Properties: If A & B are any two subsets of E then

- $\bar{E} = \varphi$ - $\bar{\varphi} = E$ - $\bar{\bar{E}} = E$

$A \subset B$ iff $\bar{B} \subset \bar{A}$

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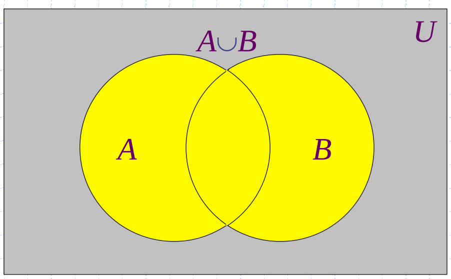
Union

Elements in at least one of the two sets:

$A \cup B = \{ x \mid x \in A$ or $x \in B \}$



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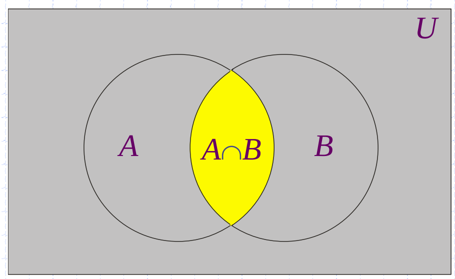
Intersection

Elements in exactly one of the two sets:

$A \cap B = \{ x \mid x \in A$ and $x \in B \}$



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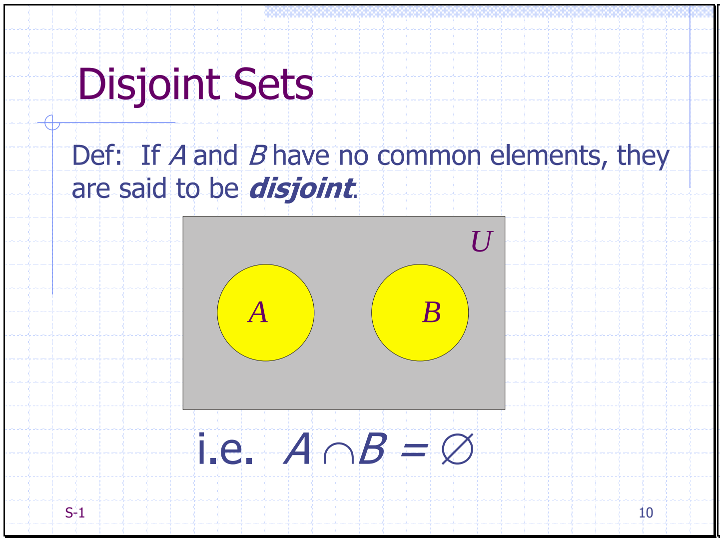
Disjoint Sets
Def: If A and B have no common elements, they are said to be disjoint.
U
A
B
i.e. A ∩ B = ∅
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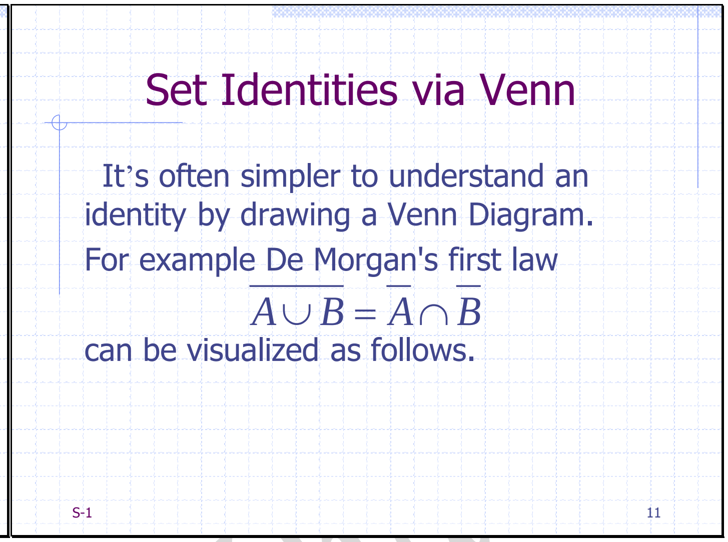
Set Identities via Venn
It's often simpler to understand an identity by drawing a Venn Diagram.
For example De Morgan's first law
$\overline{A \cup B} = \overline{A} \cap \overline{B}$
can be visualized as follows.
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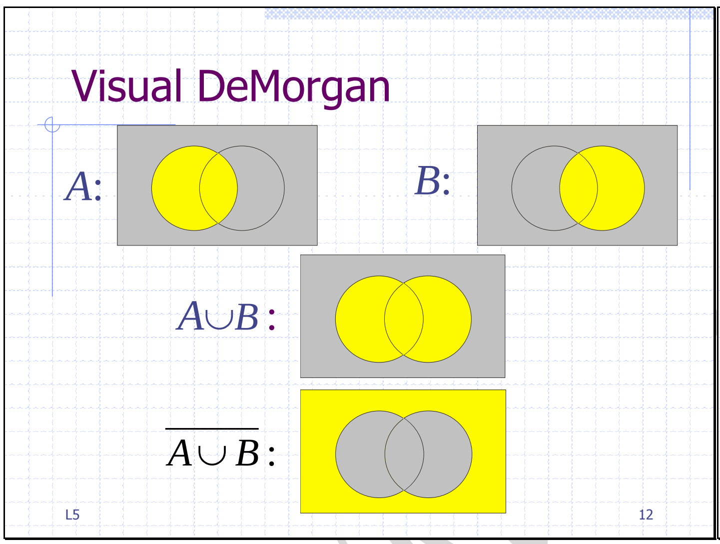
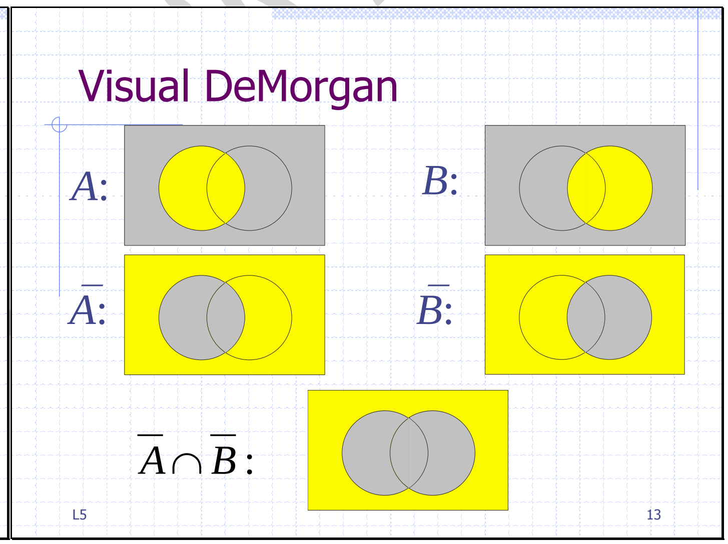
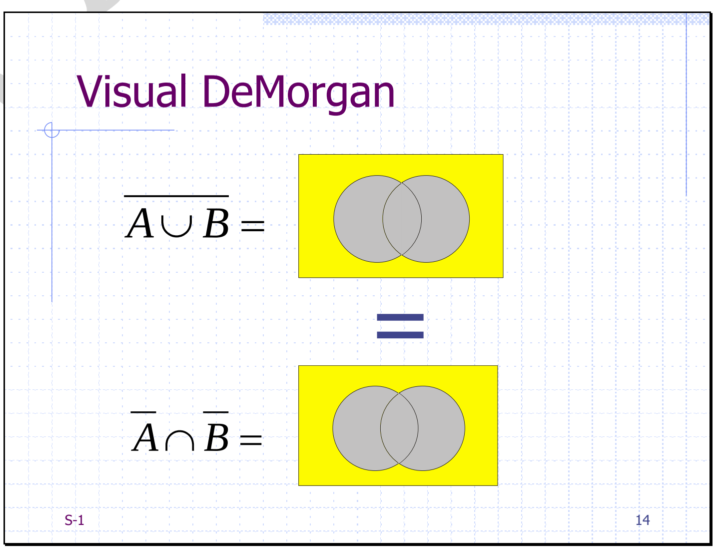

Visual DeMorgan
A:
B:
A∪B :
$\overline{A \cup B}$:
L5
12


Visual DeMorgan
A:
B:
$\overline{A}$:
$\overline{B}$:
$\overline{A} \cap \overline{B}$:
L5
13


Visual DeMorgan
$\overline{A \cup B} =$
=
$\overline{A} \cap \overline{B} =$
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