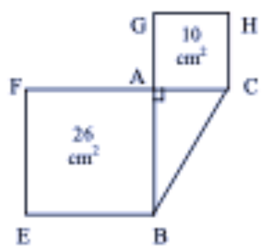
وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات	امتحانات الشهادة المتوسطة	دورة سنة ٢٠٠٤ العادية
عدد المسائل : ستة	مسابقة في الرياضيات المدة: ساعتان	الاسم: الرقم:

ملاحظة : يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو لاختزان المعلومات أو لرسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I- (2 points)

In the table below, only one among the proposed answers to each question is correct.
Write down the number of each question and give, with justification, the corresponding answer for your choice.

N°	Questions	Answers a	Answers b	Answers c
1)	$\frac{8}{15}+\frac{7}{15}\times\frac{2}{3}$ is equal to ...	$\frac{2}{3}$	$\frac{38}{45}$	$\frac{22}{15}$
2)	If each year prices increase by 10% then at the end of two years prices will increase by ...	100%	21%	20%
3)	 In the adjacent figure the area of square ABEF is 26 cm^2 and the area of square ACHG is 10cm^2. Then BC = ...	$(\sqrt{26}+\sqrt{10})$cm	$\sqrt{\sqrt{26}+\sqrt{10}}$ cm	6cm

II- (2 points)

In this problem, the unit of length is 1cm.
Given the three points A, B and C such that : $AB=\sqrt{108}$, $BC=\sqrt{48}$ and $AC=10\sqrt{3}$.

3) Calculate AB + BC giving the answer in the form $a\sqrt{3}$.
4) Are the points A, B and C collinear ? Justify.

III- (2½ points)

In this problem, the unit of length is 1 cm.
x and y are two positive numbers. ABC is a right triangle at A such that :
$AB = 2x + y$, $AC = x + y$ and $BC = 3x + y$.

The perimeter of triangle ABC is 24 and $\tan ABC = \frac{3}{4}$.

1) Justify that the preceding given is translated into the system $\begin{cases} 2x - y = 0 \\ 6x + 3y = 24 \end{cases}$
2) a- Calculate x and y showing all the steps followed.
   b- Deduce the length of the sides of triangle ABC.

IV- (3 points)

Consider the two expressions :
$A(x) = (x + 3)(4x + 7)$ and $B(x) = x^2 - 4 + (x - 2)(3x + 5)$.

4) Solve the equation $A(x) = 0$.

5) Prove that $B(x) = (x - 2)(4x + 7)$.

6) Given the expression $F(x) = \dfrac{x^2 - 4 + (x - 2)(3x + 5)}{(x + 3)(4x + 7)}$.

   a) Determine the values of x for which $F(x)$ is defined.
   b) Simplify $F(x)$; then solve the equation $F(x) = 2$.
   c) Does the equation $F(x) = -3$ admit a solution? Justify.

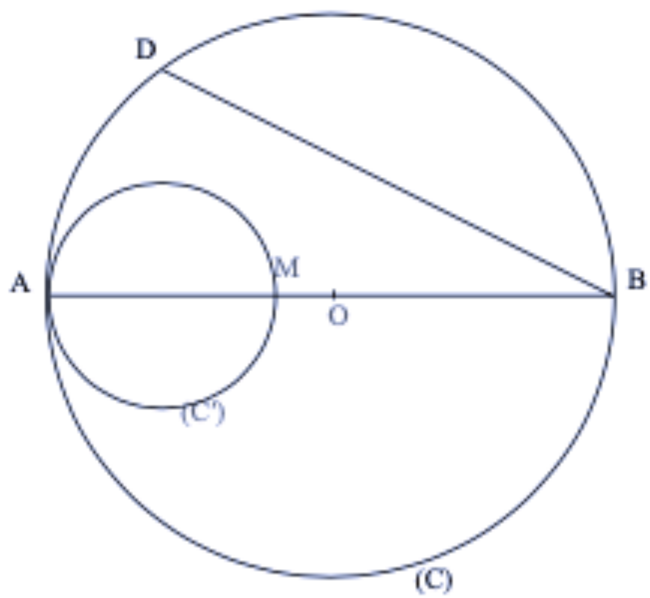
V- (6 points)

Consider in an orthonormal system of axes $x'Ox$, $y'Oy$, the points A(4 ; 2), B(-2 ; -2) and the line (d) of equation $y = -x + 4$.

1) Draw (d) and plot A and B.
2) Calculate the coordinates of point G the midpoint of segment [OA].
3) a- Determine the equation of the straight line (OA).
   b- Let (Δ) be the perpendicular bisector of segment [OA].
   Show that the equation of (Δ) is $y = -2x + 5$.
4) Let M be the intersection point of the two straight lines (Δ) and (d).
   a- Justify that MO = MA.
   b- Calculate the coordinates of M.
   c- Prove that the triangle MOA is a right isosceles triangle.

5) Designate by N the image (translate) of M by the translation of vector $\overrightarrow{OB}$.
   Prove that NB = MA.

VI- (4 ½ points)



In the opposite figure, we have :

- AB = 8 cm
- (C) is the circle of diameter [AB] and center O.
- M is the point of segment [AO] such that AM = 3cm.
- (C') is the circle of diameter [AM]
- D is a point of (C) such that BD = 7 cm.
- (C) and (C') are tangent at A.

1) Reproduce the figure.

2) Justify that ADB is a right triangle and calculate, rounded to the nearest degree, the measure of angle ABD.

3) The straight line (AD) cuts circle (C') in a second point E. Prove that (BD) and (ME) are parallel, then calculate EM.

4) The common tangent at A to (C) and (C') cuts the straight line (BD) in N. Choose two triangles and prove that they are similar, then deduce that $AN^2 = ND \times NB$.

5) Let F be the point such that $\overrightarrow{DF} = \overrightarrow{DA} + \overrightarrow{DB}$.
   Prove that the quadrilateral DAFB is a rectangle. Deduce that F belongs to circle (C).

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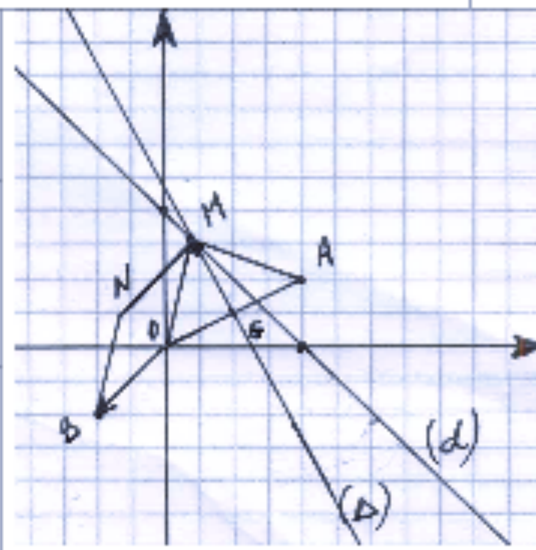
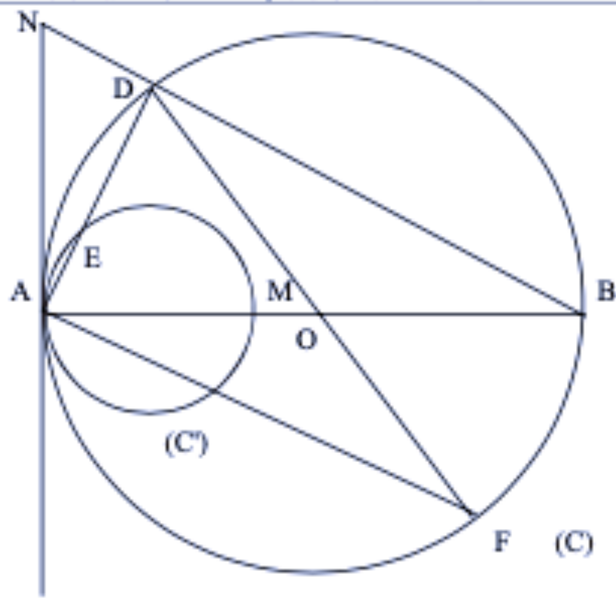
المديرية العامة للتربية
دائرة الامتحانات

مادة الرياضيات

Barème - EB9

Question		Barème provisoire		Note
I	N° 1	La réponse correcte est b. Justification : $\frac{8}{15}+\frac{7}{15}\times\frac{2}{3}=\frac{8}{15}+\frac{14}{45}=\frac{24+14}{45}=\frac{38}{45}$		
	N° 2	La réponse correcte est b. Justification : si x est le prix initial, au bout d'une année le prix est 1,1x x ; au bout de deux années le prix est 1,1 x (1,1 x) c'est-à-dire 1,21 x. Les prix auront augmenté de 21%.		
	N° 3	La réponse correcte est c. Justification : ABC est un triangle rectangle en A. Alors $BC^2 = AB^2 + AC^2$. Or $AB^2 = 26$ cm² et et $AC^2 = 10$ cm². D'où $BC^2 = 26 + 10 = 36$ cm² et BC = 6 cm.		
II	1) 2)	$AB + BC = \sqrt{48} + \sqrt{108} = 4\sqrt{3} + 6\sqrt{3} = 10\sqrt{3}$ A, B et C sont alignés car AB + BC = AC		
III	1)	• AB + AC + BC = (2x + y) + (x + y) + (3x + y) = 6x + 3y. D'où 6x + 3y = 24. C'est la deuxième équation du système. • Tan $\widehat{ABC} = \frac{AC}{AB} = \frac{x+y}{2x+y}$ (ABC est rectangle en A) D'où $\frac{x+y}{2x+y} = \frac{3}{4}$, alors 4 (x + y) = 3 (2x + y). 4x + 4y = 6x + 3y. 2x – y = 0 C'est la deuxième équation du système.		
	2)	a)	Le système donné s'écrit $\begin{cases} 2x - y = 0 \\ 2x + y = 8. \end{cases}$ D'où 4 x = 8 et x = 2 x = 2 et 2x – y = 0, alors y = 4.	
		b)	AB = 8 , AC = 6 et BC = 10.	
IV	1)	A(x) = 0, alors (x + 3) (4x + 7) = 0 (x + 3) = 0 ou 4x + 7 = 0 x = -3 ou $x = -\frac{7}{4}$.		
	2)	B(x) = (x – 2) (x + 2) + (x – 2) (3x +5) = (x-2) (x+2+3x+5) B(x) = (x-2)(4x +7).		
	3-	a)	• $F(x) = \frac{x^2 - 2 + (x-2)(3x+5)}{(x+3)(4x+7)}$ • F(x) est définie si l'on a: $x + 3 \neq 0$ et $4x + 7 \neq 0$, Alors : $x \neq -3$ et $x \neq -\frac{7}{4}$.	
		b)	• $F(x) = \frac{(x-2)+(4x+7)}{(x+3)(4x+7)} = \frac{x-2}{x+3}$	

<table>
<tr><td rowspan="2"></td><td rowspan="2"></td><td></td><td colspan="2">• F (x) = 2 alors $\frac{x-2}{x+3} = 2$
$x - 2 = 2(x + 3)$
$x = -8$</td><td></td></tr>
<tr><td>c)</td><td colspan="2">F (x) = -3 alors $\frac{x-2}{x+3} = -3$
$x - 2 = -3(x + 3)$
$x = -\frac{7}{4}$
F(x) n'est pas définie en $-\frac{7}{4}$, alors l'équation F(x) = - 3 n'admet pas de solution.</td><td></td></tr>
<tr><td rowspan="4">V</td><td>1)</td><td colspan="2">Pour tracer (d):
<table><tr><td>x</td><td>0</td><td>4</td></tr><tr><td>y</td><td>4</td><td>0</td></tr></table></td><td colspan="2" rowspan="3">
</td></tr>
<tr><td>2)</td><td colspan="2">$x_G = \frac{x_A + x_0}{2} = \frac{4}{2} = 2$
$y_G = \frac{y_A + y_0}{2} = \frac{2}{2} = 1$
G (2 ; 1)</td></tr>
<tr><td rowspan="2">3-</td><td>a)</td><td>L'équation de (0A) est de la forme y = ax.
Les coordonnées de A vérifient cette équation :
2 = 4a.
D'où a = $\frac{1}{2}$.
L'équation de (0A) est
$y = \frac{1}{2}x$</td></tr>
<tr><td>b)</td><td colspan="2">(Δ) n'est pas parallèle à y'y, alors son équation est de la forme y = ax + b
(Δ) est perpendiculaire à (0A), alors $a \times \frac{1}{2} = -1$. D'où a = -2.
L'équation de (Δ) s'écrit alors y= - 2x + b.
(Δ) passe par G, alors $y_G = -2x_G + b$
$1 = -4 + b$
$b = 5$
L'équation de (Δ) est donc y = - 2x + 5.</td><td></td></tr>
</table>

V	4)	a)	M est un point de la médiatrice de [OA], alors M est équidistant de O et A. D'où MO = MA.
		b)	$\begin{cases} y = -2x + 5 \\ y = -x + 4 \end{cases}$ $-2x + 5 = -x + 4$, alors $x = 1$ et $y = 3$. d'où M (1 ; 3).
		c)	▪ Soit a le coefficient directeur de (OM) et a ' celui de (AM). $a = \dfrac{y_M - y_O}{x_M - x_O} = 3$, $a' = \dfrac{y_A - y_M}{x_A - x_M} = \dfrac{2-3}{4-1} = -\dfrac{1}{3}$. $a \times a' = -1$, alors (OM) est perpendiculaire à (AM). ▪ D'après 4 – a) : MO = MA , alors OMA est isocèle en M. ▪ D'où OMA est rectangle et isocèle.
	5)		N est le translaté de M par la translation de vecteur $\overrightarrow{OB}$, alors $\overrightarrow{MN} = \overrightarrow{OB}$ et OMNB est un parallélogramme. D'où NB = MO. Comme on a MO = MA , alors NB = MA.
VI	1		
	2		▪ [AB] est un diamètre de (C) et D est un point de ce cercle, alors ADB est rectangle en D. ▪ $\cos \widehat{ABD} = \dfrac{BD}{AB} = \dfrac{7}{8}$. La calculatrice affiche 28^o , 95502437. La mesure de l'angle $\widehat{ABD}$ arrondie au degré près est 29^o.
	3		▪ [AM] est un diamètre de (C') et E est un point de ce cercle, alors AME est rectangle en E. ▪ (ME) ⊥ (AE) et (BD) ⊥ (AD), alors (ME) est parallèle à (BD). ▪ (DA) et (DB) se coupent en D , et (ME) est parallèle à (BD), alors et d'après la propriété de Thalès : $\dfrac{ME}{BD} = \dfrac{AM}{AB}$. D'où $ME = \dfrac{BD \times AM}{AB} = \dfrac{3 \times 7}{8}$. $ME = \dfrac{21}{8}$ cm.

	4)	▪ Choisissons les deux triangles ADN et BAN. ▪ ADN est rectangle en D et BAN est rectangle A, $\hat{N}$ est un angle commun à ces deux triangles. ▪ Ces deux triangles sont alors semblables car deux angles de l'un sont respectivement égaux à deux angles de l'autre. ▪ $\left.\begin{array}{l}\text{ADN}\\ \\ \text{BAN}\end{array}\right\} \frac{AD}{BA} = \frac{AN}{BN} = \frac{DN}{AN}$, alors $AN^2 = ND \times NB$.
	5)	▪ $\overrightarrow{DF} = \overrightarrow{DA} + \overrightarrow{DB}$, alors DAFB est un parallélogramme. Dans ce parallélogramme, $\widehat{ADB} = 90^o$. Alors DAFB est un rectangle. ▪ Dans le rectangle DAFB , $\widehat{AFB} = 90^o$. [AB] est un diamètre du cercle (C), alors F est sur ce cercle.

وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات	امتحانات الشهادة المتوسطة	دورة سنة ٢٠٠٤ الإكمالية الاستثنائية
عدد المسائل : سبعة	مسابقة في الرياضيات المدة: ساعتان	الاسم: الرقم:

ملاحظـة : يُسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو لاختزان المعلومات أو لرسم البيانات
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I- (1½ point)

Given the inequality : $2x + 1 \leq 5(x - 1) + 15$.

Solve this inequality and represent the solutions on an axis of origin O.

II- (2 points)

Consider the following numbers A, B and C .

$A = \frac{1}{5} - \left(\frac{2}{5}\right)^2$, $B = (2 - \sqrt{5})^2 + 2(8 + \sqrt{20})$, $C = \frac{-1.25 \times 8 \times 10^7 \times 10^{-4}}{4 \times 10^2}$.

1) Calculate A, B and C showing all the steps of calculation and give each result in its simplest possible form.
2) From the numbers A, B and C, choose two opposite numbers and two numbers inverses of each other.

III-(2½ points)

1) Determine the numerical values of a and b so that the numbers 1 and 2 are the roots of the polynomial $P(x) = ax^2 + bx + 2a - 3b - 9$.
2) Given the polynomial $Q(x) = (x - 1)(x - 2)$.
   a- Show that $Q(x) - 2 = x(x - 3)$.
   b- Solve the equation $Q(x) = 2$.

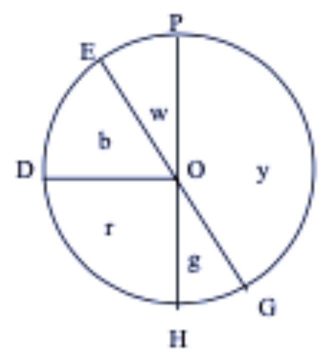
IV-(3 points)

The opposite circular diagram represents the distribution of marbles in a bag according to their colours :

red : r, green : g , yellow : y , white : w , brown : b.

[EG] and [PH] are two diameters of the circle,

$\widehat{DOE} = 60°$ and $\widehat{DOH} = 90°$.



1) Calculate the angles $\widehat{EOP}$, $\widehat{POG}$ and $\widehat{GOH}$.
2) Justify that the yellow colour is the most frequent.
3) Knowing that the number of the red marbles is 270, reproduce and complete the following table and verify that the number of marbles in the bag is 1080 :

Colour	r	g	y	w	b
Frequency	270				

4) Calculate the percentage of the red marbles.

V- (3 ½ points)

ABC is a triangle right angled at A such that AB = 4cm and AC = 6cm. M is the midpoint of [AC].

1) Calculate BM.
2) Calculate, rounded to the nearest degree, the angle $\widehat{ABM}$ and deduce the angle $\widehat{BMC}$.
3) Let E be the symmetric of B with respect to M.
   a- Place E and determine the nature of quadrilateral CBAE.
   b- Calculate the area of CBAE.
4) Let G be the symmetric of B with respect to the straight line (AC).
   a- Place G and determine the nature of quadrilateral GECA.
   b- Calculate the area of quadrilateral BCEG.

VI-(2 ½ points)

Two perpendicular straight lines (d) and (d') intersect in a point O. The circle (C) of center O and radius 4 cuts (d) in A and B. Let M be a point of (C) distinct from A and let L be the midpoint of [AM]. The line (AM) cuts (d') in N.

1) Draw a figure.
2) a- What is the nature of triangle OLA ? Justify.
   b- Find the locus of L when M moves on the circle (C).
3) a- Prove that the two triangles OAN and MAB are similar.
   b- Deduce that the product $AM \times AN$ remains constant when M moves on circle (C).

VII-(5 points)

In an orthonormal system of axes x'Ox, y'Oy, consider the point C(0 ; 3) and the straight line (D) of equation $y = \frac{1}{2}x - 2$.

1) (D) cuts x'Ox in A and y'Oy in B. Calculate the coordinates of A and B, and draw (D).
2) The perpendicular (D') drawn from C to (D) cuts the straight line (D) in I.
   a- Find the equation of (D').
   b- Calculate the coordinates of I.
3) Let E be the point such that $\overrightarrow{AE} = \overrightarrow{AB} + \overrightarrow{AC}$.
   a- What is the nature of quadrilateral ABEC ?
   b- Calculate the coordinates of E.
4) Let M(0 ; m) be a point on y'Oy , where m is a positive number.
   a- Calculate the numerical value of m such that triangle ABM is right at A.
   b- For this value of m, the circle of diameter [MB] cuts again the axis x'Ox in a point H . What are the coordinates of H ? Justify.

دورة سنة ٢٠٠٥ العادية	امتحانات الشهادة المتوسطة	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات
الاسم: الرقم:	مسابقة في الرياضيات المدة: ساعتان	عدد المسائل : سبعة

ملاحظة: يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو لاختزان المعلومات أو لرسم البيانات.
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I- (1 point)

Solve the following inequality :

$$4(2x - 1) \geq 9x - 7.$$

II- (1 ½ points)

The students of a school are distributed in the following way :

- 47 % are in the elementary section.
- 27 % are in the intermediate section.
- 130 students are in the secondary section.

1) What is the percentage of students in the secondary section ?
2) Calculate the number of students of this school.

III- (2½ points)

Given the expression : $E = (2x + 3)^2 + (x - 1)(2x + 3)$.

1) Expand and reduce E .
2) Calculate the exact value of E for $x = \sqrt{2}$.
3) Factorize E.
4) Solve the equation : $(3x + 2)(2x + 3) = 0$.

IV- (2½ points)

1) Solve the following system, showing all the steps of calculation :

$$\begin{cases} x + y = 11 \\ 2x + 5y = 34 \end{cases}$$

2) A survey was made to find the number of books read by the students of a certain class. The results are grouped in the following statistical table.

Number of read books	1	2	3	4	5	6
Number of students	5	x	4	3	y	2

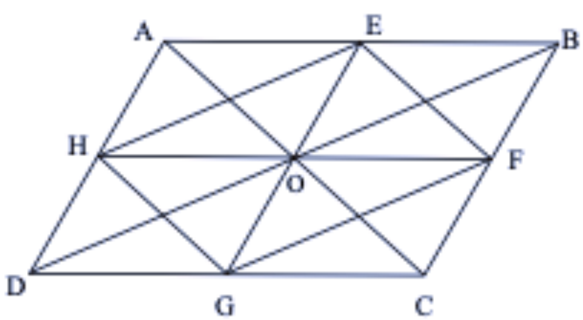
We know moreover that the number of students of this class is 25 and the mean of read books is 3.
Calculate x and y.

V- (2 ½ points)

Remark :

It is not required to reproduce the opposite figure.

In this figure, ABCD is a parallelogram of center O and the points E, F, G and H are the midpoints of the sides.



Reproduce and complete the following phrases.

1) The symmetrical of triangle GOD about point O is the triangle … .
2) The image (translate) of E by the translation of vector $\overline{AO}$ is the point … .
3) The point F is the image (translate) of point … by the translation of vector $\overrightarrow{DO}$.

4) $\overrightarrow{FE} + ... = \overrightarrow{FG}$.

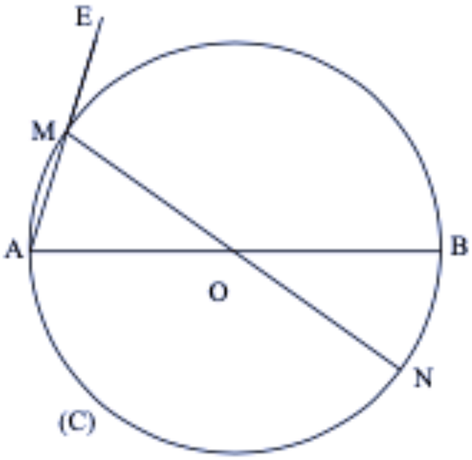
5) $\overrightarrow{AE} + \overrightarrow{AH} = ...$

6) $\overrightarrow{FE} + \overrightarrow{BC} = ...$

VI- (4 ½ points)

In the opposite figure :

- (C) is a circle of center O, [AB] is a fixed diameter of (C) such that AB = 6cm.
- [MN] is a variable diameter of (C).
- E is the symmetrical of A with respect to M.



1) Reproduce this figure.
2) a- Prove that (OM) and (BE) are parallel.
   b- Prove that (BM) is the perpendicular bisector of [AE] .
   c- Prove that triangle ABE is isosceles of principal vertex B.
   d- Prove that when M moves on (C), the point E moves on a fixed circle whose center and length of radius are to be determined.
3) Let I be the intersection point of the straight lines (EN) and (AB).
   a- Prove that the two triangles ION and IBE are similar and deduce that : IB = 2 × IO.
   b- Calculate IO and IB.
   c- Is I the center of gravity of triangle MBN ? Justify.
   d- (EN) cuts (MB) in F. Prove that (OF) is perpendicular to (MB).

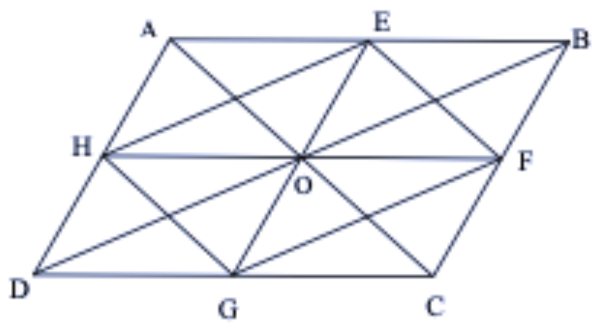
VII- (5 ½ points)

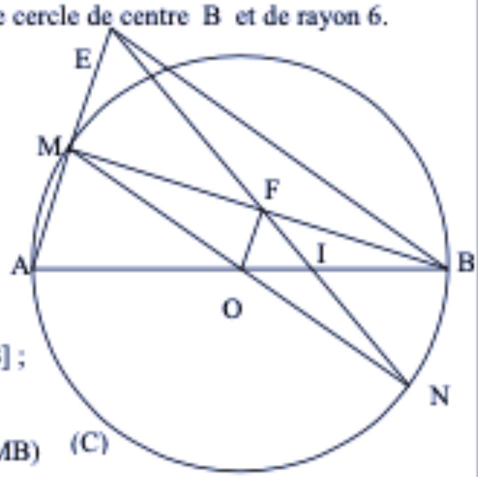
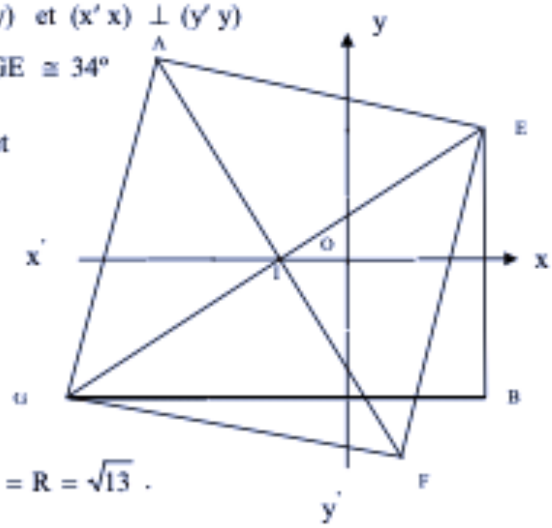
Consider in an orthonormal system of axes x′ O x , y′ O y, the points :

A(– 3 ; 3), B(2 ; – 2), G(– 4 ; – 2) and E(2 ; 2).

1) Plot the points A, B, G and E.
2) a- Justify that the straight line (BE) is parallel to (y′ y) and that the straight line (BG) is parallel to (x′ x).
   b- Prove that the triangle BGE is right angled at B.
   c- Calculate $\tan \widehat{BGE}$ and calculate, rounded to the nearest degree, the angle $\widehat{BGE}$.
3) Designate by (C) the circle circumscribed about triangle BGE, prove that its center is the point I(– 1 ; 0), and calculate the exact value of its radius.
4) Prove that A is a point of the circle (C).
5) a- Find the equation of the straight line (GE).
   b- Prove that (GE) and (AI) are perpendicular.
   c- Let F be the point such that $\overrightarrow{AE} + \overrightarrow{AG} = \overrightarrow{AF}$.
   Prove that the quadrilateral AGFE is a square.

توزيع علامات مسابقة الرياضيات

Questions		Eléments de réponses	Notes
I-		$8x - 4 \geq 9x - 7$; $-x \geq -3$ alors $x \leq 3$	¼ + ½ + ¼
II-	1)	$100 - (47 + 27) = 26$ soit 26 %	¾
	2)	$\frac{26}{100} = \frac{130}{N}$ alors $N = \frac{130 \times 100}{26} = 500$	¾
III-	1)	$E = 4x^2 + 9 + 12x + 2x^2 + 3x - 2x - 3$ $= 6x^2 + 13x + 6$	¼ ¼
	2)	$E\left(\sqrt{2}\right) = 6\left(\sqrt{2}\right)^2 + 13\sqrt{2} + 6 = 18 + 13\sqrt{2}$	½
	3)	$E = (2x + 3)(2x + 3 + x - 1)$ $= (2x + 3)(3x + 2)$	½ ¼
	4)	$2x + 3 = 0$ alors $x = -\frac{3}{2}$ ou $3x + 2 = 0$ alors $x = \frac{-2}{3}$	¾
IV-	1)	$\begin{cases} -2x - 2y = -22 \\ 2x + 5y = 34 \end{cases}$, $3y = 12$ alors $y = 4$ et $x = 7$.	½ , ¼ + ¼
	2)	$5 + x + 4 + 3 + y + 2 = 25$; $\frac{1 \times 5 + 2 \times x + 3 \times 4 + 4 \times 3 + 5 \times y + 6 \times 2}{25} = 3$ $x + y = 11$ $2x + 5y = 34$ alors $x = 7$ et $y = 4$.	½ ; ½ ½
V-	1) 2) 3) 4) 5) 6)	EOB F G $\overrightarrow{FE} + \overrightarrow{EG} = \overrightarrow{FG}$ $\overrightarrow{AE} + \overrightarrow{AH} = \overrightarrow{AO}$ $\overrightarrow{FE} + \overrightarrow{BC} = \overrightarrow{FG}$  	¼ ¼ ½ ½ ½ ½

Questions		Eléments de réponses	Notes
VI	1)	Figure	¼
	2) a-	Théorème des milieux … O milieu de [AB] et M milieu de [AE]	½
	b-	BMA = 90° et M milieu de [AE]	½
	c-	B appartient à la médiatrice de [AE]	¼
	d-	B fixe BE = BA = 6 alors E décrit le cercle de centre B et de rayon 6.	½
	3) a-	(ON) // (BE) ou …	½
		$\frac{IO}{IB} = \frac{ON}{BE} = \frac{3}{6} = \frac{1}{2}$ alors IB = 2IO	½
	b-	IB = 2IO ; IO = 1 et IB = 2	¼ + ¼
	c-	[BO] médiane et $\frac{IO}{BO} = \frac{1}{3}$ ou …	½
	d-	[MI] médiane alors F milieu de [MB] ; (OF) // (MA)	½
		Théorème des milieux alors (OF) ⊥ (MB) 	
VII	1)	Placer A, B, G et E	½
	2) a-	$x_B = x_E = 2$, $y_B = y_G = -2$	¼ + ¼
	b-	(BG) // (x′ x) ; (BE) // (y′ y) et (x′ x) ⊥ (y′ y)	½
	c-	tan BGE = $\frac{BE}{BG} = \frac{2}{3}$; BGE ≅ 34°	½ + ½
	3)	$x_I = \frac{x_G + x_E}{2} = -1$ et  $y_I = \frac{y_G + y_E}{2} = 0$, IE = R = $\sqrt{13}$.	¼ + ¼
	4)	IA = $\sqrt{13}$ = R.	½
	5) a-	a = $\frac{2}{3}$ et b = $\frac{2}{3}$ équation de (GE) : $y = \frac{2}{3}x + \frac{2}{3}$.	¾
	b-	$a_{(AI)} = \frac{y_I - y_A}{x_I - x_A} = -\frac{3}{2}$ alors $a_{(AI)} \times a_{(GE)} = -1$ donc (AI) ⊥ (GE)	½
	c-	$\overline{AE} + \overline{AG} = \overline{AF}$ donc AGFE est un parallélogramme GAE = 90° (AI) ⊥ (GE) alors AGFE est un carré.	¾

عدد المسائل : سبعة	مسابقة في : تايضايرلا ةدام المدة : ساعتان	الاسم: الرقم:

ملاحظة : يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I- *(1 point)*

Given that : $A = \dfrac{1 - \dfrac{2}{3}}{2 + \dfrac{1}{3}}$, $B = \dfrac{5 \times 10^8 \times 2 \times 10^3}{7 \times (10^4)^3}$.

Calculate A and B , showing all the steps of calculation, and deduce that A and B are two forms of the same number.

II-*(1 ½ points)*

Given that : $C = \dfrac{\sqrt{45} - \sqrt{80} + 2\sqrt{125}}{\sqrt{7} \times \sqrt{35} - 7\sqrt{5} + 3}$.

Calculate C and give the result in the form $a\sqrt{5}$ where a is an integer.

III-*(1 point)*

Let $E = \dfrac{4x + 7}{3}$.

1) Calculate the value of E for $x = \dfrac{7}{4}$.
2) Without solving the inequality $\dfrac{4x + 7}{3} < 5$, is the number $\dfrac{7}{4}$ a solution of this inequality?
Justify your answer.

IV- *(2 points)*

There are some missing numbers in the following text :
« For buying….. pencils and 2 pens we pay ….. L L , and for buying ….. pencils and 3 pens we pay 7800 L L. »
Setting up the complete given of the text in equations give the following system :

$$\begin{cases} 4x + 2y = 5600 \\ 2x + 3y = 7800 \end{cases}$$

1) Copy again the complete text according to the given system.
2) Solve, showing all the steps of calculation, the preceeding system and find the price of one pencil and the price of one pen.

V- *(4 points)*

In the plane of an orthonormal system of axes x ′ O x , y ′ O y, consider the points :

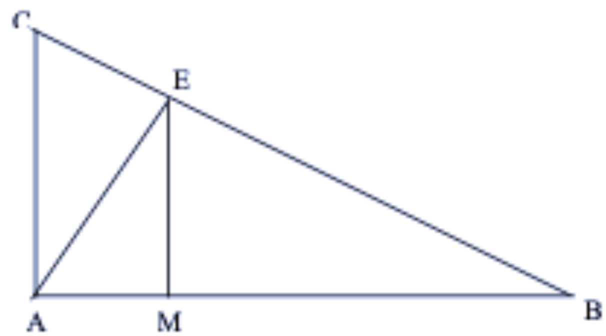
A(– 4 ; 4) , B(3 ; 3) and C(1 ; –1) .

1) Plot the points A, B and C.
2) Prove that the three points A, O and C are collinear.
3) Prove that the triangle ABC is isosceles of principal vertex A.
4) Let H be the midpoint of [BC] . Prove that (AH) is perpendicular to (BC).
5) Let N be the image (translate) of B by the translation of vector $\overrightarrow{AC}$. Prove that CABN is a rhombus.

VI- ***(5 ½ points)***

In the opposite figure where the unit of length is the centimeter :

- ABC is a triangle right angled at A
- AB = 8 and AC = 4
- M is a point of [AB] such that BM = x and $0 \le x \le 8$
- (ME) is perpendicular to (AB).



Part A

1) Prove that: $ME = \frac{1}{2}x$.
2) Calculate x so that the triangle AME is isosceles.

Part B

Consider an orthonormal system of axes x ' O x , y ' O y.

1) Draw, in this system, the straight line (d) of equation $y = \frac{x}{2}$ and the straight line (d′) of equation $y = -x + 8$.
2) Using the graph, find again the result of question 2) of **part A.**

Part C

1) Calculate the exact value of the length of side [BC] of triangle ABC.
2) Write the value of BC appearing on your calculator, then give this value rounded to the nearest 10^{-2} by default.
3) Calculate tan $\widehat{ABC}$, then calculate, rounded to the nearest degree, the angle $\widehat{ABC}$.

VII- ***(5 points)***

EBF is a triangle right angled at B such that EB = 6 cm, BF = 8 cm and FE = 10 cm. M is the midpoint of [BF] and (𝒞) is the circle of diameter [MF]. The circle (𝒞) cuts again [EF] in G. The straight lines (MG) and (EB) intersect in S.

1) Draw a figure.
2) Prove that the four points E, B, M and G belong to the same circle whose diameter is to be determined .
3) a- Prove that the two triangles EBF and MGF are similar and calculate MG and GF.
   b- Calculate the area of triangle MGF.
   c- Calculate the ratio of the areas of the two triangles EBF and MGF.
4) Let P be the point of intersection of (EM) with (SF).
   a- Prove that (EP) is perpendicular to (SF).
   b- Deduce that P is a point of circle (𝒞).

امتحانات الشهادة المتوسطة دورة سنة ٢٠٠٥ الاستثنائية

توزيع علامات مسابقة الرياضيات

Questions		Eléments de réponses	Notes
I-		$A = \frac{1}{7}$; $B = \frac{1}{7}$.	½ + ½
II-		$C = \frac{3\sqrt{5} - 4\sqrt{5} + 10\sqrt{5}}{7\sqrt{5} - 7\sqrt{5} + 3} = \frac{9\sqrt{5}}{3} = 3\sqrt{5}$.	1 ½
III-	a)	$E = \frac{14}{3}$	½
	b)	$\frac{7}{4}$ sol de l'inéquation car $\frac{14}{3} < 5$ ou ...	½
IV	1)	Pour acheter 4 crayons et 2 stylos on paye 5600 L L, et pour acheter 2 crayons et 3 stylos on paye 7800 L L.	½
	2)	$4y = 10000$; $y = 2500$; $x = 150$ prix d'un crayon 150 L L Prix d'un stylo 2500 L L .	1 ½
V-	1)	figure	½
	2)	équation (OA) : $y = - x$; C est un point de (OA) ou …	1
	3)	$AB = \sqrt{50} = 5\sqrt{2}$; $AC = \sqrt{50} = 5\sqrt{2}$; AB = AC donc ABC est un triangle isocèle en A.	1
	4)	[AH] mediane à la fois hauteur .	½
	5)	$\overrightarrow{AC} = \overrightarrow{BN}$ (CABN parallélogramme). AB = AC donc CABN est un losange ou … .	1
VI-	A- 1)	(ME) // (AC) ; $\frac{BM}{BA} = \frac{BE}{BC} = \frac{ME}{AC}$ (Thalés ou triangle semblables) $ME = \frac{x}{2}$.	¾
	2)	$8 - x = \frac{x}{2}$; $x = \frac{16}{3}$.	¾
	B- 1)	Tracé : (d) ; (d') .	1 ½
	2)	I est le point d'intersection de (d) et (d') ; $x_I = \frac{16}{3}$	½
	C- 1)	$BC^2 = 64 + 16 = 80$ $BC = 4\sqrt{5}$ cm	½
	2)	La valeur affichée est 8,94427191 La valeur approchée à 10^{-2} près par défaut est 8,94 cm	½

Questions		Eléments de réponses	Notes
Suite VI	C- 3)	Tan $\widehat{ABC} = \frac{AC}{BA} = \frac{4}{8} = 0{,}5$; ABC = $\tan^{-1}(0{,}5) = 26{,}56$; ABC ≅ 27°	1
VII-	1)	Figure	½
	2)	MGE = MBE = 90° ; GME et MBE 2 triangles de même hypotenuse [EM] B ; M ; G et E sur le même cercle de Diametre [ME].	¾
	3) a-	B = G = 90° ; F commun ; $\frac{GF}{BF} = \frac{FM}{FE} = \frac{GM}{BE} = \frac{2}{5}$ $GF = \frac{16}{5}$; $GM = \frac{12}{5}$	1¼
	3) b-	$\mathcal{A}(GMF) = \frac{GF \times GM}{2} = \frac{96}{25} = 3{,}84 \text{ cm}^2$	½
	3) c-	$\frac{A(GMF)}{A(EBF)} = \frac{GF \times GM}{BF \times BE} = \frac{4}{25} = \left(\frac{2}{5}\right)^2$ ou …	¾
	4) a-	[SG] et [BF] sont 2 hauteurs du triangle ESF alors [EP] est la 3ème hauteur.	¾
	4) b-	MPF = 90 ° [MF] diametre de ($\mathcal{C}$) donc P est un point de ($\mathcal{C}$).	½

وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات

امتحانات الشهادة المتوسطة

دورة سنة ٢٠٠٦ العادية

مسابقة في مادة الرياضيات
المدة: ساعتان

الاسم:
الرقم:

ملاحظة : - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو إختزان المعلومات أو رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة.

I- (2 points)

In the table given below, only one among the proposed answers to each question is correct.
Write down the number of each question and give, with justification, the answer corresponding to it.

N°	Questions	Answers		
		a	b	c
1°	$\frac{4}{5} - \frac{3}{5} \times \frac{10}{6} =$	$-\frac{1}{5}$	$\frac{26}{25}$	$\frac{1}{3}$
2°	$3^{14} - 3^{12} =$	3^2	$3^{12} \times 8$	6
3°	x is an acute angle such that $\cos x = \frac{1}{3}$, then $\sin x =$	$\frac{2}{3}$	$\frac{\sqrt{2}}{3}$	$\frac{2\sqrt{2}}{3}$
4°	An object costs 270LL. Its price is increased by 5%. The new price of the object is :	275 LL	270.05LL	283.5 LL

II- (1½ points)

Given the two numbers A and B :

$$A = \frac{3.6 \times 10^3 \times 10^{-5}}{9 \times 10^2} \quad \text{and} \quad B = \left(2+\sqrt{5}\right)^2 + \sqrt{5}\left(1+2\sqrt{5}\right).$$

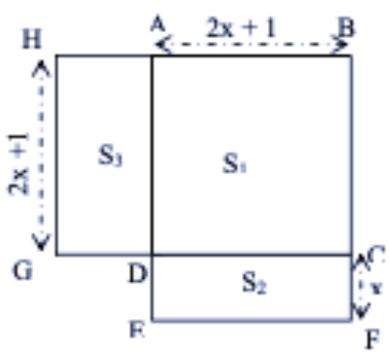
1) Write A in the form $a \times 10^n$ where a and n are two integers, then write A in the form of a decimal number.

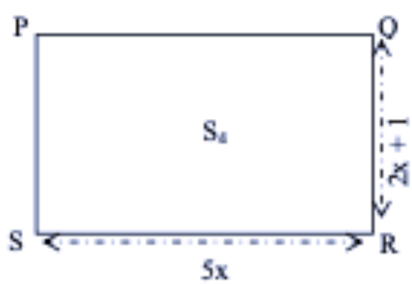
2) Write B in the form $b + c\sqrt{5}$ where b and c are two integers.

III- (2½ points)

1) Consider the expression : $E(x) = 4x^2 - 1 + (2x+1)^2 + x(2x+1)$.
Show that $E(x) = 5x\,(2x+1)$.

2) In the figure below :





- x is a measure of length in centimeters and $2x - 1 > 0$.
- ABCD is a square of area S_1 .
- DCFE , HADG and PQRS are three rectangles of areas S_2 , S_3 and S_4 respectively.

a- Express S_1 and S_2 in terms of x .

b- Knowing that $S_1 + S_2 + S_3 = S_4$ and using the preceding results, calculate AH in terms of x .

IV- (2 points)

If we add 5 to each term of the irreducible fraction $\frac{x}{y}$, we get a fraction equal to $\frac{7}{8}$. If we subtract 3 from each term of this fraction, we get a fraction equal to $\frac{3}{4}$.

1) Show that the preceding information is translated into the following system of two equations with two unknowns :

$$\begin{cases} 8x - 7y = -5 \\ 4x - 3y = 3 \end{cases}$$

2) Solve this system, showing all the steps of calculation, and find the fraction $\frac{x}{y}$.

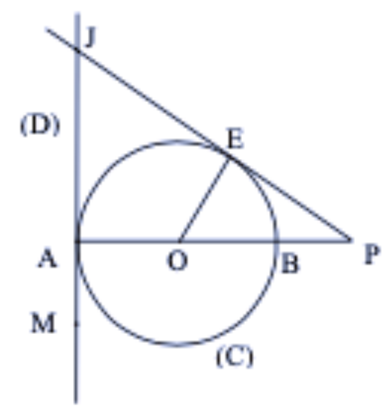
V- (6½ points)

Consider in an orthonormal system of axes $x'Ox$, $y'Oy$, the points $A(-3, 6)$, $B(5, 2)$ and $E(1, -2)$ and the straight line (d) of equation $y = x - 3$.

1) Plot the points A , B and E.
2) Verify by calculation, that E and B are two points of straight line (d). Draw (d).
3) a- Write the equation of the straight line (AE). Deduce that the points E, A and O are collinear.
   b- Are the straight lines (d) and (AE) perpendicular? Justify your answer.
4) Designate by M and N the respective images (translates) of A and C by the translation of vector $\overrightarrow{EB}$, and designate by (D) the image (translate) of $(y'y)$ by the same translation.
   a- Prove that B, N and M are collinear.
   b- Calculate the coordinates of $\overrightarrow{EB}$.
   c- Calculate the coordinates of N .
   d- Draw (D) and find its equation.
5) a- Show that AEBM is a parallelogram and not a rectangle.
   b- The diagonals of AEBM intersect in J. Calculate the coordinates of J.

VI- (5½points)

In the opposite figure:



- (C) is a circle of center O and diameter [AB]
- OA = OB = 3 cm
- P is the point of [AB) such that OP = 5 cm
- E is a point of (C) such that PE = 4 cm
- (D) is the tangent at A to (C)
- M is a variable point on (D)
- (PE) cuts (D) in J.

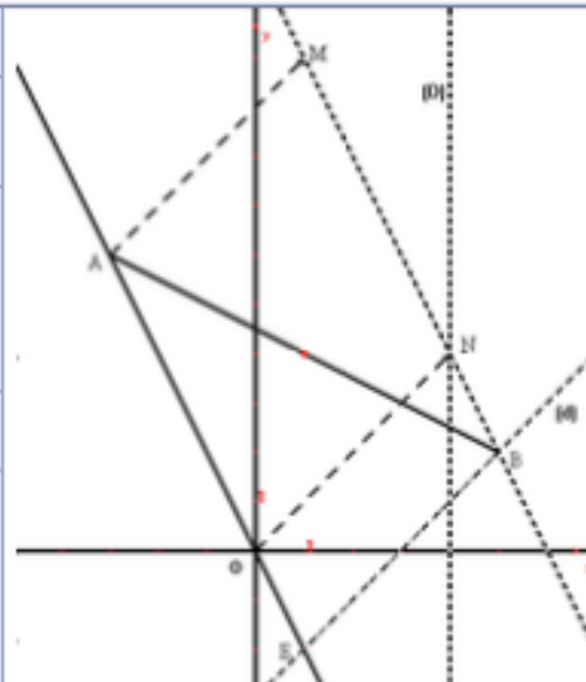
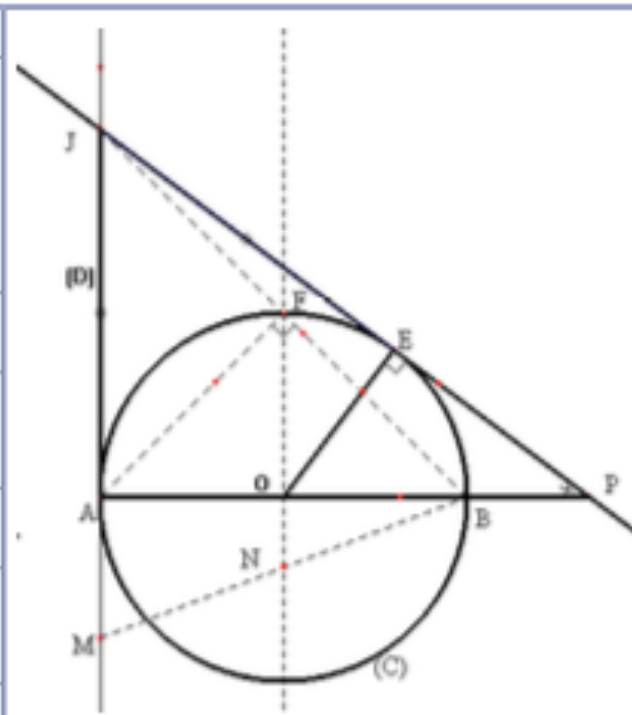
1) Reproduce this figure. It will be used and completed by the remaining parts of this problem.
2) a- Prove that (PE) is tangent to (C) at E . Deduce that JE = JA.
   b- Calculate tan $\widehat{OPE}$ and round to the nearest degree the angle $\widehat{OPE}$.
3) Let JE = JA = x and JP = x + 4 where x is a measure of length in centimeters.
   a- Apply Pythagoras theorem to triangle APJ and calculate x.
   b- Deduce that triangle ABJ is a right isosceles triangle.
4) (JB) cuts (C) in a second point F. Prove that F is the midpoint of [JB] and that (FO) is the perpendicular bisector of [AB].
5) Let N be the midpoint of [MB].
   Find the locus of N when M moves on (D).

دورة سنة ٢٠٠٦ العادية

وزارة التربية والتعليم العالي	امتحانات الشهادة المتوسطة
المديرية العامة للتربية
دائرة الامتحانات

توزيع علامات مسابقة الرياضيات

I	1)		$\frac{4}{5}-\frac{3}{5}\times\frac{10}{6}=-\frac{1}{5}$	½
	2)		$3^{14}-3^{12}=3^{12}\times 8$	½
	3)		$\sin^2 x+\cos^2 x=1\ ;\sin^2 x=\frac{8}{9}\ ;\ \sin x=\frac{2\sqrt{2}}{3}$	½
	4)		Le nouveau prix: $270+\frac{270\times 5}{100}=283{,}5$ L.L	½
II	1)		$A=\frac{3{,}6\times 10^{3}\times 10^{-5}}{9\times 10^{2}}=...=4\times 10^{-5}$ $A=0{,}00004$	¾
	2)		$B=...=19+5\sqrt{5}$	¾
III	1)		$E(x)=.....\ =5x(2x+1)$	1
	2)	a-	$s_1=(2x+1)^2\ ;\ s_2=x(2x+1)$	¾
		b-	$s_1+s_2+s_3=s_4$ $(2x+1)^2+x(2x+1)+AH\cdot(2x+1)=5x(2x+1)$ $AH=2x-1$	¾
IV	1)		$\begin{cases}\frac{x+5}{y+5}=\frac{7}{8}\\ \frac{x-3}{y-3}=\frac{3}{4}\end{cases}\ ;\ \begin{cases}8x-7y=-5\\ 4x-3y=3\end{cases}$	1
	2)		$\begin{cases}8x-7y=-5\\ -8x+6y=-6\end{cases}\ ;\ y=11\ \text{et}\ x=9\ ;\ \frac{x}{y}=\frac{9}{11}$	1

V	1)		Placer A, B et E		½
	2)		(d): $y = x - 3$ -2=1-3 donc E est sur (d) 2=5-3 donc B est sur (d)		¾
	3)	a-	Equation de (AE) : $y = ax + b$ $\begin{cases} 6 = -3a + b \\ -2 = a + b \end{cases}$; $a = -2$, $b = 0$ $y = -2x$, passe par O Alors A, E et O sont alignées.		1
		b-	pente de (d) . pente $(AE) = -2$ donc (d)n'est pas perpendiculaire à (AE)		½
	4)	a-	$\overrightarrow{AM} = \overrightarrow{EB}$ donc AEBM et un parallélogramme $\overrightarrow{ON} = \overrightarrow{EB}$ donc OEBN est un parallélogramme Alors B ; M et N sont alignées (ou…)		1
		b-	$\overrightarrow{EB}$ (4 ; 4)		½
		c-	$\overrightarrow{ON} = \overrightarrow{EB}$ alors N (4 ; 4)		½
		d-	x = 4 (D)		¾
	5)	a-	$\overrightarrow{AM} = \overrightarrow{EB}$; AEBM est un parallélogramme (AE) n'est pas perpendiculaire à (EB) alors AEBM n'est pas un rectangle.		½
		b-	J milieu de [AB] ; $J(1;4)$		½
VI	1)		figure		¼
	2)	a-	$PE^2 + OE^2 = 16 + 9 = 25$ $OP^2 = 25$ donc OPE est un triangle rectangle en E. (PE) perpendiculaire à OE alors (PE) est tangente au cercle. JE = JA (…)		1
		b-	$\tan O\hat{P}E = \frac{3}{4}; O\hat{P}E = 36°,8 \approx 37°$		¾
	3)	a-	$PJ^2 = AJ^2 + AP^2$; $(x+4)^2 = x^2 + 64$; $x = 6$		1
		b-	$AB = AJ = 6$; $J\hat{A}B = 90°$ JAB est un triangle rectangle isocèle		½
	4)		$A\hat{F}B = 90°$; [AF] hauteur à la fois médiane , donc F milieu de [BJ] (FO) médiatrice de [AB].		1
	5)		N décrit la médiatrice de [AB]		1

دورة سنة ٢٠٠٦ الاستثنائية	امتحانات الشهادة المتوسطة	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات
الاسم: الرقم:	مسابقة في مادة الرياضيات المدة: ساعتان	عدد المسائل: ستة

ملاحظة: يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو لاختزان المعلومات أو لرسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة).

I) ***(1 ½ pts)***

Write each of the following numbers in the form of a fraction as simple as possible:

$$A = \frac{7}{3} - \frac{8}{3} \times \frac{5}{2} \quad ; \qquad B = \frac{\frac{5}{3} - 1}{1 - \frac{1}{6}} \quad ; \qquad C = \frac{8 \times 10^7 \times 1.5}{3 \times 10^9} \ .$$

II) ***(2 ½ pts)***

Given the two numbers X and Y:

$$X = \sqrt{32} - 3\sqrt{2} + 2\sqrt{18} \quad ; \qquad Y = \sqrt{50} - \sqrt{72} + 3\sqrt{2}\,.$$

1) Write X in the form $a\sqrt{2}$ and Y in the form $b\sqrt{2}$ where a and b are two integers to be calculated.

2) Deduce that $X \times Y = 28$.

3) Prove that the apposite table is a proportionality table.

X	$4\sqrt{3} + 2\sqrt{5}$
$4\sqrt{3} - 2\sqrt{5}$	Y

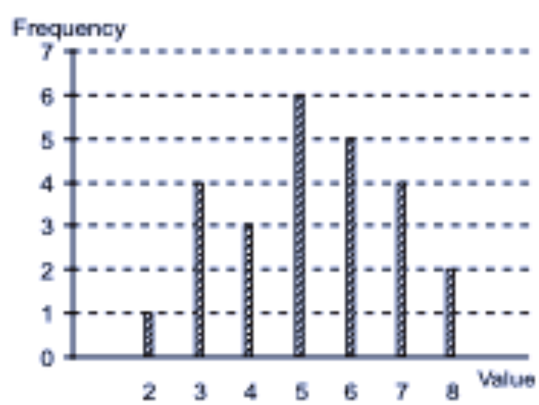
III) ***(2 ½ pts)***

The opposite bar graph represents a statistical series.

1) Calculate the total frequency.

2) Represent this series in a table showing the frequencies, and the relative frequencies in percentage.

3) Calculate the mean of this series.



IV) ***(2 ½ pts)***

To buy two copybooks and one pen we must pay 2750 LL, and to buy four copybooks and three pens we must pay 7750 LL. The preceding information is translated into the following system: $\begin{cases} 2x + y = 2750 \\ 4x + 3y = 7750 \end{cases}$.

1) What does x and y represent in this system?
2) Which information is translated by the equation $4x + 3y = 7750$?
3) Solve the preceding system, showing the followed steps in detail, to find the price of a copybook and the price of a pen.

V) ***(6 pts)***

Consider in an orthonormal system of axes $x'Ox, y'Oy$, the points :

$A(-2; 2)$; $B(3; 1)$ and $E(0; -1)$.

1) Plot the points A, B and E.
2) Write an equation of the line (BE).
3) Knowing that $AB = \sqrt{26}$ and $BE = \sqrt{13}$, calculate AE and prove that the triangle ABE is an isosceles right triangle at E.
4) Let (C) be the circle circumscribed about triangle ABE. Calculate the radius of (C) and the coordinates of its center J.
5) Designate by F the image (translate) of A by the translation of vector $\overrightarrow{EB}$.
   a) Prove that AEBF is a square.
   b) Deduce that F is a point of (C).
   c) Calculate the coordinates of F.

VI) ***(5 pts)***

Consider a semi-circle (C) of diameter [AB], of center O and of radius R. Let E be the midpoint of segment [OB].

The perpendicular bisector of [OB] cuts (C) in G. Let K be a variable point on segment [EG]. The straight line (BK) cuts (C) in a second point M.

1) Draw a figure.
2) Prove that $OB = OG = GB$. Deduce the measure of the angle $\widehat{BOG}$.
3) Calculate, in terms of R, the area of triangle AGB.
4) a) Prove that the two triangles BEK and BMA are similar.
   b) Deduce that $BK \times BM = BA \times BE$.
5) Designate by N the midpoint of [AM].
   Prove that, when K describes [EG], N moves on a circle whose diameter is to be determined.

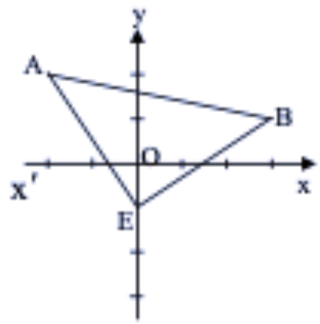
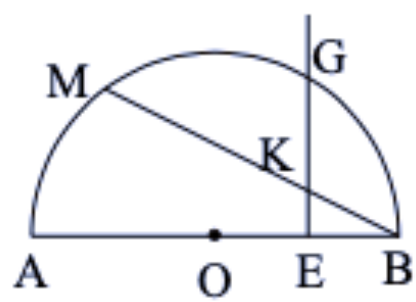
امتحانات الشهادة المتوسطة   دورة سنة ٢٠٠٦ الاستثنائية

توزيع علامات مسابقة الرياضيات

<table>
<tr><th colspan="2">Questions</th><th>Eléments de réponses</th><th>Notes</th></tr>
<tr><td>I-</td><td></td><td>$A = \frac{7}{3} - \frac{20}{3} = \frac{-13}{3}$.
$B = \frac{\frac{5-3}{3}}{\frac{6-1}{6}} = \frac{2}{3} \times \frac{6}{5} = \frac{4}{5}$.
$C = \frac{4}{100} = \frac{2}{50} = \frac{1}{25}$</td><td>½
½
½</td></tr>
<tr><td>II-</td><td>1-

2-
3-</td><td>$X = \sqrt{32} - 3\sqrt{2} + 2\sqrt{18} = 4\sqrt{3} - 3\sqrt{2} + 6\sqrt{2} = 7\sqrt{2}$.
$Y = 5\sqrt{2} - 6\sqrt{2} + 3\sqrt{2} = 2\sqrt{2}$.
$x \times y = 28$.
$(4\sqrt{3} - 2\sqrt{5})(4\sqrt{3} + 2\sqrt{5}) = 28 = xy$.</td><td>¾
½
¼
1</td></tr>
<tr><td>III-</td><td>1-
2-

3-</td><td>$N = 25$.
<table><tr><td>valeurs</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>Total</td></tr><tr><td>effectifs</td><td>1</td><td>4</td><td>3</td><td>6</td><td>5</td><td>4</td><td>2</td><td>25</td></tr><tr><td>fréquences en %</td><td>4</td><td>16</td><td>12</td><td>24</td><td>20</td><td>16</td><td>8</td><td>100</td></tr></table>
$\overline{X} = \frac{2\times1 + 3\times4 + 4\times3 + 5\times6 + 6\times5 + 7\times4 + 8\times2}{25} = \frac{130}{25} = 5.2$</td><td>½
¼
¾
1</td></tr>
<tr><td>IV-</td><td>1-
2-
3-</td><td>x est le prix d'un cahier ; y est le prix d'un stylo.
Le prix de 4 cahiers et de 3 stylos est 7750 LL.
$-4x - 2y = -5500$
$4x + 3y = 7750$
$y = 2250$; $x = 250$
Le prix d'un cahier est 250 LL et celui d'un stylo est 2250 LL.</td><td>½
½
1 ½</td></tr>
</table>

Questions		Eléments de réponses	Notes
V-	1-		¾
	2-	Equation de (BE) : $a = \frac{2}{3}$; $b = -1$ d'où $y = \frac{2}{3}x - 1$.	1
	3-	$AE = \sqrt{13}$; ABE est un triangle rectangle isocèle.	1¼
	4-	Rayon de (C) = $\frac{\sqrt{26}}{2}$; $J(\frac{1}{2} ; \frac{3}{2})$	1
	5-a)	AEBF est un carré .	1
	b)	F est un point de (C).	½
	c)	F(1 ; 4).	½
VI-	1-	Figure 	½
	2-	OB = OG = BG = R $\widehat{BOG} = 60°$	1
	3-	L'aire de AGB = $\frac{R^2\sqrt{3}}{2}$.	1
	4-a)	BEK et BMA sont semblables car …	1
	b)	$\frac{BE}{BM} = \frac{EK}{MA} = \frac{BK}{BA}$. alors BK × BM = BA × BE	½
	5-	N se déplace sur le cercle de diamètre [AO].	1

عدد المسائل : ستة	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

إرشادات عامة : - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو إختزان المعلومات أو رسم البيانات .
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة.

I- *(2 points)*

Questions 1) and 2) of this exercise are independent.

1) Given the two numbers A and B defined by :

$$A = \frac{13}{7} - \frac{3}{7} \times \frac{14}{9} \quad , \quad B = 2\sqrt{36} + 5\sqrt{12} - 9\sqrt{75} + 4\sqrt{27}.$$

Show all the steps of the following calculations :

a) Calculate A and give the result in the form of an irreducible fraction.

b) Write B in the form $a + b\sqrt{3}$ where a and b are two integers.

2) x is any acute angle, establish the following equalities :

a) $(1 + \tan^2 x)\cos^2 x = 1.$

b) $(\cos x + \sin x)^2 - 2\cos x \sin x = 1$.

II- *(2points)*

A statistical series is given in the opposite table where a, b, c and d are integers.

1) Calculate the numerical value of each of the numbers a, b, c and d.

2) Calculate the mean of this statistical series.

Values	5	7	8	12	**Total**
Frequencies	12	18	a	15	75
Relative frequencies in %	16	c	d	20	b

III- *(2points)*

In what follows, designate by x the price of a pen in L L and y the price of a copybook in L L. To buy one pen and one copybook we pay 2500 L L. If the price of a pen is decreased by 30% and the price of a copybook is decreased by 20% the amount we pay becomes 1900 L L.

1) Prove that the preceding information is translated into the following system : $\begin{cases} x + y = 2500 \\ 7x + 8y = 19000 \end{cases}$

2) Solve the preceding system, showing the followed steps in detail, and find the price of one pen and the price of one copybook.

IV- *(3 points)*

Part A

1) Verify the equality: $2(x - 3)(x + 7) = 2x^2 + 8x - 42$.

2) Solve the equation : $2x^2 + 8x - 42 = 0$.

Part B

In this part, the unit of length is the centimeter.

ABC is a triangle such that AB = x , AC = x + 4 and BC = $\sqrt{58}$, **where x is an integer strictly greater than 1.**

1) Can we find a value for x such that triangle ABC is right angled at C ? Justify.

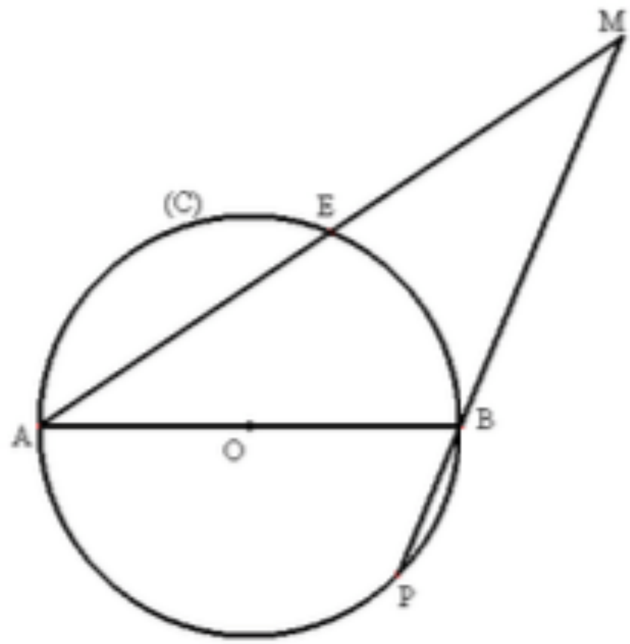
2) Calculate x so that triangle ABC is right angled at A. (You can use the results of **part A**).

3) Calculate x so that the perimeter of triangle ABC is less than or equal 18. (**In this question,** you can consider 7.6 as an approximate value of $\sqrt{58}$).

V- *(5 points)*

Consider a circle (C) of center O and diameter [AB] such that AB = 6 cm. E is a variable point of (C) and M is the symmetric of A with respect to E.
The straight line (BM) cuts circle (C) in a second point P (see the figure below).
Designate by J the point of intersection of (BE) with (AP), T the point of intersection of (AB) with (MJ) and S the midpoint of [MB].

1) Draw a figure.
2) Prove that triangle ABE is right.
3) a) Prove that triangle ABM is isosceles of principal vertex B.
   b) On what line does S move when E describes the circle (C) ?
4) Prove that triangle ABM is an enlargement of triangle OBS and precise the scale factor of this enlargement.
5) a) Prove that (AT) is perpendicular to (MJ).
   b) Prove that the points E, B, T and M belong to the same circle. Determine a diameter of this circle.



VI- *(6 points)*

In the plane of an orthonormal system x′ O x, y′ O y, where the unit of length is the centimeter, consider the straight line (d) of equation $y = -\frac{3}{2}x - 1$ and the points $A(-4\,;5)$, $B(6\,;3)$ and $G(0, -1)$.

1) Plot the points A, B and G.

2) Verify by calculation, that A and G are two points of (d), then draw (d).

3) Write an equation of the straight line (BG) and deduce that the straight lines (d) and (BG) are perpendicular.

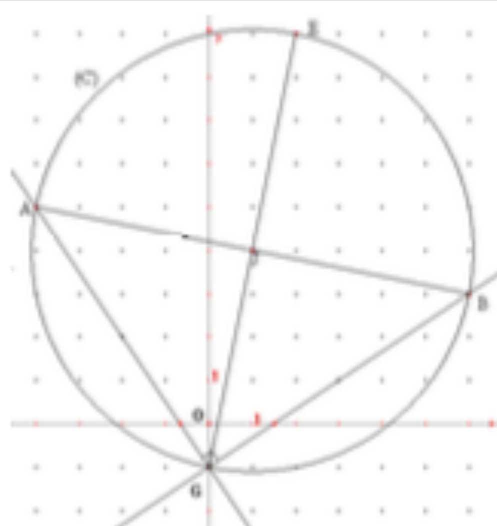
4) Knowing that $AG = 2\sqrt{13}$. Calculate BG and deduce that AGB is an isosceles right triangle.

5) Let (C) be the circle circumscribed about triangle ABG. Calculate the radius of (C) and the coordinates of its center J.

6) Designate by E the point defined by $\overrightarrow{GE} = \overrightarrow{GA} + \overrightarrow{GB}$.
   a) Prove that AGBE is a square.
   b) Calculate the coordinates of E.
   c) Prove that E is a point of (C).

توزيع علامات مسابقة الرياضيات

I	1.a)	$A=\frac{13}{7}-\frac{2}{3}=\frac{25}{21}$.	½
	1.b)	$B=12+10\sqrt{3}-45\sqrt{3}+12\sqrt{3}=12-23\sqrt{3}$.	½
	2.a)	$(1+\tan^2 x)\cos^2 x=\cos^2 x+\frac{\sin^2 x}{\cos^2 x}\cos^2 x=\cos^2 x+\sin^2 x=1$.	½
	2.b)	$(\cos x+\sin x)^2-2\cos x\sin x=\cos^2 x+\sin^2 x+2\cos x\sin x-2\cos x\sin x=1$.	½
II	1	a = 30 ; b = 100 ; c = 24 ; d = 40.	1 ¼
	2	$\bar{x}=\frac{606}{75}=8{,}08$.	¾
III	1	$\begin{cases}x+y=2500\\0{,}7x+0{,}8y=1900\end{cases}$ then $\begin{cases}x+y=2500\\7x+8y=19000.\end{cases}$	¼, ½, ¼
	2	x = 1000 and y = 1500 The price of one pen is 1000LL ; The price of one copybook is 1500LL.	¾ ¼
IV	A.1)	$2(x-3)(x+7)=2(x^2+4x-21)=2x^2+8x-42$	½
	A.2)	x = 3 ; x = -7	½
	B.1)	No, because x < x + 4 , so [AB] can not be a hypotenuse. or : $x^2=(x+4)^2+58$ gives $x=-\frac{74}{8}$. This value of x is rejected because x is negative. …	½
	B.2)	$BC^2=AB^2+AC^2$ so $2x^2+8x-42=0$, then x = 3 or x = -7 ; -7 rejected.	¾
	B.3)	$x+x+4+7{,}6\le 18$; $x\le 3{,}2$ so x = 2 or x = 3.	¾
V	1	Figure 	¼

V	**2**	[AB] is a diameter E is a point of the circle so triangle AEB is right at E.		½
	3.a)	In triangle ABM, [BE] is a median and a height so the triangle ABM is isosceles of principal vertex B.		¾
	3.b)	S moves on the circle of center B and radius BS = 3cm.		¾
	4	(OS) // (AM) ; $\frac{BM}{BS}=\frac{BA}{BO}=\frac{AM}{OS}=2$ because … ; The scale factor is 2.		¾
	5.a)	In triangle AMJ, B is the orthocenter so (JE) is perpendicular to (AM).		1
	5.b)	MEB and MTB are two right triangles of the same hypotenuse [BM]. So, they are inscribed in the circle of diameter [BM].		1
VI	**1**	A , B and G.		½
	2	$-\frac{3}{2}x_A-1=5=y_A$; A is a point of (d). $-\frac{3}{2}x_G-1=-1=y_G$; G is a point of (d). Draw (d).		¾
	3	Equation of (BG) : $y=\frac{2}{3}x-1$ $a_{(BG)}\times a_{(d)}=-1$; (BG) ⊥ to (d).		¾ ¼
	4	$BG=2\sqrt{13}$ BG = AG and (BG) ⊥ (AG) so AGB is a right isosceles triangle.		½ ½
	5	$r=\frac{AB}{2}=\sqrt{26}$ because [AB] is a diameter of (C). J (1 ; 4) (J midpoint of [AB]).		½ ½
	6.a)	$\overrightarrow{GE}=\overrightarrow{GA}+\overrightarrow{GB}$ so AGBE is a parallelogram. AGB is an isosceles right triangle so AGBE is a square.		¾
	6.b)	E (2 ; 9).		¾
	6.c)	E is the 4th vertex of the square ; so E is a point of (C).		¼

عدد المسائل : سبعة	مسابقة في تايضايرلا ةدام المدة: ساعتان	الاسم: الرقم:

ملاحظة : يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو لاختزان المعلومات أو لرسم البيانات .
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I- (1 point)

Given that : $A=\frac{8}{7}-\frac{3}{7}\times\frac{14}{6}$ and $B=\frac{2.1\times10^4\times10^{-5}}{3\times10^2}$.

1) Write A in the form of an irreducible fraction.
2) Write B in the form $a\times10^n$ where a and n are two integers.

II- (1½ points)

A bag contains a number of balls distributed in the following way:

10 % of the balls are red
15 % of the balls are white
$\frac{2}{5}$ of the balls are green
42 balls are black

1) Find the percentage of the green balls and that of the black balls.
2) Calculate the total number of balls in the bag.

III- (2½ points)

Given that $P(x)=4x^2-9+(x-2)(2x+3)$ and $Q(x)=(2x+3)(x-1)$.

1) Prove that $P(x)=(2x+3)(3x-5)$.
2) Solve the equation $Q(x)=0$.
3) Let $F(x)=\frac{P(x)}{Q(x)}$.
   a- For what values of x, is F(x) defined ?
   b- Simplify F(x), then solve the equation $F(x)=\sqrt{2}$, and write the solution in the form $\frac{a+b\sqrt{2}}{c}$ where a, b and c are integers.

IV- (3 points)

A video-club offers its customers two choices A and B. Each choice is formed of a fixed sum paid in advance which is called subscription and another sum to be paid for each rented cassette (Film).

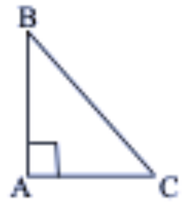
	Subscription in LL	**Price in LL paid for each rented cassette**
Choice A	60 000	900
Choice B	42 000	1 500

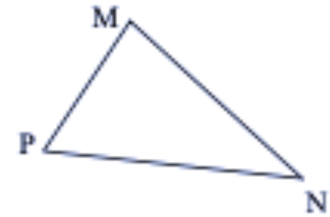
1) A customer who wants to rent 20 cassettes chooses choice A. How much should he pay?
2) Designate by x the number of cassettes that a second customer desires to rent.
   a- Express in terms of x, the price $S_1(x)$ that this customer should pay if he chooses choice A and the price $S_2(x)$ that he should pay if he chooses choice B.
   b- Starting from which number does the rented cassettes of choice A become more advantageous than choice B?
   (we advise you to start by solving the inequality $S_1(x)\leq S_2(x)$.)
3) A third customer has chosen choice B and paid 93 000LL.
   a- What is the number of cassettes rented to this customer?
   b- Which choice is better for him? Justify.

V- (2½ points)

In the opposite figure (not drawn to scale):

- ABC is a triangle right angled at A such that AB = 6 cm and $\tan \widehat{ACB} = \frac{3}{2}$.





- MNP is a triangle similar to ABC such that $\frac{MN}{AB} = \frac{MP}{AC} = \frac{5}{4}$.

1) Find rounded to the nearest degree, the measure of angle $\widehat{ACB}$ and write on your paper the measure of $\widehat{ACB}$ appearing on your calculator.
2) Prove that the triangle MNP is right angled at M and that $\widehat{ACB} = \widehat{MPN}$.
3) Calculate NP.

VI- (3 points)

Consider a semi-circle (C) of center O, radius R and diameter [AB]. Let M be a point on (C) distinct from A and B. The tangent at M to (C) cuts the tangent at A in point N and the tangent at B in point P. (OP) cuts [MB] in D and (ON) cuts [AM] in E.

1) Draw a figure.
2) Prove that D is the midpoint of [MB] and that E is the midpoint of [MA].
3) Calculate ED in terms of R.
4) Prove that ODME is a rectangle.
5) Let J be the midpoint of [DE]. Prove that, when M moves on (C), J moves on a semi-circle whose center and radius are to be determined.

VII- (6½ points)

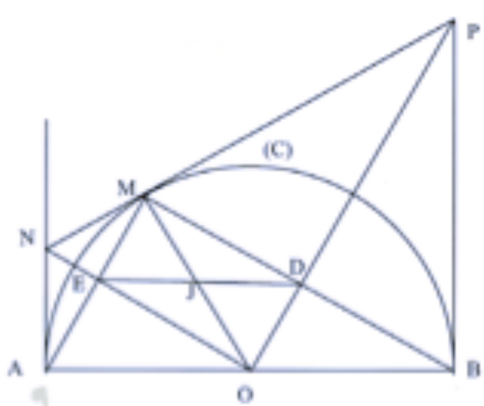
Consider, in an orthonormal system of axes x'Ox and y'Oy where the unit of length is the centimeter, the points A(0 ; – 4) , E(0 ; 1) , F(4 ; – 1) and the straight line (d) of equation $y = -\frac{1}{2}x + 1$.

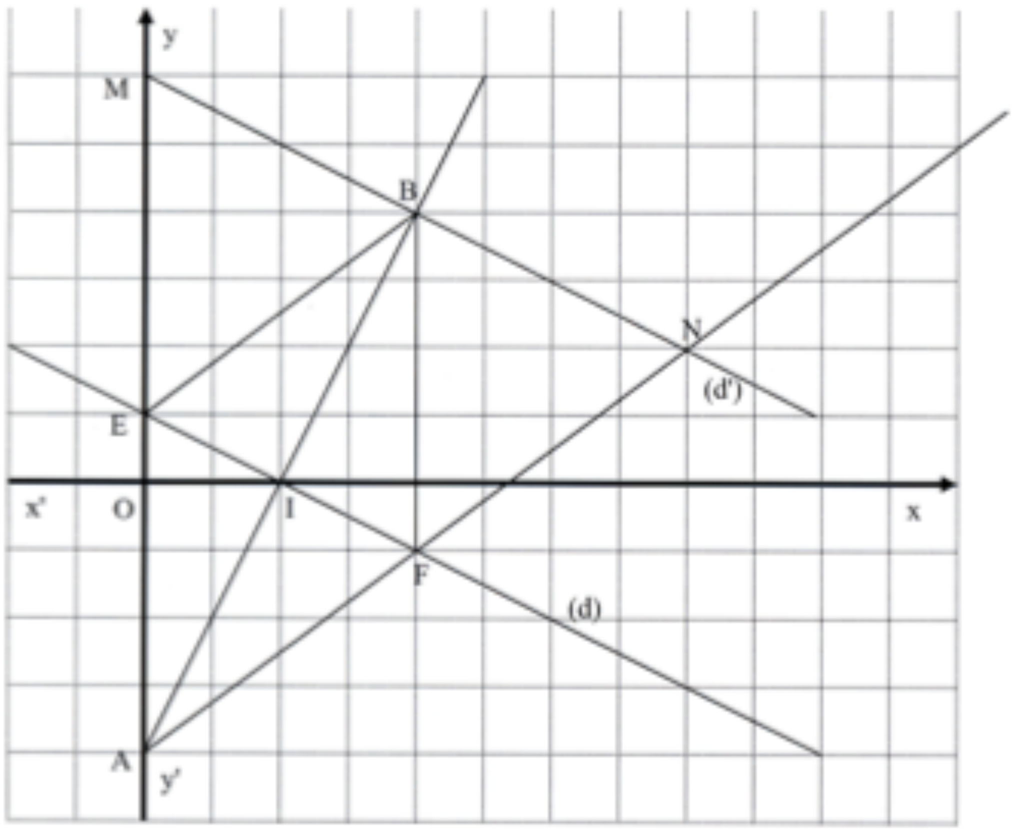
1) Plot the points A, E and F.
2) Verify by calculation, that E and F are two points of (d), then draw (d).
3) Prove that I(2 ; 0) is the midpoint of [EF].
4) We know that $EF = 2\sqrt{5}$.
   a- Calculate AE and AF. Deduce that triangle AEF is isosceles of principal vertex A.
   b- Is the straight line (AI) perpendicular to (EF)? Justify.
5) Let B be the symmetric of A with respect to I.
   a- Prove that AFBE is a rhombus.
   b- Calculate the coordinates of B.
6) Let (d') be the straight line passing through B and parallel to (d). Determine the equation of (d').
7) (AE) and (AF) intersect (d') in M and N respectively. Prove that EMNF is an isosceles trapezoid and calculate its area.

وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات

امتحانات الشهادة المتوسطة
توزيع علامات مسابقة الرياضيات

دورة سنة ٢٠٠٧ الاستثنائية

Questions		Short answers	Marks
I	1	$A = \frac{8}{7} - 1 = \frac{1}{7}$.	½
	2	$B = \frac{21 \times 10^{-2}}{3 \times 10^{2}} = 7 \times 10^{-4}$	½
II	1	The percentage of green balls $\frac{2}{5} \times 100 = 40\%$ The percentage of black balls $100 - (10 + 15 + 40) = 35\%$	½ ½
	2	The number of balls in the bag is 120.	½
III	1	$P(x) = (2x - 3)(2x + 3) + (x - 2)(2x + 3) = (2x + 3)[(x - 3) + (x - 2)]$. $= (2x + 3)(3x - 5)$ or by expansion	½
	2	$Q(x) = 0$; $x = -\frac{3}{2}$ or $x = 1$.	½
	3a	F(x) is defined for $x \neq -\frac{3}{2}$ and $x \neq 1$.	½
	3b	$F(x) = \frac{3x - 5}{x - 1}$; $3x - 5 = \sqrt{2}(x - 1)$; $x = \frac{5 - \sqrt{2}}{3 - \sqrt{2}} = \frac{13 + 2\sqrt{2}}{7}$	¼ ; ¾
IV	1	$60\,000 + 900 \times 20 = 60\,000 + 18\,000 = 78\,000$ LL	½
	2a	$S_1(x) = 60\,000 + 900x$ $S_2(x) = 42\,000 + 1\,500x$	½ ¼
	2b	$S_1(x) \leq S_2(x)$; $60\,000 + 900x \leq 42\,000 + 1\,500x$; $x \geq 30$ cassettes.	¾
	3a	$42\,000 + 1\,500x = 93\,000$; $x = 34$ cassettes.	½
	3b	No, the number of cassettes is greater than 30.	½
V	1	$\widehat{ACB} = 56{,}30993247$ calculator 56° is rounded …	½ ¼
	2	$\widehat{PMN} = 90°$ because........ ; $\widehat{ACB} = \widehat{MPN}$ because ...	½ ; ¼
	3	$\tan \widehat{ACB} = \frac{3}{2} = \frac{6}{AC}$ so $AC = 4$. $BC = \sqrt{52} = 2\sqrt{13}$. $\frac{NP}{BC} = \frac{5}{4}$ then $NP = \frac{5\sqrt{13}}{2}$	½ ½
VI	1	fig  	½

continued VI	2	D is the midpoint of [MB] because ….. E is the midpoint of [MA] because …	½ ¼
	3	ED = R	½
	4	ODME is a rectangle because …..	¾
	5	J moves on the semi-circle of center O and of radius $\frac{R}{2}$.	½
VII	1	 A ; E et F	¾
	2	E is a point of (d) because …, F is a point of (d) because … . drawing of (d).	¼ ; ¼ ; ¼
	3	$x_I = \frac{x_E + x_F}{2} = 2$; $y_I = \frac{y_E + y_F}{2} = 0$.	½
	4a	AE = AF = 5 AEF is an isosceles triangle of principal vertex A.	¼ ; ½ ¼
	4b	(AI) is perpendicular to (EF) because …	½
	5a	AFBE is a rhombus because …	½
	5b	B(4 ; 4)	½
	6	$y = -\frac{1}{2}x + b$ because the slope of (d) = slope of (d') = $-\frac{1}{2}$ $y = -\frac{1}{2}x + 6$ because (d') passes by B.	½ ½
	7	EMNF is an isosceles trapezoid because … $\boldsymbol{A}_{EMNF} = \frac{(MN + EF) \times IB}{2} = \frac{6\sqrt{5} \times 2\sqrt{5}}{2} = 30\,cm^2$	½ ½

وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات	الشهادة المتوسطة	دورة سنة ٢٠٠٨ العادية
عدد المسائل ستة	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

إرشادات عامة : - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة.

I- (3 points)

Consider the polynomial $p(x) = (x-2)^2 - (2-x)(x+4)$

1) Factorize $p(x)$.

2) Expand and reduce $p(x)$.

3) **a.** Expand and reduce $2(x-3)(x+2)$.

b. Calculate $p(3)$.

c. Solve the equation $p(x) = 8$.

II- (2 points)

Given the two numbers A and B :

$A = (\sqrt{3}+\sqrt{2})^2 + (\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})$; $B = \sqrt{50}+\sqrt{150}+\sqrt{96}+\sqrt{54}-5\sqrt{2}$.

1) Calculate A and write it in the form of $a + b\sqrt{6}$ where a and b are integers.

2) Calculate B and write it in the form of $x\sqrt{6}$ where x is an integer.

3) By using the answers in questions 1 and 2, rationalize the denominator of the expression $\frac{A}{B}$ and simplify the answer.

III- (2 points)

In what follows,we have the survey of the scores of 30 students of a class :

12 ; 18 ; 15 ; 11 ; 14 ; 7
14 ; 12 ; 11 ; 8 ; 18 ; 15
7 ; 18 ; 12 ; 14 ; 17 ; 10
14 ; 11 ; 10 ; 18 ; 17 ; 12
7 ; 12 ; 15 ; 8 ; 14 ; 17.

1) Use the above survey to construct a table of scores containing the frequencies and the increasing cumulative frequencies.

2) What is the percentage of the students who got a score less than 13?

3) Calculate the mean of the class scores.

IV- (2 points)

Consider an isosceles triangle ABC, such that AB = AC = 3 cm, and BC = 2 cm
M is the midpoint of [BC].

1) **a.** Calculate AM.

b. Calculate $\sin \widehat{ABC}$.

2) **a.** Calculate the area S of the triangle ABC.

b. Show that $2S = BA \times BC \times \sin \widehat{ABC}$.

V- (6 points)

In an orthonormal system of axes (x'ox, y'oy), Consider the points $E(3;3)$, $F(2;-2)$ and $G(-2;4)$.

1) Locate the points E, F and G.

2) **a.** Given that $EG=\sqrt{26}$ calculate EF and FG.

b. Deduce that the triangle EFG is isosceles and right angled at E.

3) Let (C) be the circle circumscribed about triangle EFG .

a. Find the radius R of (C).

b. Calculate the coordinates of the point I , the center of (C), and deduce that I is on$(y'y)$.

c. Show that $P(-3,3)$ belongs to the circle (C).

4) **a.** Calculate the coordinates of point L, the translate (image) of E under the translation of vector $\overrightarrow{OP}$.

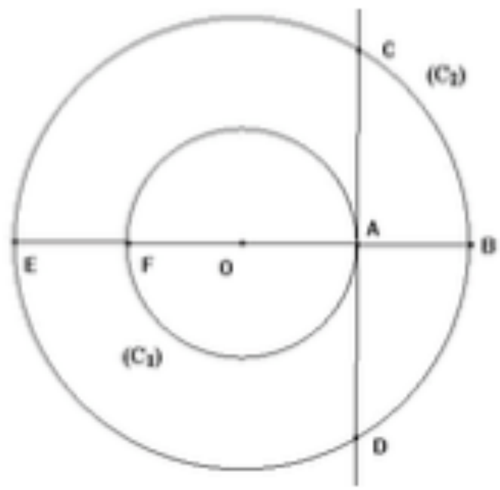
b. Determine the equation of the straight line (OE).

c. Determine the equation of the straight line (d'), the translate (image) of (OE) under the translation of vector $\overrightarrow{OP}$.

d. Show that P, G and L are collinear.

VI- (5 points)

In the figure below we have the 2 circles (C_1) and (C_2), of the same center O and their radii are $R_1 = 2cm$ and $R_2 = 4cm$ respectively. A straight line passing through O cuts (C_1) at F and A, and cuts (C_2) at B and E. The tangent to circle (C_1) at A cuts the circle (C_2) at C and D.



1) Show that (CD) is the perpendicular bisector of [OB].

2) Determine the nature of the triangle OBC.

3) Show that the quadrilateral OCBD is a rhombus.

4) Let P be the midpoint of [CE].

a. Calculate OP and deduce that P belongs to the circle (C_1).

b. Show that D, O and P are collinear.

c. Show that (OP) is perpendicular to (CE) and deduce that (CE) is tangent to (C_1).

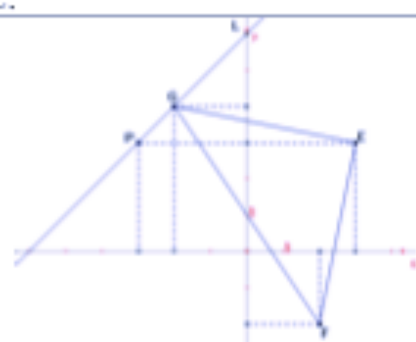
d. Show that (CO) is perpendicular to (DE).

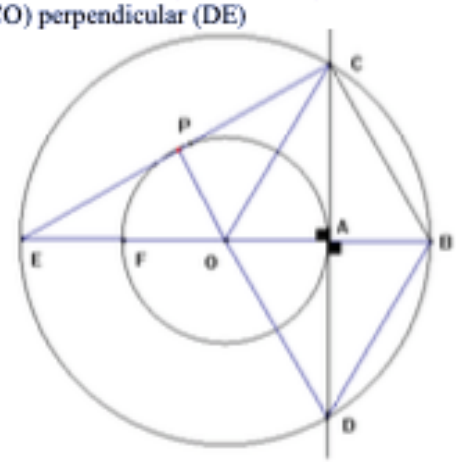
5) Show that the triangle EBC is an enlargement of the triangle EOP and precise the center and ratio of this enlargement.

وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات

الشهادة المتوسطة
مسابقة الرياضيات

دورة سنة ٢٠٠٨ العادية
اسس التصحيح

<table>
<tr><th>Part of the Q</th><th>Answer</th><th>Mark</th></tr>
<tr><td>I.1</td><td>$p(x) = (x-2)^2 - (2-x)(x+4)$
$p(x) = (x-2)(x-2+x+4) = (x-2)(2x+2)$
$p(x) = 2(x-2)(x+1)$.</td><td>0.50</td></tr>
<tr><td>I.2</td><td>$p(x) = 2x^2 + 2x - 4x - 4$
$p(x) = 2x^2 - 2x - 4$.</td><td>0.50</td></tr>
<tr><td>I.3.a</td><td>$2(x-3)(x+2) = 2(x^2 + 2x - 3x - 6) = 2x^2 - 2x - 12$.</td><td>0.50</td></tr>
<tr><td>I.3.b</td><td>$p(3) = 2(3-2)(3+1) = 8$.</td><td>0.50</td></tr>
<tr><td>I.3.c</td><td>If $p(x) = 8$, then $2x^2-2x-4=8$ so $2x^2 - 2x - 12=0$ $x=-2$ $x=3$</td><td>1</td></tr>
<tr><td>II.1</td><td>$A = 6 + 2\sqrt{6}$.</td><td>0.75</td></tr>
<tr><td>II.2</td><td>$B = 12\sqrt{6}$.</td><td>0.75</td></tr>
<tr><td>II.3</td><td>$\frac{A}{B} = \frac{6+2\sqrt{6}}{12\sqrt{6}} = \frac{6+2\sqrt{6}}{12\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{6\sqrt{6}+12}{12\times 6} = \frac{\sqrt{6}+2}{12}$</td><td>0.50</td></tr>
<tr><td>III.1</td><td><table>
<tr><td>score</td><td>7</td><td>8</td><td>10</td><td>11</td><td>12</td><td>14</td><td>15</td><td>17</td><td>18</td><td>Total</td></tr>
<tr><td>Frequency</td><td>3</td><td>2</td><td>2</td><td>3</td><td>5</td><td>5</td><td>3</td><td>3</td><td>4</td><td>30</td></tr>
<tr><td>Cumulative frequency</td><td>3</td><td>5</td><td>7</td><td>10</td><td>15</td><td>20</td><td>23</td><td>26</td><td>30</td><td></td></tr>
</table></td><td>0.75</td></tr>
<tr><td>III.2</td><td>15 students have a score less than 13 the percentage is 50%</td><td>0.25</td></tr>
<tr><td>III.3</td><td>$\overline{X} = \frac{7\times3+8\times2+10\times2+11\times3+12\times5+14\times5+15\times3+17\times3+18\times4}{30}\ \frac{388}{30} = 12.9$</td><td>0.50</td></tr>
<tr><td>IV.1.a</td><td>The triangle AMB is right at M and
BM = 1 cm.
$AB^2 = AM^2 + BM^2$
$9 = AM^2 + 1$;
So, $AM^2 = 8$ and
$AM = 2\sqrt{2}$
</td><td>0.50</td></tr>
<tr><td>IV.1.b</td><td>$\sin \widehat{ABC} = \frac{AM}{AB} = \frac{2\sqrt{2}}{3}$.</td><td>0.50</td></tr>
<tr><td>IV.2.a</td><td>S = Area of ABC = $\frac{1}{2} BC \times AM = \frac{1}{2} \times 2 \times 2\sqrt{2} = 2\sqrt{2}\ cm^2$.</td><td>0.50</td></tr>
<tr><td>IV.2.b</td><td>$2S = 4\sqrt{2}$
$BA \times BC \times \sin \widehat{ABC} = 3 \times 2 \times \frac{2\sqrt{2}}{3} = 4\sqrt{2}$.
Hence, $2S = BA \times BC \times \sin \widehat{ABC}$.</td><td>0.50</td></tr>
<tr><td>V.1</td><td>
Figure</td><td>0.50</td></tr>
<tr><td>V.2.a</td><td>$EF = \sqrt{26} = EG$
$FG = \sqrt{52}$</td><td>1</td></tr>
</table>

V.2.b	$FG^2 = EF^2 + EG^2$ Therefore EFG is a right isosceles triangle	**0.50**
V.3. a	Radius= $\frac{FG}{2} = \frac{\sqrt{52}}{2} = \sqrt{13}$	**0.50**
V.3. b	I(0 ;1) belongs to (y' y)	**0.75**
V.3. c	$IP = \sqrt{13} =$ Radius, therefore P belongs to the circle (C).	**0.25**
V.4. a	$\overrightarrow{OP} = \overrightarrow{EL}$; L(0;6)	**0.75**
V.4. b	Equation of (OE) : y = x	**0.25**
V.4. c	(d') has a translate (PL) : y = x + 6	**1**
V.4. d	G, P and L are points on (d')	**0.50**
VI.1	A is the midpoint of [OB] and (CD) is perpendicular to (OA) then (CD) is the perpendicular bisector of [OB].	**0.75**
VI.2	OC = BC and OB = OC then OBC is an equilateral triangle.	**0.50**
VI.3	OD = BD and OB = OD then OBD is an equilateral triangle therefore OC = BC = OD = BD then OCBD is a rhombus.	**0.50**
VI.4.a	Triangle ECB we have : O midpoint of [EB] and P midpoint of [CE] then (OP) // (BC) (midpoint theorem) and $OP = \frac{BC}{2} = \frac{4}{2} = 2 = R_2$ then P belongs to circle (C_1).	**0.75**
VI.4.b	(OD) // (BC) since ODBC is a rhombus and (OP) // (BC) (midpoint theorem); then at O we can have only 1 parallel to (BC) then P, O and D are collinear.	**0.50**
VI.4.c	[OP] is the median and height of the isosceles triangle COE then (OP) is the perpendicular to (CE) and (CE) perpendicular at P to OP, then (CE) is tangent to (C_2).	**0.75**
VI.4.d	In the triangle CED ; we have [EA] height, [DP] height then O orthocenter therefore [CO] is the 3rd height then (CO) perpendicular (DE) 	**0.50**
VI.5	Center E and ratio 2.	**0.75**

عدد المسائل: ستة	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

* ارشادات عامة : - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة.

I- (2 points)

The questions 1), 2) and 3) are independent of each other.

1) Write the following expression in the form of a decimal fraction, showing the steps of your calculation: $\dfrac{-2.4\times5^2+3(9.3-4.3)^2}{2\times2.5\times60}$.

2) Given that $a=\dfrac{3}{2}$, $b=\dfrac{3}{4}$ and $c=\dfrac{4}{5}$. Calculate $(b-a)$, $(a-bc)$ and $\dfrac{ac}{b}$.

3) x is a natural integer:

a. Write in terms of x the natural integer which is just before x and the integer that is just after x.

b. Calculate the product of these three natural integers in terms of x .

c. Use the above result to calculate: $9\times10\times11$.

II- (3 points)

1) Consider the polynomial $P(x) = a\,x^2 - 4(x+5)$.

a. Calculate a such that -2 is a root of $P(x)$.

b. Let $E(x) = 4(x^2-4)-(x+2)^2$; verify that $E(x) = 3x^2-4x-20$.

c. Factorize E(x).

d. Solve the equation $E(x) = 0$.

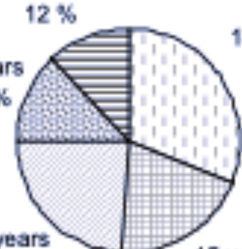
2) Suppose that $x = 2\sqrt{2}+1$

a. Calculate x^2 and $2x+7$, and then compare the two numbers obtained.

b. Verify that $x-2=\dfrac{7}{x}$.

III- (2 ½ points)

The 300 students in a school are distributed into 5 categories according to their ages, which are respectively 14,15, 16,17 and 18 years. The circular diagram below shows the percentage frequencies of this distribution.



1) Copy and complete the following table: .

age	14	15	16	17	18	
frequency				39		Total=300
Cumulative frequency						

2) Determine the mean age of these students.

3) Determine the number of students having ages strictly less than 17 years.

IV- (2 points)

A person bought a computer for 1 200 000 LL. He paid one quarter of this price as a down-payment. The remaining amount of the price is to be paid in 10 monthly payments:

Some of these payments are 150 000 LL each, and the others are 50 000 LL each.

Let x be the number of the 150 000 LL payments.

1) Calculate the down-payment, and the remaining amount of the price of this computer.
2) Calculate the number of the 150 000 LL payments, and that of the 50 000 LL payments.

V-(5 ½ points)

Consider a circle (C) with center O, diameter [AB] and radius 3 cm.
(d) is the tangent at A to (C) and F is a point on (d) such that AF = 4 cm.
E is the orthogonal projection of A on (OF). (AE) cuts again (C) at a point L.

1) **a.** Draw a figure.
   b. Calculate OF and $\cos \widehat{OFA}$.
2) **a.** Show that the two triangles OAF and BLA are similar. Write the similarity ratio.
   b. Use this ratio to calculate BL and AL.
   Deduce OE and AE.
3) (LO) cuts again (C) at K, and (BK) meets (d) at S.
   a. Determine the nature of quadrilateral BLAK.
   b. Compare $\widehat{SAK}$ and $\widehat{ABK}$.
   c. Determine $\cos \widehat{SAK}$ in the triangle SAK, then calculate AS.
   d. Express the cosine of the angle $\hat{B}$ in each of the two right triangles ABK and ABS, then deduce the relation $AB^2 = BK \times BS$.

VI- (5 points)

In the orthonormal system of axes x'Ox, y'Oy , given the two lines:

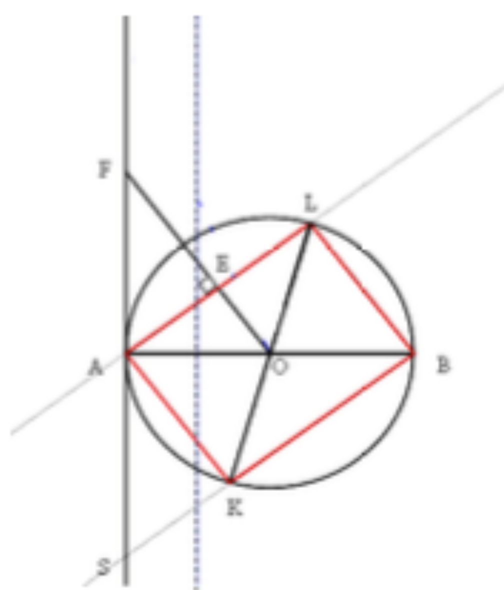
(D_1) of equation $y = 3x + 6$ and (D_2) of equation $y = -\frac{x}{3} + 3$.

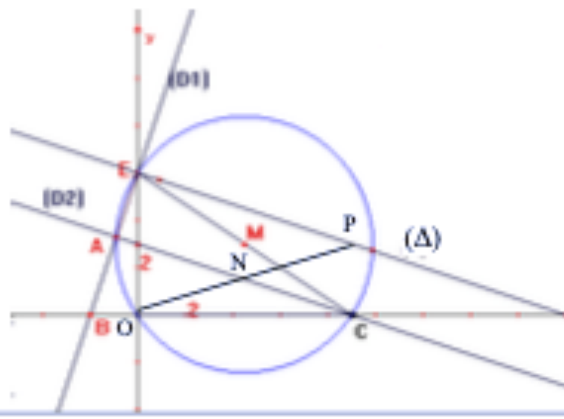
1) Plot the two lines (D_1) and (D_2).
2) Prove that (D_1) and (D_2) are perpendicular.
3) The two lines (D_1) and (D_2) intersect at A. Calculate the coordinates of the point A.
4) Plot the point C(9;0) and show that C is on (D_2).
5) (D_1) cuts x'Ox at a point B. Calculate the coordinates of B.
6) Let E be the point of intersection of (D_1) and y'Oy.
   a. Find the coordinates of the point E.
   b. Calculate the coordinates of the center M of the circle circumscribed about the triangle AEC.
   c. Find an equation of the straight line (Δ) the translate of (D_2) under the translation with vector $\overrightarrow{AE}$.
   d. Let P be a variable point on (D_2), and let N be the symmetric of O with respect to P.
   Determine the locus of N.

مشروع معيار التصحيح

مسابقه في مادة الرياضيات
المدة: ساعتان

الاسم:
الرقم:

<table>
<tr><th colspan="2">Part of Q.</th><th>Answer</th><th>Mark</th></tr>
<tr><td rowspan="5">I</td><td>1</td><td>$\frac{5^2(3-2,4)}{5\times60}=\frac{0,6}{2\times6}=\frac{1}{20}$</td><td>0.50</td></tr>
<tr><td>2</td><td>$b-a=\frac{-3}{4}$; $a-bc=\frac{9}{10}$ and $\frac{ac}{b}=\frac{8}{5}$.</td><td>0.75</td></tr>
<tr><td>3.a</td><td>The integer before x is $(x-1)$,and that after x is $(x+1)$</td><td>0.25</td></tr>
<tr><td>3.b</td><td>$(x-1)\times x\times(x+1)=x^3-x$</td><td>0.25</td></tr>
<tr><td>3.c</td><td>$9\times10\times11=1000-10=990$.</td><td>0.25</td></tr>
<tr><td rowspan="6">II</td><td>1.a</td><td>$a=3$.</td><td>0.25</td></tr>
<tr><td>1.b</td><td>$E(x)=4x^2-16-x^2-4x-4=3x^2-4x-20$.</td><td>0.50</td></tr>
<tr><td>1.c</td><td>$E(x)=(x+2)(4x-8-x-2)=(x+2)(3x-10)$</td><td>0.75</td></tr>
<tr><td>1.d</td><td>$x=-2$ or $x=\frac{10}{3}$.</td><td>0.50</td></tr>
<tr><td>2.a</td><td>$x^2=9+4\sqrt{2}$ and $2x+7=9+4\sqrt{2}$.</td><td>0.50</td></tr>
<tr><td>2.b</td><td>$2\sqrt{2}+1-2=\frac{7}{2\sqrt{2}+1}$; $\left(2\sqrt{2}-1\right)\left(2\sqrt{2}+1\right)=7$</td><td>0.50</td></tr>
<tr><td rowspan="3">III</td><td>1</td><td><table><tr><td>Age</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td></td></tr><tr><td>frequency</td><td>93</td><td>60</td><td>72</td><td>39</td><td>36</td><td>300</td></tr><tr><td>Cumulative frequency</td><td>93</td><td>153</td><td>225</td><td>264</td><td>300</td><td></td></tr></table></td><td>1.75</td></tr>
<tr><td>2</td><td>Mean age $=\frac{14\times93+15\times60+16\times72+17\times39+18\times36}{300}=\frac{4665}{300}=15,55$.</td><td>0.50</td></tr>
<tr><td>3</td><td>The number of students who are younger than 17 is 225.</td><td>0.25</td></tr>
<tr><td rowspan="2">IV</td><td>1</td><td>$\frac{1}{4}\times1200000=300\,000$ LL the remaining amount is 900 000 LL</td><td>0.75</td></tr>
<tr><td>2</td><td>The 150 000LL payments are 3, and the 50 000LL are 6.</td><td>1.25</td></tr>
<tr><td rowspan="3">V</td><td>1.a</td><td>See the figure

</td><td>0.50</td></tr>
<tr><td>1.b</td><td>$OF^2=OA^2+AF^2=25$, thus $OF=5$ cm. $\cos\widehat{OFA}=\frac{AF}{OF}=\frac{4}{5}$.</td><td>0.50</td></tr>
<tr><td>2.a</td><td>$\widehat{OAF}=\widehat{BLA}=90°$.</td><td>1</td></tr>
</table>

		(OF) // (BL), then $\widehat{ABL} = \widehat{AOF}$ (corresponding angles). Therefore OAF and BLA are similar by (AA). Similarity ratio : $\frac{OA}{BL}=\frac{OF}{BA}=\frac{AF}{LA}$.	
	2.b	$\frac{3}{BL}=\frac{5}{6}$, then BL = $\frac{18}{5}$ cm ; $\frac{5}{6}=\frac{4}{AL}$, then AL = $\frac{24}{5}$ cm. OE = $\frac{1}{2}$BL = $\frac{9}{5}$ cm ; and AE = $\frac{1}{2}$AL = $\frac{12}{5}$ cm.	**1**
	3.a	BLAK is a rectangle.	**0.50**
	3.b	$\widehat{SAK} = \widehat{ABK} = \frac{1}{2}\overset{\frown}{AK}$.	**0.50**
	3.c	In the triangle SAK, cos $\widehat{SAK} = \frac{AK}{AS} = \frac{4}{5}$; AS = $\frac{9}{2}$	**0.75**
	3.d	cos $\widehat{ABK} = \frac{AK}{AB}$; cos $\widehat{ABS} = \frac{AB}{BS}$ so $\frac{AK}{AB} = \frac{AB}{BS}$ then $AB^2 = AK \times BS$.	**0.75**
	1	$(D_1) : y = 3x + 6$. $(D_2) : y = -\frac{x}{3} + 3$. x: 0, 1 ; y: 6, 9 (for D_1) x: 0, 3 ; y: 3, 2 (for D_2) 	**1**
VI	**2**	The slope of (D_1) is a = 3, and the slope of (D_2) is a'= $-\frac{1}{3}$, therefore $a \times a' = -1$ therefore $(D_1) \perp (D_2)$	**0.50**
	3	(D_1) and (D_2) intersect at A. Therefore the coordinates of A are the solution of the system : $\begin{cases} y=3x+6 \\ y=-\frac{x}{3}+3 \end{cases}$, therefore y = y , hence A($-\frac{9}{10}$;$\frac{33}{10}$)	**0.50**
	4	C(9,0) and (D_2) ; y= $-\frac{x}{3}$ + 3 ; 0 = $-\frac{9}{10}$ + 3, hence C belongs to (D_2).	**0.25**
	5	B (– 2 ; 0)	**0.25**
	6.a	E (0 ; 6)	**0.25**
	6.b	ACE is a right triangle at A, the center M of the circle circumscribed around this triangle is the midpoint of [EC] , therefore M ($\frac{9}{2}$; 3).	**0.50**
	6.c	(Δ)//(D_2) by translation , then a = a'= $-\frac{1}{3}$; but (Δ) passes through E(0 ; 6) therefore y = $-\frac{1}{3}$ x + 6.	**1**
	6.d	The locus of P is the line (Δ) .	**0.75**

وزارة التربية والتعليم العالي الشهادة المتوسطة الدورة العادية للعام 2009
المديرية العامة للتربية
دائرة الامتحانات

مسابقة في مادة الرياضيات	الاسم:
المدة ساعتان	الرقم:

ارشادات علمة :- يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة .

I- (2 points)

Given :

$A = 2\sqrt{27} + 3\sqrt{75} - 3\sqrt{48}$ and $B = \dfrac{22}{\sqrt{18} - \sqrt{8}}$.

1) Write A in the form $a\sqrt{3}$ and B in the form $b\sqrt{2}$ where a and b are two integers.
2) Compare A and B and justify.
3) Show that $A - B = \dfrac{1}{A + B}$.

II- (2 points)

The questions 1) and 2) of this exercise are independent.

1) A class contains 30 students where 40 % of them are boys. Another class contains 20 students where 60 % of them are boys.
   The students of these two classes meet together in the computer room.
   Calculate the number and the percentage of boys in this room.
2) All the articles of a certain shop are subject to an increase of 20 % on their prices.
   Denote by x the original price of an article and by y its new price after the increase.
   a. Find y as a function of x.
   b. If the new price of a calculator is 30 000 LL, what is its original price?

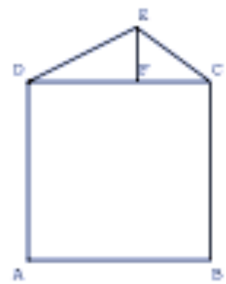
III- (3 points)

A first bunch of flowers is formed by 3 roses and 4 tulips and it costs 4800 LL.
A second bunch of flowers is formed by 5 roses and 6 tulips and it costs 7500 LL.
Denote by x the price of one rose, and by y the price of one tulip.

1) Write a system of two equations modeling the previous information.
2) Solve the previous system in showing the steps of calculation. Determine the price of one rose and that of one tulip.
3) A client buys a bunch formed by 10 flowers and he pays 6450 LL.
   Calculate the number of roses and that of tulips in this bunch .

IV-(3 points)

Consider the polynomial $P(x) = (x+9)^2 - 3(x-1)(x+9)$.
1) Factorize $P(x)$.
2) In this part,the unit of length is the centimeter.
In the figure to the right, ABCD is a square, DEC is a triangle such that $CF = 9$, $DF = x$ and the height $EF = x - 1$ with $x>1$.
Calculate x such that the area of the square ABCD is 6 times the area of the triangle CED.



V- (5 points)

Consider in an orthonormal system of axes x'ox and y'oy the line (d) with equation $y = 3x + 2$ and the two points $A(1\,;5)$ and $B(-2\,;-4)$.

1) Show that the points A and B belong to the line (d).
2) Locate A and B and plot (d).
3) Let (d') be the perpendicular bisector of [AB] and H the midpoint of [AB].
   a. Calculate the coordinates of H.
   b. Determine the equation of (d').
4) Let $M(-5; 2)$ be a point on (d').
   a. Given that $MA = 3\sqrt{5}$, justify that $MB = 3\sqrt{5}$.
   b.Calculate AB and deduce that AMB is a right isosceles triangle.
5) Consider the point P on the line (d') so that P is distinct from M and AP = AM.
   a. Locate P, and show that BP = BM.
   b. What is the nature of the quadrilateral MAPB ? Justify.

VI- (5 points)

Consider a circle (C) with center O, diameter [AB] and radius 2 cm. T is a point on (C) so that AT = 2 cm , and M is the symmetric of O with respect to A.

1) a. Make a figure.
   b. Prove that (MT) is tangent to (C).
   c. Calculate MT.
   d.Prove that MTB is an isosceles triangle.
2) E is the meeting point of (MT) and the tangent at B to (C).
   a. Prove that T is the midpoint of [EM].
   b. (TO) intersects (C) at F, calculate EF.
   c. Calculate to the nearest degree the angle $\widehat{EFT}$.
3) N is a variable point on (C) and S is the image of N under the translation with vector $\overrightarrow{AM}$.
   a. Prove that ASNO is a rhombus.
   b. K is the midpoint of [MS], prove that K moves on a fixed circle whose diameter is to be determined.

I- (2 points)

Part of Q.	correction	Mark
1	A=$9\sqrt{3}$, B=$11\sqrt{2}$	1
2	A>B	0.5
3	$A^2=243; B^2=242; A^2-B^2=1$	0.5

II- (2 points)

1	Number of boys of the room: $\frac{30\times40}{100}+\frac{20\times60}{100}=24$. Percent of boys is: $\frac{24}{50}\times100=48$that is 48 %.	**1**
2.a	y = 1,2 x.	**0.50**
2.b	The original price is: $\frac{30000}{1,2}=25000$ LL.	**0.50**

III- (3 points)

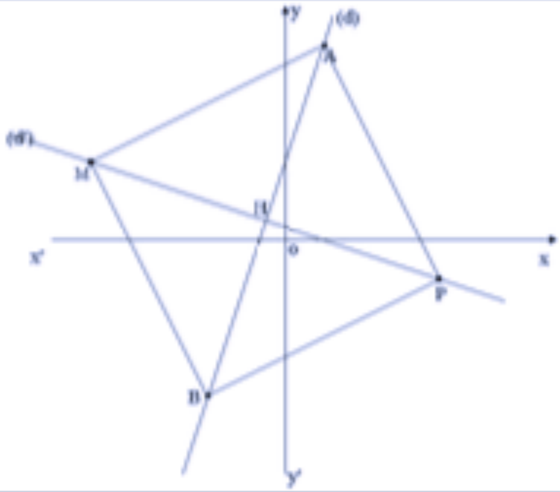
1	$\begin{cases}3x+4y=4800\\5x+6y=7500\end{cases}$	**0.75**
2	x = 600 et y = 750 The price of one rose is 600 LL and that of one tulip is 750 LL.	**1**
3	the number of roses is 7 and that of tulips is 3..	**1.25**

IV- (3 points)

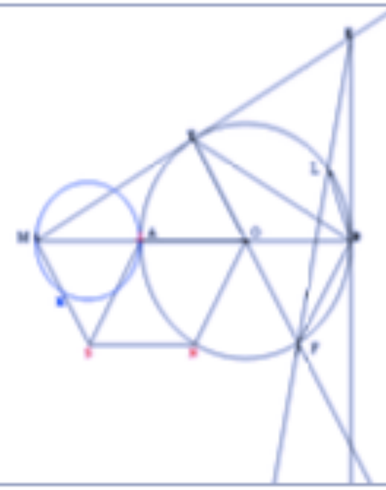
1	P(x) = (x + 9) (– 2x + 12) = -2(x+9)(x-6)	**1**
2	The area of the square = $(x+9)^2$; 6 time the area of the triangle CED = 3(x – 1) (x + 9). P(x) = 0 x = – 9 uinacceptable x = 6 acceptable	**2**

V- (5 points)

1	A belongs to (d).; B belongs to (d)	0.50
2	(d) (AB)	0.50
3.a	H $\left(-\frac{1}{2};\frac{1}{2}\right)$.	0.50
3.b	a'= $-\frac{1}{3}$ and (d') passes through H therefore ; $y=-\frac{1}{3}x+\frac{1}{3}$.	1
4.a	MA=$3\sqrt{5}$ = MB	0.50
4.b	$AB^2=90$ then AB=$\sqrt{90}=3\sqrt{10}$, MAB,MAB is a right isosceles triangle.	0.75
5.a	MA = MB = $3\sqrt{5}$ = AP and P belongs to (d') therefore PA = PB then PB = BM.	0.75
5.b	MAPB is a square.	0.50

VI- (5 points)

1.a		0.50
1.b	In the triangle MTO, [TA] is a median and TA = 2 = $\frac{1}{2}$ MO. Hence, MTO is right at T and (MT) is tangent to (C).	0.75
1.c	$MT^2 = MO^2 - OT^2 = 16 - 4 = 12$, then $MT = 2\sqrt{3}$.	0.50
1.d	$TB = MT = 2\sqrt{3}$, so the triangle MTB is isosceles with vertex T.	0.50
2.a	TAO is an equilateral triangle. $\widehat{ETB} = \widehat{EBT} = \widehat{TAB} = 60°$. Hence, the triangle ETB is equilateral. So, ET = BT, but BT = TM, then TE = TM. Therefore T is the midpoint of [EM].	0.75
2.b	$EF^2 = ET^2 + TF^2 = 12 + 16 = 28$; thus, $EF = 2\sqrt{7}$.	0.50
2.c	$\widehat{TFE} \approx 41°$ or $40°$.	0.50
3.a	ASNO is a parallelogram with OA = ON, then it is a rhombus.	0.50
3.b	MAS is an isosceles triangle and [AK] is a median, then $\widehat{AKM} = 90°$. Hence, K moves on the circle with diameter [AM].	0.50

وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات

الشهادة المتوسطة

دورة سنة 2009 الإستثنائية

عدد المسائل : خمسة	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

إرشادات عامة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الاجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة .

I. (3 points)

Consider five distinct points O, I, K, L and S such that:

$$OI = \frac{5}{2} - \cos 60° \quad , \quad OK = \frac{2\times10^{3}\left[\left(2\times10^{-1}\right)^{2} + \left(6\times10^{-2}\right)\right]}{5\times10^{-1}\times2\times10^{2}}$$

$$OL = 20\left(\frac{5}{6} - \frac{2}{15} - \frac{3}{5}\right) \quad \text{and} \quad OS = \frac{(\sqrt{10}-\sqrt{2})(\sqrt{10}+\sqrt{2})}{\sqrt{16}}.$$

1) Calculate, in showing the details of calculation OI, OK, OL and OS and write each result in the form of a natural number.
2) Show that the points I, K, L and S belong to the same circle with center O.

II. (3 points)

The following table represents the grades over 60 of 20 students:

Grade	45	48	52	56	58	60
Frequency	3	6	4	2	1	4
Increasing cumulative frequency						

1) What is the relative frequency of the grade 52?
2) Complete the previous table.
3) Plot the increasing cumulative frequency polygon.
4) What is the percentage of students having a grade greater than 57 ?
5) Determine the average grade (mean) of the students.

III. (4 points)

Given $E(x) = (2x-1)^2 + (x-2)(1-2x)$ and $F(x) = ax^2 + bx - 2$.
1) Factorize E(x).
2) Calculate a and b such that F(1) = 5 and F (-2) = 20.
3) Let $Q(x) = 6x^2 + x - 2$. Verify that Q(x) = (2x-1) (3x+2).
4) Let $P(x) = \frac{E(x)}{Q(x)}$.

a) Determine the values of x so that P(x) is defined. Then simplify P(x).
b) Solve the equation P(x) = 0.
c) Does the equation $P(x) = \frac{3}{7}$ have a solution? Justify.

IV. (5 points)

In an orthonormal system x'Ox ,y'Oy with 1cm as unit of length, consider the points $A(-2\ ;\ 3)$, $B(1;-1)$, $C(9;\ 5)$, and the line (d) with equation $y = 2x - 13$.

1) Locate the points A, B , C and plot the line (d).

2) Calculate the coordinates of N the intersection point of (d) with the axis x'Ox.

3) Prove that the triangle ABC is right angled at B.

4) Let M be the midpoint of [AC], calculate the coordinates of M.

5) Show that N is the translate of C by the translation with vector $\overrightarrow{MB}$.

6) Prove that the quadrilateral BMCN is a rhombus.

V. (5 points)

In the opposite figure:

- (S) is a semicircle with center O and radius R
- [EF] is the diameter of (S)
- A is a point on (EF) so that OA = 2 R
- (d) is a variable line through A that intersects (S) at B and C
- The tangents at B and C to (S) intersect at M.

1) Justify that (OM) is the perpendicular bisector of [BC].

2) (OM) intersects [BC] at I. let P be the orthogonal projection of M on (OA).

   a. Show that the triangles OIA and OMP are similar.

   b. Prove the relation $OA \times OP = OM \times OI$.

3) a. Let (S') be the circle circumscribed about the triangle CIM. Show that (OC) is tangent to (S').

   b. Use two similar triangles to prove that $OM \times OI = R^2$.

   c. Calculate OP in terms of R. Deduce the locus of M as (d) varies.

4) **In this question, suppose that OBMC is a square.**

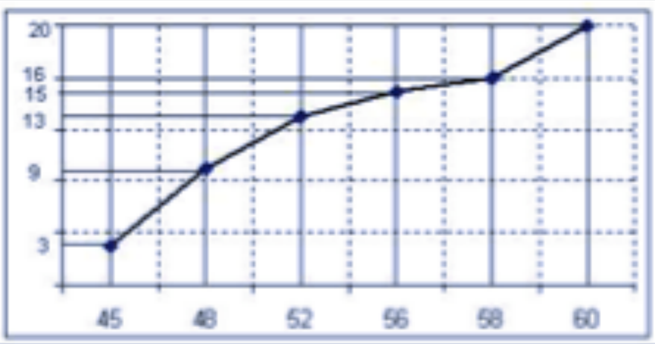
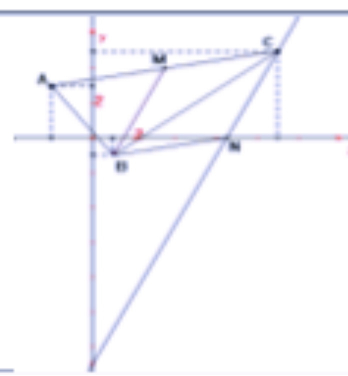
   a. Calculate the lengths BC and MP in terms of R.

   b. Calculate the exact value of $\tan \widehat{MAP}$.

   c. Calculate the value of $\widehat{MAP}$ to the nearest degree.

مشروع معيار التصحيح

مسابقة في مادة الرياضيات
المدة : ساعتان

<table>
<tr><th colspan="2">Q.</th><th>Corrigé</th><th>Note</th></tr>
<tr><td rowspan="2">I</td><td>1</td><td>$OI = \frac{5}{2} - \frac{1}{2} = 2$. $OL = 20\left(\frac{25-4-18}{30}\right) = 20\left(\frac{3}{30}\right) = 2$.
$OS = \frac{(\sqrt{10}-\sqrt{2})(\sqrt{10}+\sqrt{2})}{\sqrt{16}} = \frac{10-2}{4} = 2$. $OK = \frac{2\times10^{3}\left[(4\times10^{-2})+(6\times10^{-2})\right]}{5\times2\times10} =$
$\frac{2\times10^{3}\left[10^{-2}(4+6)\right]}{10^{2}} = \frac{2\times10\times(10^{-2}\times10)}{1} = \frac{2\times10\times10^{-1}}{1} = 2.$</td><td>2.50</td></tr>
<tr><td>2</td><td>OI = OK = OL = OS = 2 , therefore the points I , K , L and S belong to the same circle with center O and radius 2.</td><td>0.50</td></tr>
<tr><td rowspan="5">II</td><td>1</td><td>Relative frequency 52 $= \frac{4}{20} = 0.2$</td><td>0.50</td></tr>
<tr><td>2</td><td><table><tr><td>Grade</td><td>45</td><td>48</td><td>52</td><td>56</td><td>58</td><td>60</td></tr><tr><td>Frequency</td><td>3</td><td>6</td><td>4</td><td>2</td><td>1</td><td>4</td></tr><tr><td>Commulative frequency</td><td>3</td><td>9</td><td>13</td><td>15</td><td>16</td><td>20</td></tr></table></td><td>0.50</td></tr>
<tr><td>3</td><td>
</td><td>0.75</td></tr>
<tr><td>4</td><td>$\frac{5}{20}\times100 = 25\%$</td><td>0.75</td></tr>
<tr><td>5</td><td>$\bar{x} = 52.05$.</td><td>0.50</td></tr>
<tr><td rowspan="6">III</td><td>1</td><td>E(x) = (2x-1)(2x-1-x+2)=(2x-1)(x+1)</td><td>0.50</td></tr>
<tr><td>2</td><td>F(1) = a + b – 2 = 5; a + b = 7 F (-2) = 4a-2b-2 = 20 ; then a = 6 and b =1</td><td>1.25</td></tr>
<tr><td>3</td><td>(2x-1) (3x+2)= $6x^2 + x - 2$.</td><td>0.50</td></tr>
<tr><td>4a</td><td>a) $P(x) = \frac{E(x)}{Q(x)} = \frac{(2x-1)(x+1)}{(2x-1)(3x+2)}$ is defined if $x \neq \frac{1}{2}$ and $x \neq -\frac{2}{3}$; then $P(x) = \frac{x+1}{3x+2}$</td><td>0.50</td></tr>
<tr><td>4b</td><td>b) $x = -1$</td><td>0.50</td></tr>
<tr><td>4c</td><td>$\frac{x+1}{3x+2} = \frac{3}{7}$;2x=1;then $x = \frac{1}{2}$, rejected.</td><td>0.75</td></tr>
<tr><td>IV</td><td>1</td><td>Locate A,B and C
For the line (d):
<table><tr><td>x</td><td>5</td><td>8</td></tr><tr><td>y</td><td>–3</td><td>3</td></tr></table></td><td>1.25</td></tr>
</table>

	2	N (6,5; 0).	0.25
	3	ABC is right at B because…	1
	4	M(3.5 ; 4).	0.50
	5	Coordinates of the vector $\overrightarrow{MB}$: $x_{\overrightarrow{MB}} = x_B - x_M = 1 - 3{,}5 = -2{,}5$; $y_{\overrightarrow{MB}} = y_B - y_M = -1 - 4 = -5$ Coordinates of the vector $\overrightarrow{CN}$: $x_{\overrightarrow{CN}} = x_N - x_C = 6{,}5 - 9 = -2{,}5$; $y_{\overrightarrow{CN}} = y_N - y_C = 0 - 5 = -5$. Then $\overrightarrow{MB} = \overrightarrow{CN}$ so N is the translate of C with respect to $T_{\overrightarrow{MB}}$.	1
	6	$\overrightarrow{MB} = \overrightarrow{CN}$,then BMCN is a parallelogram and we have ABC is a right triangle, [BM] is the median relative to the hypotenuse therefore $BM = \frac{AC}{2} = MC$, hence BMCN is a rhombus.	1
V	1	(OM) is the perpendicular bisector of [BC] because…	0.50
	2a	OMP and OIA are similar $\widehat{OIA} = \widehat{OPM} = 90^\circ$ and $\widehat{POI}$ is a common angle.	0.50
	2b	$\frac{OM}{OA} = \frac{MP}{AI} = \frac{OP}{OI}$; then $OA \times OP = OM \times OI$	0.50
	3a	The circle circumscribed about the triangle MCI, has a diameter [MC] and (MC) $\perp$ (OC) then (OC) a tangent to (S')	0.50
	3b	The two triangles OCI and OCM are similar since : $\widehat{OCM} = \widehat{OIC} = 90^\circ$ and $\widehat{COI}$ is a common angle. $\frac{OI}{OC} = \frac{OC}{OM}$, then $OM \times OI = OC^2 = R^2$.	0.75
	3c	$OM \times OI = OA \times OP = R^2$, then $OP = \frac{R^2}{2R} = \frac{R}{2}$. O, A and P are fixed. The locus of M is the perpendicular at P to [OA].	1
	4a	$BC = OM = R\sqrt{2}$. In the right triangle OPM : $MP^2 = OM^2 - OP^2 = 2R^2 - \frac{R^2}{4} = \frac{7R^2}{4}$; $MP = \frac{R\sqrt{7}}{2}$.	0.75
	4b	$\tan \widehat{MAP} = \frac{MP}{AP} = \frac{\frac{R\sqrt{7}}{2}}{\frac{3R}{2}} = \frac{\sqrt{7}}{3}$.	0.25
	4c	$\widehat{MAP} = \tan^{-1}\frac{\sqrt{7}}{3} = 41^\circ$ (rounded down) or 42° (rounded up).	0.25