وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات	الشهادة المتوسطة	دورة العام 2010 العادية

عدد المسائل: ستة	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

إرشادات عامة :- يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات

-يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة

I- (2 points)

Consider the three numbers A, B and C.

$$A = \frac{3}{5} - \frac{1}{5} \times \left(\frac{5}{2} + 2\right), \qquad B = \frac{3\times10^{3}\times1.2\times10^{-5}}{15\times10^{3}} \quad \text{and} \quad C = \sqrt{63} - 2\sqrt{28} + \sqrt{700}.$$

In the following three questions, the steps of calculation must be shown.

1) Calculate A and write the answer as a fraction in the simplest form, then as a decimal.
2) Write B in scientific notation.
3) Write C in the form $a\sqrt{b}$ where a and b are two integers, then give to the nearest thousandth an approximate value of C.

II- (3 points)

Given $A(x) = 2(2x-3)(x-4) + (8x^2-18) - 2(2x-3)^2$.

1) Show that $8x^2 - 18 = 2(2x-3)(2x+3)$.
2) Factorize A(x).
3) Solve the equation $A(x) = 0$.
4) Let $B(x) = 2x^2 + 8x + 8$. Factorize B(x).
5) Let $F(x) = \frac{A(x)}{B(x)}$.
   a. Find x so that F(x) is defined.
   b. Simplify F(x), then solve the equation $F(x) = 3$.

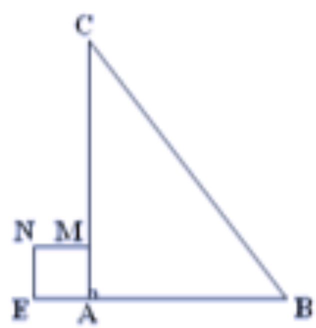
III- (2 points)

In this exercise, the unit of length is the centimeter.

In the adjoining figure, $\widehat{BAC} = 90°$,

$AB = 4$, $AM = 1$, $AC = x$ $(x > 0)$ and

AMNE is a square.



1) Express in terms of x the area of the triangle ABC.
2) Given the information: the sum of areas of the triangle ABC and the square AMNE is greater than 11.
   a. Write an inequality modeling the previous information.
   b. Solve this inequality, then compare AB and AC.

IV- (3 points)

200 people are surveyed about their favorite football team. The following table represents the results of this survey.

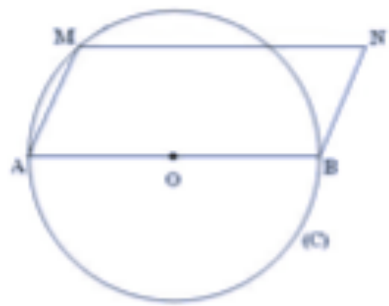
Team	Italy	Brazil	Spain	Algeria	Total
Frequency	60	40	a	30	200
% relative frequency	30	b	c	d	100
Central angle	e	f	126°	g	360°

1) Calculate a, b, c, d, e, f and g .
2) Draw the bar graph of frequencies.
3) Construct the corresponding circle graph.

V- (5 points)

In the adjoining figure:

- (C) is a circle with center O and radius R
- [AB] is a diameter of (C)
- M is a variable point on (C)
- AMNB is a parallelogram.



1) Copy this figure which will be completed in the other parts of the problem.
2) Let E be the symmetric of N with respect to B.
   a. Prove that AMBE is a rectangle.
   b. Prove that the points M, O and E are collinear, and deduce that E is a point on (C).
   c. Prove that MEN is an isosceles triangle.
3) (NO) intersects [MB] at G.
   a. Prove that (EG) intersects [MN] at its midpoint.
   b. Prove that the two triangles GOB and GNM are similar, then calculate $\frac{GN}{GO}$.
4) O' is the point defined so that $\overrightarrow{BO'} = \overrightarrow{OB}$.
   a. Show that MOO'N is a parallelogram.
   b. As M moves on (C), prove that N moves on a fixed circle with center and radius should be determined.

VI- (5 points)

In an orthonormal system of axes x'Ox and y'Oy, consider the points A (1; 1) and B (3; 2).

1) Locate the points A and B and calculate AB.
2) Verify that the equation of (AB) is $y = \frac{1}{2}x + \frac{1}{2}$.
3) The circle (C) with center A and radius AB intersects (AB) at another point D. Calculate the coordinates of D and deduce that D is on the axis x'Ox.
4) Let (d) be the tangent at B to the circle (C).
   a. Determine the equation of (d).
   b. Verify that (d) intersects y'Oy at the point E (0 ; 8).
   c. Calculate the coordinates of F, the intersection point of (d) and the axis x'Ox.
5) Let H be the translate of E by the translation with vector $\overrightarrow{BA}$. Calculate the coordinates of the point H.
6) What is the nature of the quadrilateral ABEH? Justify.
7) Calculate the value of the angle $\widehat{EAB}$ rounded to the nearest degree.

وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات

الشهادة المتوسطة

دورة العام 2010 العادية

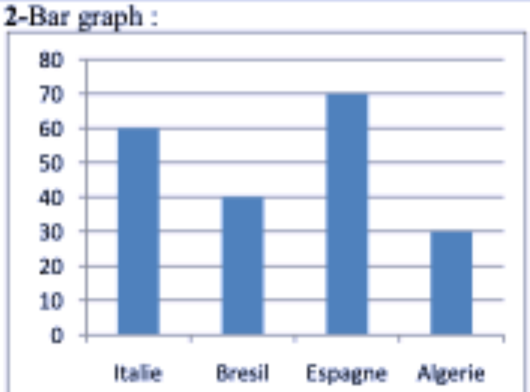
مشروع معيار التصحيح

مسابقة في مادة الرياضيات
المدة : ساعتان

I	1	$A = \frac{3}{5} - \frac{1}{2} - \frac{2}{5} = \frac{1}{5} - \frac{1}{2} = -\frac{3}{10}$. $A = -0{,}3$.	0.75
	2	$B = \frac{1{,}2 \times 10^{-5}}{5} = 2{,}4 \times 10^{-6}$.	0.50
	3	$C = 3\sqrt{7} - 4\sqrt{7} + 10\sqrt{7} = 9\sqrt{7}$. C=23,812	0.75
II	1	$8x^2 - 18 = 2(4x^2 - 9) = 2(2x - 3)(2x + 3)$.	0.50
	2	$A(x) = 2(2x - 3)[(x - 4) + (2x + 3) - (2x - 3)]$ $= 2(2x - 3)[x - 4 + 2x + 3 - 2x + 3]$ $= 2(2x - 3)(x + 2)$.	0.75
	3	$A(x) = 2(2x - 3)(x + 2) = 0$ $2x - 3 = 0$; $x = \frac{3}{2}$ or $x + 2 = 0$; $x = -2$.	0.50
	4	$B(x) = 2x^2 + 8x + 8 = 2(x^2 + 4x + 4) = 2(x + 2)^2$.	0.50
	5.a	F(x) is defined for $x + 2 \neq 0$ and $x \neq -2$.	0.25
	5.b	$F(x) = \frac{2(2x-3)(x+2)}{2(x+2)^2} = \frac{2x-3}{x+2}$; F(x)=3, x = - 9	0.50
III	1	Area of ABC = $\frac{1}{2}$ AB × AC = $\frac{4x}{2} = 2x$.	0.50
	2.a	Area of AMNE = $AM^2 = 1$ if Area ABC + Area AMNE > 11,then $2x + 1 > 11$.	0.50
	2.b	$2x + 1 > 11$, ,then $2x > 10$ and $x > 5$ sincex > 5, ,then x > 4 andAC > AB.	1
IV	1	a = 70, b = 20, c = 35, d = 15. $e = 108°$, $f = 72°$, $g = 54°$.	1.75

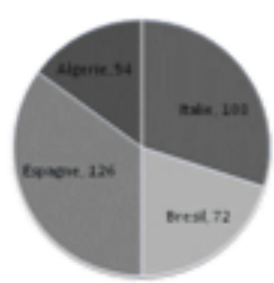
IV – 2 (0.50)

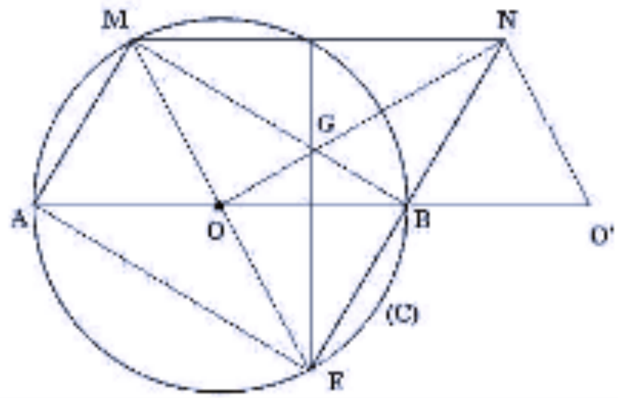
2-Bar graph :



IV – 3 (0.75)

3Circle graph :



	1		0.25
V	2.a	EB = AB (= BN) et (EB) // (AM), moreover $\widehat{AMB}=90^o$, then AMNB is a rectangle	0,75
	2.b	O is the midpoint of the diagonal [AB], then O is the midpoint of [ME]. Since OM = OE = radius, then E is a point on (C	0.75
	2.c	B is the midpoint of [EN], and $\widehat{MBE}=90^o$, then MEN is an isosceles triangle with vertex M.	0.50
	3.a	In the triangle MEN, [MB] is a median and [NO] is also a median, then G is the centroid of this triangle. Hence, [EG] is the third median through G to [MN].	0.75
	3.b	GOB and GNM are similar … $\frac{GN}{GO}=2$	0.75
	4.a	$\overrightarrow{BO'}=\overrightarrow{OB}$, then $\overrightarrow{AB}=\overrightarrow{OO'}$. Since $\overrightarrow{AB}=\overrightarrow{MN}$, then $\overrightarrow{OO'}=\overrightarrow{MN}$. Therefore MOO'N is a parallelogram.	0. 50
	4.b	Refer to (4.a) O'N = OM = R, since O' is fixed and R is constant, then N moves on the circle with center O' and radius R.	0.75
	1	 AB= $\sqrt{5}$	0.75
VI	2	The equation of (AB) is y = $\frac{1}{2}x+\frac{1}{2}$	0.50
	3	D (-1; 0) ;D is a point on x'ox	0. 50
	4.a	$a_{(AB)}=\frac{1}{2}$, then $a_{(d)}=-2$, (d) passes through B ,then the equation is y = -2x+8	0.75
	4. b	E (0 ; 8) is a point on (d).	0.25
	4. c	F (4 ; 0)	0.25
	5	$\overrightarrow{BA}(-2;-1)$; $\overrightarrow{BA}=\overrightarrow{EH}$ then H (-2;7)	0.75
	6	$\overrightarrow{BA}=\overrightarrow{EH}$ and the angle ABE = 90° then ABEH is a rectangle	0.50
	7	tanEAB=3,value by calculator : 71,5650…°.rounded value of EAB :71° or72°.	0.75

عدد المسائل :خمسة

مسابقة في مادة الرياضيات
المدة: ساعتان

الاسم:
الرقم:

ارشادات عامة :- يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة.

I- (3 points)

Dans le tableau suivant, une seule réponse proposée à chaque question est correcte. Ecrire le numéro de chaque question et donner, avec justification, la réponse correspondante.

N°	Questions	Réponses		
		a	b	c
1	Le prix d'un article subit une augmentation de 8% et devient 1 350 LL Le prix initial de cet article est:	1 458 LL	1 242 LL	1 250 LL
2	OAB est un triangle, C et F sont deux points tels que: $\overrightarrow{OA}+\overrightarrow{OB}=\overrightarrow{OC}$ et $\overrightarrow{AF}=\overrightarrow{OC}$. Donc:	B est le milieu de [CF]	C est le milieu de [BF]	F est le milieu de [BC]
3	Si $2^{15}-x=2^{14}$, alors x =	2^{14}	2	2^{7}
4	Dans un repère orthonormé d'axes x'Ox et y'Oy, on donne le point A(3; 1) et la droite (d) d'équation y = 2 . L'équation de la droite (d') passant par A et perpendiculaire à (d) est :	$y=-x+4$	$y=1$	$x=3$

II- (2 points)

Une librairie offre une réduction de 10 % sur ses articles.

La somme des prix initiaux d'un stylo et d'un agenda est trois fois le prix initial du stylo.

La somme des prix réduits du stylo et de l'agenda est 54 000 LL.

1) Traduire les informations ci-dessus par un système de deux équations à deux inconnues.

2) Résoudre ce système et trouver le prix initial d'un stylo et celui d'un agenda.

III- (4 points)

On donne: $P(x)=-x^2+6x-8$ et $Q(x)=(x-2)^2-3(x-2)$.

1) Montrer que $P(x)=(x-2)(4-x)$, puis résoudre l'équation $P(x)=0$.

2) Factoriser Q(x).

3) On donne $F(x)=\dfrac{Q(x)}{P(x)}$.

a. Pour quelles valeurs de x, l'expression F(x) est-elle définie?

b. Simplifier F(x), puis résoudre l'équation F(x) = 1.

c. L'équation $F(x)=-\dfrac{3}{2}$ admet-elle une solution? Pourquoi?

IV- (6 points)

Dans un repère orthonormé d'axes x'Ox et y'Oy, on donne la droite (d) d'équation $y = -2x + 3$ et les points A (0; – 2) et E (6; 1).

1) Placer A et E.

2) La droite (d) coupe y'Oy en G.

a. Calculer les coordonnées de G.

b. Placer G et tracer (d).

3) a. Montrer que $y = \frac{1}{2}x - 2$ est l'équation de (AE).

b. Montrer que (d) est perpendiculaire à (AE) au point B(2; – 1).

c. Montrer que le triangle rectangle GBE est isocèle.

4) On désigne par I le milieu de [GE] et par M le symétrique de B par rapport à I.

a. Montrer que le quadrilatère BGME est un carré.

b. Calculer les coordonnées de M.

5) On désigne par (d') le translaté de la droite (AE) par la translation de vecteur $\overrightarrow{BG}$

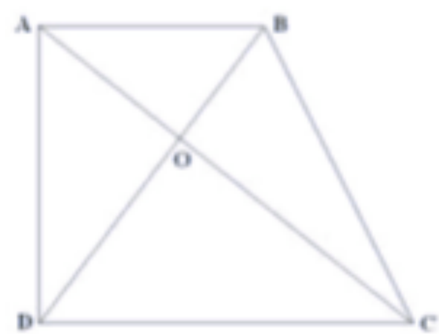
a. Tracer (d').

b. Trouver l'équation de (d').

V- (5 points)

Dans la figure ci-contre où l'unité de longueur est le cm :

- ABCD est un trapèze rectangle
- AB = 3 ; AD = 4 ; CD = 5
- les droites (AB) et (CD) sont parallèles
- les droites (AC) et (BD) se coupent en O.



1) Reproduire la figure.

2) Montrer que le triangle BCD est isocèle de sommet principal D.

3) Calculer l'aire du trapèze ABCD.

4) Montrer que l'on a : $OA \times OD = OC \times OB$.

5) Les droites (AD) et (BC) se coupent en S. Montrer que les angles $\widehat{CBD}$ et $\widehat{ABS}$ sont égaux.

6) On pose SA = x.

a. Montrer que : $\frac{x}{x+4} = \frac{3}{5}$, puis calculer SA.

b. Déterminer la valeur arrondie au degré près de l'angle $\widehat{ASB}$.

c. Soit H le milieu de [BC]. Montrer que les quatre points A, B, H et D appartiennent à un même cercle dont on déterminera un diamètre.

وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات

الشهادة المتوسطة

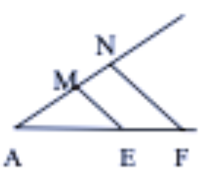
دورة العام 2011 العادية

عدد المسائل: ستة	مسابقة في مادة الرياضيات المدة ساعتان	الاسم: الرقم:

ارشادات عامة :- يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة .

I- (2 points)

In the following table, only one of the proposed answers to each question is correct.
Write the number of each question and its corresponding answer. Justify your choice.

N°	Questions	Proposed answers		
		a	b	c
1	If $\left(x+\frac{1}{x}\right)^2=15$ then $x^2+\frac{1}{x^2}=\ldots$	225	13	$\sqrt{15}$
2	n is a natural number $\left(\frac{4}{5}\right)^{n+1}\times\left(\frac{5}{4}\right)^{n}=\ldots$	8×10^1	1^{2n+1}	8×10^{-1}
3	In an orthonormal system, the two lines $(D_1): y=(2-\sqrt{5})x-5$ and $(D_2): y=(2+\sqrt{5})x+5$ are …	parallel	perpendicular	intersecting at B(0 ; 5)
4	(ME) and (NF) are two parallel lines , then $\frac{NF}{ME}=\ldots.$ 	$\frac{AN}{AM}$	$\frac{AE}{AF}$	$\frac{AN}{MN}$

II- (2 points)

Given : $A=\frac{8}{3}-\frac{5}{3}\times\frac{21}{15}$; $B=\frac{3.4\times10^{-3}\times5\times\left(10^2\right)^3}{4\times10^{-3}}$ and $C=\frac{\left(1-\sqrt{3}\right)^2}{\left(2+\sqrt{3}\right)^2}$.

Show all the details of the following :

1) Calculate A and write the answer as a fraction in the simplest form.
2) Calculate B and write the answer in scientific notation.
3) Write C in the form $a-b\sqrt{3}$ where a and b are two integers.

III- (3 points)

An agency for renting cars proposes to its customers the following two offers A and B:

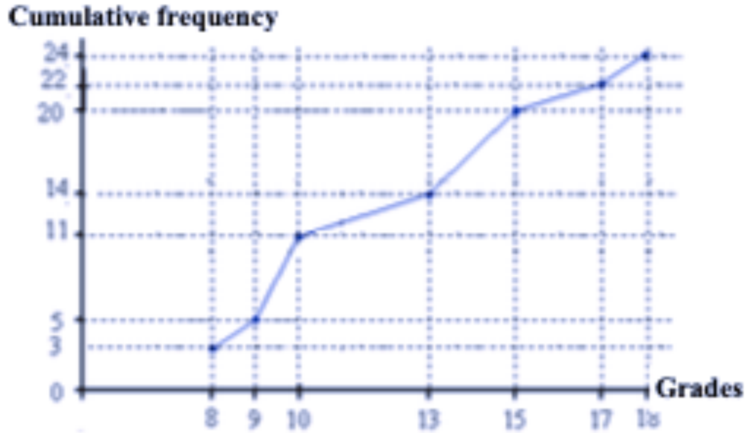
	Deposit	Charge per km
Offer A	50 000 LL	600 LL
Offer B	42 000 LL	700 LL

Denote by x the number of kilometers traveled by a car that a customer rents.

1) Find in term of x, the amount S paid by this customer if he selects the offer A, and the amount S' if he selects the offer B.
2) Calculate x so that S is equal to S' .
3) Starting what traveled distance is the offer A more advantageous than the offer B? justify.
4) A second customer selects offer A and pays 410 000 LL. What is the traveled distance by this customer?

IV- (3 points)

The adjacent graphic represents the cumulative frequency polygon of the students' grades in a certain class.



1) What is the number of students of this class?

2) Complete the following table:

Grades	8	9	10	13	15	17	18
Cumulative frequency	3	5					
Frequency	3	2					

3) Write as percent the relative frequency of grade 10.

4) What is the average grade of the students of this class ?

V- (5 points)

In an orthonormal system of axes, x'Ox and y'Oy, consider the line (D) with equation $y = -2x-3$ and the two points A (-2 ; 1) and B (6 ; 5).

1) Verify that (D) passes through A.

2) Plot A and B and draw (D).

3) Determine the equation of (AB) and deduce that (D) is perpendicular to (AB).

4) Calculate, rounded to the nearest degree, the value of the acute angle that (AB) makes with x'Ox .

5) The line (D) intersects y'Oy at C. Find the coordinates of C.

6) Let (S) be the circle circumscribed about the triangle ABC. I is the center of this circle . Calculate the coordinates of I.

7) The line (D') is the parallel through C to (AB) . (D') intersects the circle (S) at another point E.

a. What is the nature of the quadrilateral ABEC? Justify.

b. Calculate the coordinates of the point E.

c. (d) is the tangent at A to (S). Find the equation of (d).

VI- (5 points)

ABE is an isosceles triangle with vertex B, and so that BE = BA = 6 cm and $\widehat{ABE} = 140^\circ$.
The circle (C) with diameter [BE] and center O intersects (AB) at another point F.

1) Make a figure.

2) What is the nature of the triangle BEF? Justify.

3) I is the midpoint of [AE]. Show that I is a point on (C).

4) **a.** Calculate $\widehat{BAE}$ and $\widehat{EBF}$.

b. Find to the nearest thousandth an approximate value of BF.

5) Prove that the two triangles ABI and AEF are similar, and deduce that $AB \times AF = 2 \times AI^2$.

6) G is the translate of E under the translation with vector $\overrightarrow{FB}$.

a. Show that EFBG is a rectangle but not a square.

b. Prove that G, O and F are collinear.

وزارة التربية والتعليم العالي
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مشروع معيار التصحيح	مسابقة في مادة الرياضيات المدة ساعتان	الاسم: الرقم:

Answer the six following exercises:

First exercise (2 points)

Part of the Q	Answer	Mark
1	$(x+\frac{1}{x})^2 = x^2+\frac{1}{x^2}+2 = 15$ therefore $x^2+\frac{1}{x^2} = 13$ (b).	0.50
2	$\left(\frac{4}{5}\right)^{n+1} \times \left(\frac{5}{4}\right)^{n} = \frac{4^1}{5^1} = \frac{4}{5} = 0{,}8 = 8 \times 10^{-1}$ (c).	0.50
3	$a \times a' = (2-\sqrt{5})(2+\sqrt{5}) = 4-5 = -1$. therefore (D_1) and (D_2) are perpendicular. (b).	0.50
4	Using Thalès : $\frac{NF}{ME} = \frac{AN}{AM}$. (c).	0.50

Second exercise (2 points)

Part of the Q	Answer	Mark
1	$A = -\frac{5}{3}$.	**0.50**
2	$B = 4\times10^5 = 0.4\times10^6$.	**0.75**
3	$C = \frac{(4-2\sqrt{3})(7-4\sqrt{3})}{49-48} = 52-30\sqrt{3}$.	**0.75**

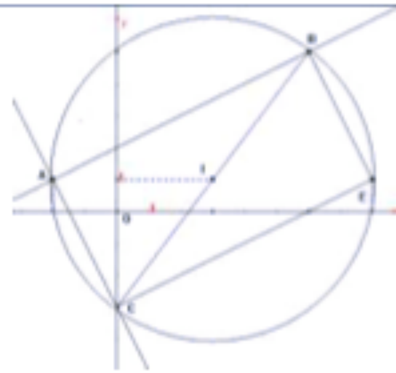
Third exercise (3 points)

Part of the Q	Answer	Mark
1	The number of the students is: 24.	**0.50**
2	**Cumulative frequency**: 8, 9, 10, 13, 15, 17, 18; **Grades**: 3, 5, 6, 3, 6, 2, 2	**1.50**
3	$\frac{2}{24}\times 100=\frac{25}{3}$, then $\frac{25}{3}$%.	**0.50**
4	The average grade $\overline{X}$: $\overline{X}=\frac{(8\times 3)+(9\times 2)+(10\times 6)+(13\times 3)+(15\times 6)+(17\times 2)+(18\times 2)}{24}$ $\overline{X}=\frac{301}{24}\approx 12.54.$	**0.50**

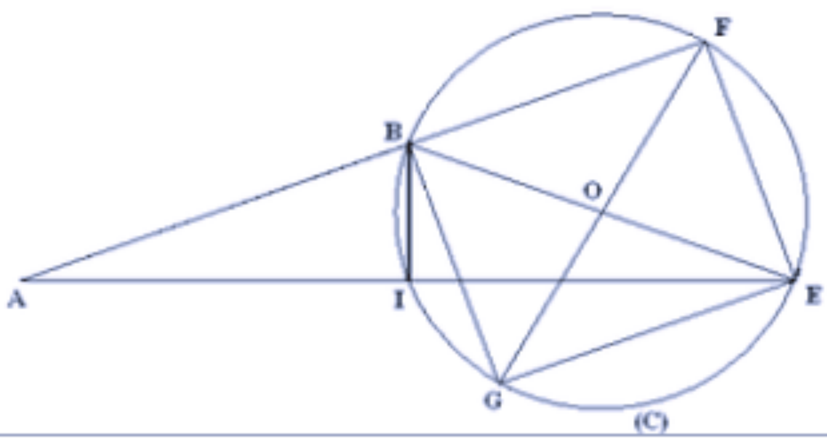
Fourth exercise (3 points)

Part of the Q	Answer	Mark
1	S = 50000 + 600x ; S' = 42000 + 700x	**1**
2	S = S' then x = 80. The traveled distance is 80 km.	**0.50**
3	S < S' then x > 80 ; the offer A is better than B for every distance greater than 80 km.	**0.75**
4	410000 = 50000 + 600 x x = 600.	**0.75**

Fifth exercise (5 points)

Part of the Q	Answer	Mark
1	 Fig.	1.25
2	The coordinates of A verify the equation (D)	0.5
3	$a_{(AB)} = \frac{y_B - y_A}{x_B - x_A} = \frac{4}{8} = \frac{1}{2}$, where the equation of (AB) : $y = \frac{1}{2}x + 2$. (D) is perpendicular (AB) (product of the slopes equals -1)	1
4	C = (0;-3)	0.25
5	I (3 ;1)	0.5
6.a	ABEC is a rectangle since ……..	0.5
6.b	E (8;1)	0.5
7	$\alpha = \tan^{-1}\frac{1}{2} = 27^\circ$	0.5

Sixth exercise (5 points)

Part of the Q	Answer	Mark
1	La figure: 	0.50
2	BEF is right at F.	0.50
3	(BI) $\perp$ (AB), then $\widehat{BIE} = 90^\circ$. Hence I is on (C).	0.75
4.a	$\widehat{ABE} = 180^\circ - 40^\circ = 140^\circ$; $\widehat{EBF} = 40^\circ$.	0.50
4.b	EBF is right at F , $\cos \widehat{EBF} = \frac{BF}{BE}$ $BF = 6 \times \cos 40^\circ$; $BF \approx 4.596$.	0.75
4.c	$EF^2 = BE^2 - BF^2$, $EF \approx 3.8$ (Or : $EF = BE \times \sin 40^\circ$).	0.50
5.a	EFBG is a parallelogram and $\widehat{BFE} = 90^\circ$, then it is a rectangle. Moreover EF $\neq$ BF, then it is not a square.	0.75
5.b	[GF] is a diagonal of the rectangle EFBG, then O is the midpoint of [GF].	0.75

الدورة الإستثنائية للعام 2011	الشهادة المتوسطة	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات

الاسم: الرقم:	مسابقة في مادة الرياضيات المدة ساعتان	عدد المسائل: خمسة

ارشادات علمة :- يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة .

I- (2 points)

1) Calculate the greatest common divisor (GCD) of 154 and 112.

2) Write the fraction $\frac{154}{112}$ in its simplest form.

3) Let $m = \frac{154}{112} + \frac{1}{8}$.

a. Write m as a fraction in the simplest form .

b. Is the number m decimal? Justify.

II- (3 points)

The two parts **A** and **B** of this exercise are independent.

<u>Part **A**</u>

Given $P(x) = (3x-2)(x+2) - (3x-2)^2$.

1) a. Develop and reduce P(x).

b. Calculate $P(\sqrt{5})$.

2) a. Factorize $P(x)$.

b. Solve the equation $P(x) = 0$.

<u>Part **B**</u>

Given two real numbers x and y such that $xy = 2\sqrt{3}$ and $x + y = 2 + 2\sqrt{3}$.

1) Calculate $x^2y + xy^2$.Give the result in the form $a + b\sqrt{3}$ where a and b are two integers.

2) Calculate $x^2 + y^2$.

III- (3 points)

To buy **three** copybooks and **two** pens we must pay 4 500 LL. To buy **six** copybooks and **three** pens we must pay 7 500 LL. This information is translated into the following system:

$$\begin{cases} 3x + 2y = 4\,500 \\ 6x + 3y = 7\,500 \end{cases}$$

1) What does x and y represent in this system?

2) Solve the previous system, showing the details of the steps you follow, to find the price of a copybook and that of a pen.

3) A student bought a pack which contains copybooks and pens, and he paid 11 000 LL. Calculate the number of copybooks and the number of pens in this pack knowing that the sum of these two numbers is 12.

IV- (6 points)

In an orthonormal system of axes x'Ox and y'Oy, consider the points A(3; 4) , B(3;– 1), C(1; 3) and the line (d) with equation $y = -2x + 5$.

1) Plot the points A, B and C.

2) Prove that B and C are two points on (d), then draw (d).

3) a. Find the equation of the line (CA).

b. prove that the triangle ABC a right at C .

4) D is the point defined by $\overrightarrow{CD} = \overrightarrow{CA} + \overrightarrow{CB}$.

a. Prove that CADB is a rectangle.

b. Calculate the coordinates of D.

5) E is the symmetric of C with respect to A.

a. What is the nature of the quadrilateral ABDE? Justify.

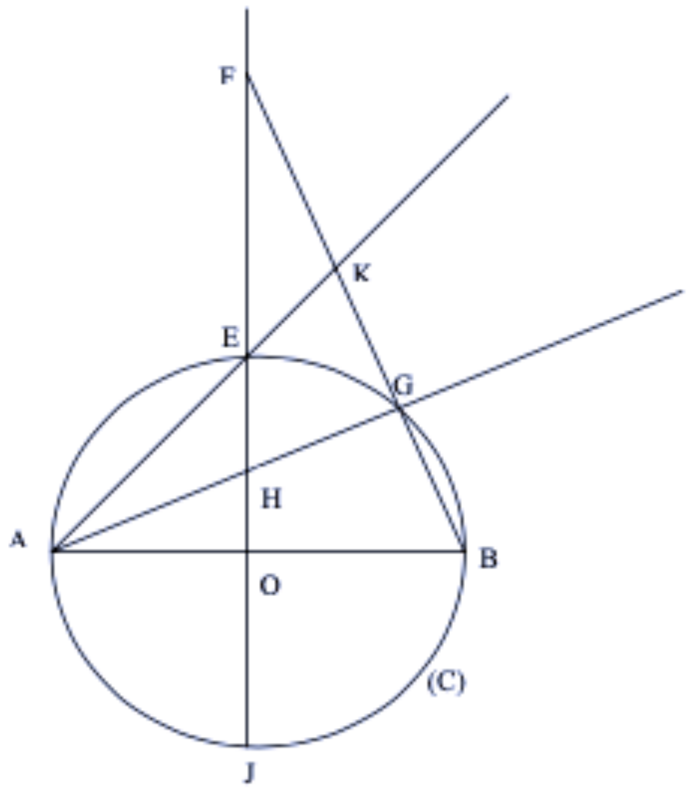
b. Prove that CDE is an isosceles triangle.

c. Prove that (DE) is parallel to y'Oy and write the equation of (DE).

V- (6 points)

In the fallowing figure:

- (C) is a circle with diameter [AB], center O and radius 3 cm
- The perpendicular at O to (AB) intersects (C) at E and J
- The bisector of the angle $\widehat{EAB}$ intersects [OE] at H and intersects (C) at another point G
- The line (BG) intersects (AE) at K and (OE) at F.



1) Reproduce the figure.

2) Verify that $\widehat{BAG} = \frac{45^\circ}{2}$.

3) Prove that the triangle ABK is isosceles with vertex A.

4) Calculate AE and EK.

5) Prove that the two triangles AOH and AGB are similar. Deduce the value of the product AH × AG.

6) **a.** Using $\cos\left(\frac{45^\circ}{2}\right)$ in the triangle AOH, calculate AH to the nearest hundredth.

b. Deduce an approximate value of the similarity ratio of the triangles AOH and AGB.

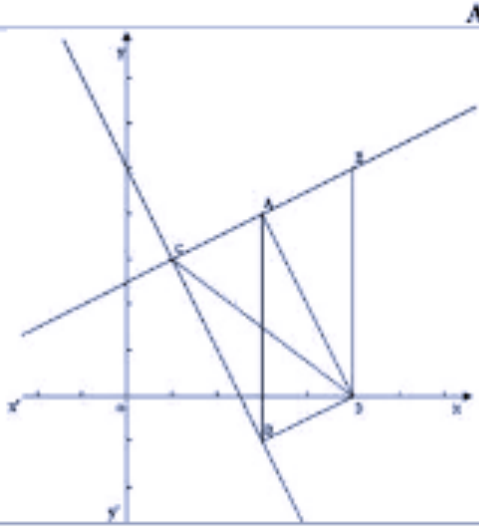
7) (BH) and (AF) intersect at I, prove that I is a point on (C).

الدورة الإستثنائية للعام 2011	الشهادة المتوسطة	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات
الاسم: الرقم:	مسابقة في مادة الرياضيات المدة ساعتان	مشروع معيار التصحيح

I(2points)	Answer	Mark
1	$154 = 2 \times 11 \times 7$ $112 = 2^4 \times 7$; GCD (154; 112) = $2 \times 7 = 14$.	0.75
2	$\frac{154}{112} = \frac{11}{8}$.	0.50
3.a	$m = \frac{154}{112} + \frac{1}{8} = \frac{12}{8} = \frac{3}{2}$	0.75
3.b	m = 1,5 ; m is a decimal number .	

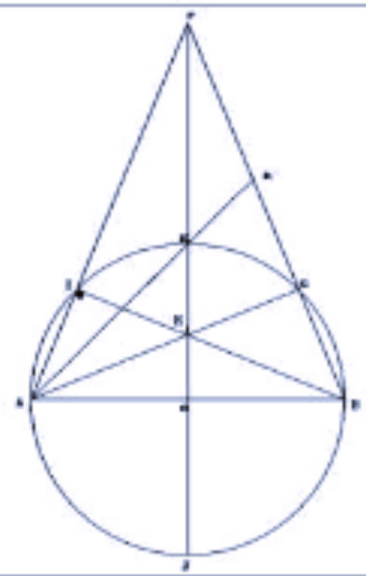
II(3points)		Answer	Note
A	1.a	$P(x) = -6x^2 + 16x - 8$.	0.5
	1.b	$P(\sqrt{5}) = 16\sqrt{5} - 38$	0.5
	2.a	$P(x) = (3x-2)(-2x+4) = -2(3x-2)(x-2)$	0.5
	2.b	$x = \frac{2}{3}$ or $x = 2$	0.5
B	1	$x^2y + xy^2 = xy(x+y) = 2\sqrt{3}(2+2\sqrt{3}) = 4\sqrt{3} + 12$.	0.5
	2	$x^2 + y^2 = (x+y)^2 - 2xy = (2\sqrt{3}+2)^2 - 4\sqrt{3} = 16 + 4\sqrt{3}$.	0.5

III.(3points)	Answer	Mark
1	x is the price of a copybook ; y is the price of a pen .	0.50
2	$\begin{cases} -6x - 4y = -9000 \\ 6x + 3y = 7500 \end{cases}$ y = 1500 and x = 500. The price of a copybook is 500LL and of a pen is 1500LL.	1.25
3	Let a be the number of pens ; (12 – a) the number of copybooks . 1500a + 500(12 – a) = 11000.Hence a = 5 and 12 – a = 7	1.25

IV(6points)	Answer	Mark
1		0.50
2	$Y_B = -2x_B + 5$; then B is on (d). $Y_C = -2x_C + 5$; then C is on (d), hence (d) is (BC),	0,75

3.a	(CA), $y = ax + b$ $\begin{cases} 3a + b = 4 \\ a + b = 3 \end{cases}$ $a = \frac{1}{2}$, $b = \frac{5}{2}$ $y = \frac{x}{2} + \frac{5}{2}$	**1**
3.b	$(-2) \times \frac{1}{2} = -1$ then : (d) $\perp$ (AC). ABC is right at C.	**0.50**
4.a	Given $\overrightarrow{CD} = \overrightarrow{CA} + \overrightarrow{CB}$ then CADB is a parallelogram . Since $\hat{C} = 90^o$, , then CADB is a rectangle .	**0.50**
4.b	$\overrightarrow{BD} = \overrightarrow{CA}$; then $x_D - x_B = x_A - x_C$ and $y_D - y_B = y_A - y_C$, thus D(5; 0)	**0.75**
5.a	Since AE = BD and (AE) parallel to (BD).Then ABDE is a parallelogram	**0.75**
5.b	A is the midpoint of [CE] and (DA) is perpendicular to (CE), then CDE is an isosceles triangle with vertex D.	**0.50**

5.c	ABDE is a parallelogram, then (DE) // (AB).
	 Since $x_A = x_B$, then (AB) // y'Oy.
	 Therefore (DE) is parallel to y'Oy. The equation of (DE) x = 5.
		0.75

V(6points)	**Answer**	**Mark**
1		**0.50**
2	$BAC = \frac{45^0}{2}$ ….	**0.50**
3	$\widehat{AGB} = 90°$; [AG] is at the same time bisector and altitude in the triangle BAK. Hence BAK is isosceles with vertex A.	**0.75**
4	$AE = 3\sqrt{2}$ cm, $EK = AK - AE = (6 - 3\sqrt{2})$cm.	**0.75**
5	$\hat{O} = \hat{G} = 90°$ and $\widehat{OAH} = \widehat{BAG}$. $\frac{AO}{AG} = \frac{AH}{AB} = \frac{OH}{GB}$, then $AH \times AG = AO \times AB$; $AH \times AG = 18$.	**1.25**
6.a	AOH right at O and $\widehat{OAH} = \frac{45^0}{2}$.cos $\widehat{OAH} = \frac{OA}{AH}$, $AH \approx 3.25$	**0.75**
6.b	$\frac{AH}{AB} \approx \frac{3.25}{6}$, hence 0.54 is an approximate value of the ratio..	**0.50**
7	[AG] and [OF] are two altitudes of the triangle ABF; Hence [BI] is the third altitude. Therefore $\widehat{AIB} = 90^o$ and I is a point on (C).	**1**

وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات	الشهادة المتوسطة	دورة العام ٢٠١٢ العادية

عدد المسائل :ستة	مسابقة في مادة الرياضيات المدة ساعتان	الاسم: الرقم:

ارشادات عامة :- يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.

- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة .

I- (3 points)

In the table below, only one of the proposed answers to each question is correct. Write down the number of each question ,and give with justification, its corresponding answer .

N°	Question	Proposed answers		
		a	b	c
1	In an orthonormal system, the two lines with equations $y = 2x + 3$ and $2y + x = 1$ are…	intersecting at the point (1,5)	perpendicular	parallel
2	The original price of an article is 30 000 LL ,and its price after the discount is 27 600 LL . The percentage of discount is…	10 %	18 %	8 %
3	A triangle ABC is right at B. If AB=3 and $\widehat{ACB} = 40^\circ$, then AC = …	$\frac{3}{\sin 40^\circ}$	$3\sin 40^\circ$	$\frac{\sin 40^\circ}{3}$
4	The natural numbers, solutions of 3x-1<8 are…	1 ;2 ;3	0 ;1;2 ;3	0 ;1;2
5	A triangle ABC is right at A and M is the midpoint of [BC]. If AM = AB = 6 then AC =…	12	$6\sqrt{3}$	$9\sqrt{3}$
6	ABCD is a parallelogram and E is the translate of D under the translation with vector $\overrightarrow{BA}$. Therefore…	C is the midpoint of [DE]	$\overrightarrow{DB} + \overrightarrow{DE} = \overrightarrow{DA}$	$\overrightarrow{DE} + \overrightarrow{DA} = \overrightarrow{DB}$

II-(2 points)

Consider the three numbers A , B and C :

$$A = \left(\sqrt{5}+1\right)^2 + \left(\sqrt{5}-1\right)^2 ; \; B = \frac{3}{4} + \frac{5}{4} \times \frac{7}{15} \text{ and } C = \frac{\left(10\sqrt{3}\right)^4}{25\times 10^3 \times 3 + 10^3 \times 6 \times 37.5}.$$

In the following questions , the steps of calculation must be shown

1) Prove that A is a natural number.

2) Write B as a fraction in its simplest form.

3) Prove that C is a decimal number.

III- (2 points)

In what follows, are the scores of a student in five tests: 10 ; 8 ; 13 ; x and y.

The difference between x and y is 7. The average (mean) of these five scores is 12.

1) Write a system of two equations with two unknowns modeling the given situation .

2) Solve the obtained system .

IV- (3 points)

Consider the expressions: $A(x) = 4x^2 - 9$ and $B(x) = (2x-3)^2 - (2x-3)(x-5)$.

1) Factorize A(x).

2) **a-** Verify that $B(x) = (2x-3)(x+2)$.

b- Solve the equation $B(x) = 0$.

3) Consider the expression $F(x) = \dfrac{A(x)}{B(x)}$.

a. For what values of x, F(x) is defined ?

b. Simplify F(x) and solve $F(x) = 3$.

c. Calculate $F(\sqrt{5})$ and write the answer in the form $a - b\sqrt{5}$. (a and b are natural numbers)

V- (5 points)

In an orthonormal system of axes x'Ox, y'Oy, where the unit of length is the centimeter, consider the points A (1;-2), B(2;1), C(5;0) and the line (d) with equation $y = 3x - 5$.

1) **a.** Verify that (d) is passing through A and B.

b. Plot the points A, B, C and draw (d).

2) **a.** Determine the equation of the line (BC).

b. Calculate the lengths AB and BC.

c. Prove that ABC is a right isosceles triangle with vertex B.

3) Let (G) be the circle circumscribed about the triangle ABC and D is the point defined by $\overrightarrow{BD} = \overrightarrow{AC}$.

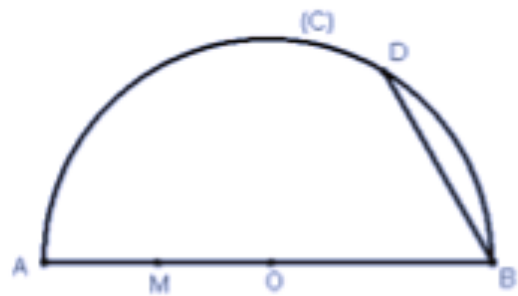
a. Calculate the coordinates of D.

b. Prove that (DB) is tangent to (G).

4) (d') is the translate of (d) under the translation with vector $\overrightarrow{AC}$. Find the equation of (d').

VI- (5 points)

Consider a semicircle (C) with centre O and radius R . [AB] is the diameter of (C) and D is a point on (C) so that BD = R.



Let M be the midpoint of [OA]. The perpendicular bisector of [OA] intersects [AD] at E, (BD) at F and (C) at K.

1) Reproduce and complete the figure.

2) Calculate the angles of the triangle ABD, then calculate AD in terms of R.

3) **a.** Prove that the two triangles ADB and FMB are similar.

b. Calculate BF in terms of R.

4) (BE) intersects (AF) at J. Show that J is on (C).

5) Prove that the points B, M, J and F are on the same circle whose center and radius should be determined.

6) **a.** Show that the triangle OKA is equilateral.

b. Prove that (OK) passes through the midpoint of [AF].

مشروع معيار التصحيح

مسابقة في مادة الرياضيات
المدة ساعتان

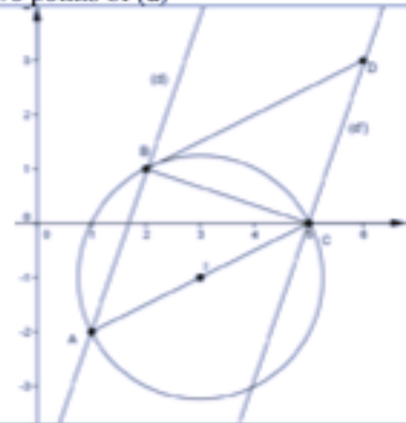
الاسم:
الرقم:

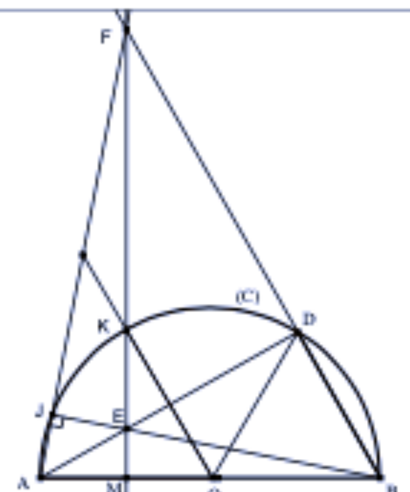
	Part of the Q	Answer	Mark
I (3pts)	1	$(2)(-0.5) = -1$, therefore (b)	0.5
	2	$30\,000 - 27\,600 = 2400 = \frac{30000 \times x}{100}$; $x=8$ (8 %), therefore (c)	0.5
	3	$\frac{3}{\sin 40^\circ}$ therefore (a)	0.5
	4	0;1;2 therefore (c)	0.5
	5	ABM is equilateral, then BC=12 and AC=$\frac{12\sqrt{3}}{2} = 6\sqrt{3}$. therefore (c)	0.5
	6	DEAB is a parallelogram, $\overrightarrow{DB} + \overrightarrow{DE} = \overrightarrow{DA}$ therefore (c)	0.5

	Part of the Q	Answer	Mark
II (2pts)	1	A=12	0.75
	2	B=$\frac{4}{3}$	0.5
	3	C =0.3	0.75

	Part of the Q	Answer	Mark
III (2pts)	1	$\frac{x+10+8+13+y}{5} = 12$; so, $x + y + 31 = 60$ Thus , $x + y = 29$ The system is $\begin{cases} y = x+7 \\ x+y=29 \end{cases}$	1.25
	2	$x = 11$, so $y = 18$.	0.75

	Part of the Q	Answer	Mark
IV (3pts)	1	$A(x) = (2x-3)(2x+3)$	0. 5
	2.a	$B(x) = (2x-3)(x+2)$.	0.5
	2.b	$B(x) = 0$ $x = -2$ or $x = \frac{3}{2}$	0.5
	3. a	F(x) is defined $x \neq -2$ and $x \neq \frac{3}{2}$	0.25
	3.b	$F(x) = \frac{2x+3}{x+2}$ $F(x) = 3$; $x = -3$.	0.75
	3.c	$F(\sqrt{5}) = \frac{2\sqrt{5}+3}{2+\sqrt{5}} = 4 - \sqrt{5}$.	0.5

V (5pts)	1. a	A and B are two points of (d)	**0.5**
	1. b		**0.75**
	2. a	$(BC): y = -\frac{1}{3}x + \frac{5}{3}$	**0.75**
	2. b	$AB = \sqrt{10}$; $BC = \sqrt{10}$	**0.75**
	2. c	$AB = \sqrt{10}$; $BC = \sqrt{10}$ and (AB) is perpendicular to (BC) then ABC is a right isosceles triangle at B.	**0.5**
	3.a	$\overrightarrow{AC}(4;2); D(x;y); \overrightarrow{BD}(x-2; y-1)$ then $D(6;3)$	**0. 5**
	3.b	(BD) is perpendicular to (IB)	**0.75**
	4	(d'): y = 3x - 15	**0.5**

VI (5pts)	1	 Figure	**0. 5**
	2	$\widehat{ADB} = 90^0$, $\widehat{ABD} = 60^0$, $\widehat{BAD} = 30^0$: $AD = R\sqrt{3}$.	**1**
	3.a	The two triangles are right, and angle $\widehat{B}$ is common	**0.5**
	3.b	BF=3R	**0.5**
	4	(BE) is the third altitude in the triangle AFB, then ABJ is a right triangle at J, then J belongs to the circle	**0.5**
	5	$\widehat{FJB} = \widehat{FMB} = 90^0$ B,M,J and F are on the same circle.with center rmidpoint of [BF], radius= $\frac{3R}{2}$	**0.75**
	6.a	AKO is an equilateral triangle since KA=KO=OA=R.	**0.5**
	6.b	(OK) parallel to (BF),O rmidpoint of [AB], (OK) passes through the midpoint of [AF].	**0.75**

وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات	الشهادة المتوسطة	الدورة الإستثنائية للعام 2012
عدد المسائل: ستة	مسابقة في مادة الرياضيات المدة ساعتان	الاسم: الرقم:

ارشادات عامة :- يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات

-يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة

I- (2 points)

Consider the following numbers A, B and C:

$$A=\left(\frac{2}{3}\right)^2+\frac{1}{3}\ ;\ B=\frac{5}{3}-\frac{2}{3}\div\left(1+\frac{3}{2}\right)\ ;\ C=\frac{18\times10^8}{8\times10^7\times3.5}$$

1) Write, showing all the steps of calculation , each of the numbers A, B and C as a fraction in its simplest form.

2) Out of the found fractions, indicate that which is decimal. Justify.

II- (3 points)

Consider the following expressions:

$E=(x+9)^2-25$; $G=(x+4)(x+14)-2(x+4)^2$.

1) Verify that $E=(x+4)(x+14)$ and factorize G.

2) The adjoining diagram is that of an apartment in the form of a square with side (x+9) meters $(x \geq 0)$.

It is formed of a salon, a room and a kitchen.

The room is a square with side 5 meters and the kitchen is also a square with side (x+4) meters.

a. Express ,in terms of x, the area A of the apartment and calculate the area of A_1 of the room.

b. Determine A_2 ,the sum of areas of the salon and the kitchen.

c. Express ,in terms of x, the area A_3 of the kitchen. Determine x so that A_2 is the double of A_3.

III- (3 points)

The following three questions are independent:

1) Solve the following equation and give the answer in the form $a\sqrt{b}$ where a and b are two integers : $\sqrt{2}(x-1)=2(x-2)+3\sqrt{2}$.

2) The measures, in cm, of sides of a triangle ABC are : $AB=\sqrt{7}+1$, $BC=\sqrt{7}-1$ and $AC=4$.
Show that ABC is a right triangle.

3) An article costs 18 000LL. If its price is subject to a discount of 12%, followed by a raise of 15%, what is therefore the new price of this article?

IV- (2 points)

The owner of a bookshop proposes the following offer to his clients:

"The first five CD are rented at the rate of 600 LL each, and the others are rented at the rate of 500 LL each".

A client has rented x CD and paid a sum less than 9 000 LL.(x>5).

1) Show that the previous information are modeled by the following inequality:
$500x + 500 < 9\,000$.

2) Solve this inequality and find the greatest value of x.

V- (5 points)

In an orthonormal system of axes x'Ox, y'Oy, consider the points $A(-1;0)$; $B(0;2)$ and $E(3;-2)$.

1) Plot A, B, and E in this system.

2) **a.** Prove that BE = 5.

b. Let I be the midpoint of [BE]. Calculate the coordinates of I.

c. Calculate AI and deduce that the triangle ABE is right at A.

3) Denote by (C) the circle circumscribed about triangle ABE and by (t) the tangent at B to (C).

a. Verify that the slope of (BE) is equal to $-\frac{4}{3}$.

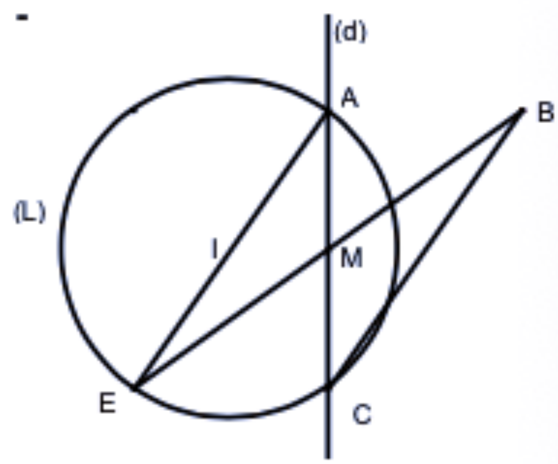
b. Write the equation of (t).

c. (t) intersects x'Ox at F. calculate, rounded to the nearest degree, the measure of the angle $\widehat{BFI}$.

VI- (5 points)

In the adjoining figure :

- A and B are two fixed points
- (d) is the perpendicular at A to (AB)
- C is a variable point on (d)
- M is the midpoint of [AC]
- E is the symmetric of B with respect to M
- (L) is the circle with diameter [AE] and center I.



1) Reproduce this figure.

2) Prove that the quadrilateral ABCE is a parallelogram.

3) Let F be the translate of B under the translation with vector $\overrightarrow{CA}$. Show that E, A and F are collinear .

4) (L) intersects (AB) at a second point G .

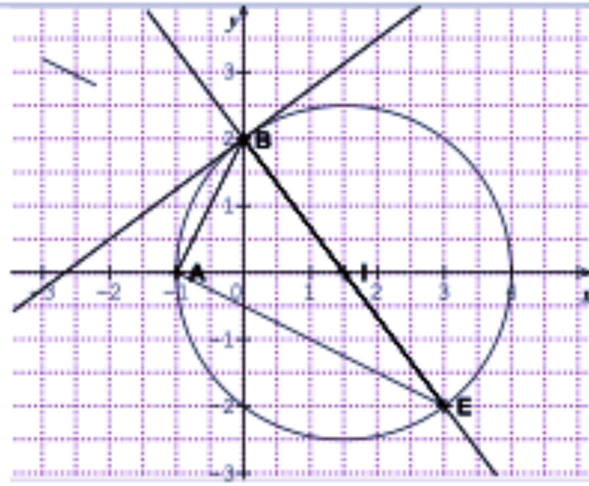
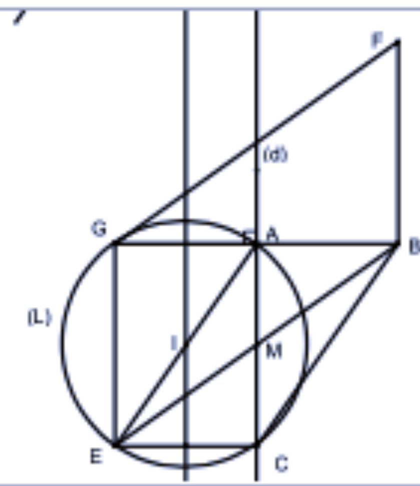
a. Prove that ACEG is a rectangle. Deduce that G is the translate of A under the translation with vector $\overrightarrow{BA}$.

b. Prove that the two triangles AGM et BGF are similar .

5) What is the locus of point I as C moves on (d) ?

الدورة الإستثنائية للعام 2012	الشهادة المتوسطة	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات
	مسابقة في مادة الرياضيات	مشروع معيار التصحيح

	Part of Q.	correction	Note
I	1	$A=\frac{7}{9}$; $B=\frac{5}{3}-\frac{4}{15}=\frac{7}{5}$; $C=\frac{45}{7}$.	1. 5
	2	B is decimal, (denominator is 5).	0. 5
II	1	E= (x+9+5)(x+9-5) =(x+4)(x+14) G=(x+4)[x+14-2(x+8)]=(x+4)(-x+6)	1
	2.a	$A=(x+9)^2$ $A_1=25$	0.5
	2.b	$A_2=(x+9)^2-25$	0.5
	2.c	$A_3=(x+4)^2$. $A_2-2A_3=0$, G=0, x= -4 not acceptable x = 6 acceptable	1
III	1	$x(\sqrt{2}-2)=4\sqrt{2}-4$; $x=\frac{4(\sqrt{2}-1)}{\sqrt{2}-2}$ then $x=-2\sqrt{2}$	1
	2	$(\sqrt{7}+1)^2=8+2\sqrt{7}$; $(\sqrt{7}-1)^2=8-2\sqrt{7}$ $AB^2+BC^2=AC^2$, then ABC is right at B.	1
	3	The discount price is 15 840LL. Then its new price after raise is 18 216 LL.	1
IV	1	$5\times600+(x-5)\times500<9000$; so ,$500x+500<9000$.	1.25
	2	$500x<8\,500$; then $x<17$. The greatest value of x is 16	0.75

V		 A,B and E	0.5
	2.a	BE = 5.	0.75
	2.b	I ($\frac{3}{2}$; 0),	0.5
	2.c	AI = $\frac{5}{2} = \frac{BE}{2}$, BAE right at A.	1
	3.a	Slope of (BE) = $-\frac{4}{3}$	0.5
	3,b	Slope of (t) = $\frac{3}{4}$,(t) passes through B(0 ;2) its equation is $y = \frac{3}{4}x + 2$	1
	3.c	$\tan^{-1}\left(\frac{3}{4}\right) \approx 36^\circ,8$ so $\widehat{BFI} \approx 37^\circ$	0.75
VI	1		0.25
	2	[BE]and [CA] bisect each other , ABCE parallelogram.	0.75
	3	E ,A and F collinear bec ….	1
	4.a	ACEG is a rectangle bec …… : $\overrightarrow{AG} = \overrightarrow{BA}$ bec…	1
	4.b	$\frac{AM}{BF} = \frac{AG}{BG} = \frac{1}{2}$; $\widehat{GAM} = \widehat{GBF} = 90^\circ$	1
	5	I moves on then perpendicular bisector of [AG] bec…	1

وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات	الشهادة المتوسطة	دورة سنة 2013 العادية
عدد المسائل ستة	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

ملاحظة : يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I- (2 points)

The following 3 questions are independent of each other :
(All steps of calculation must be shown in each exercise)

1) Given : $A = \dfrac{5\sqrt{18} - 2\sqrt{98}}{2\sqrt{3} \times \sqrt{24} - 4\sqrt{2}}$.
   a. Write A as a fraction in its simplest form.
   b. Write A in scientific notation.

2) α is an acute angle. Prove that : $(1-\sin^2\alpha)\cdot\tan^2\alpha = \sin^2\alpha$.

3) Determine the real number x so that the following table represents a proportion:

$3+\sqrt{5}$	x
$\frac{7}{5}$	$3-\sqrt{5}$

II- (3 points)

40 students were surveyed about the number of books they read last month.
The following table represents the results of the survey:

Number of read books	0	2	4	6	7	Total
Frequency	2	7	10	9	12	40
Relative frequency in%			25			100
Central angle of a circle graph			90°			360°

1) Determine the mean of this series.
2) Copy the table above, then complete it.
3) What is the number of students who have read **at least** 6 books?

III- (3 points)

Given that $A(x) = (2x-3)^2 - (x-6)^2$ and $B(x) = 2(x-3)^2 + 9 - x^2$.

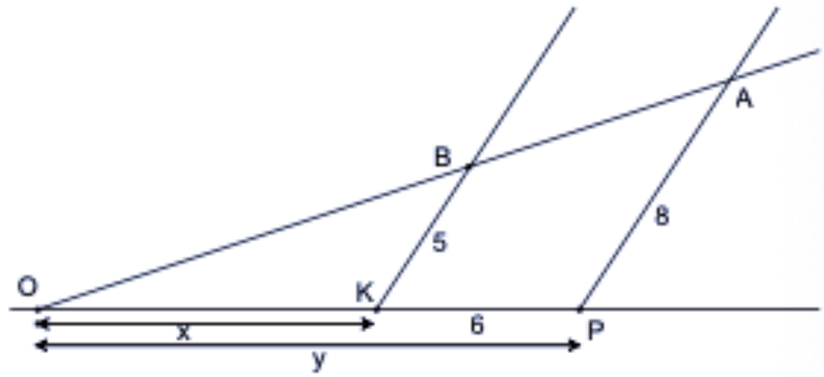
1) a. Expand and reduce A(x).
   b. Calculate $A(1+\sqrt{2})$. Write the answer in the form $a + b\sqrt{2}$ where a and b are two integers .
2) Factorize A(x).
3) Verify that $B(x) = (x-3)(x-9)$.
4) Let $F(x) = \dfrac{A(x)}{B(x)}$
   a. For what values of x, is F(x) defined?
   b. Simplify F(x),then solve the equation $F(x) = -1$.

IV- (2 points)

1) Solve the following system, showing the calculation details :

$$\begin{cases} x - y = -6 \\ 8x - 5y = 0 \end{cases}$$

2) In the opposite figure, the unit of length is the centimeter:
   - The points O, K and P are collinear
   - The points O, B and A are collinear
   - (KB) and (PA) are parallel
   - OK = x, OP = y, KB=5, KP =6 and PA=8.



Calculate the length OP.

V-(5.5 points)

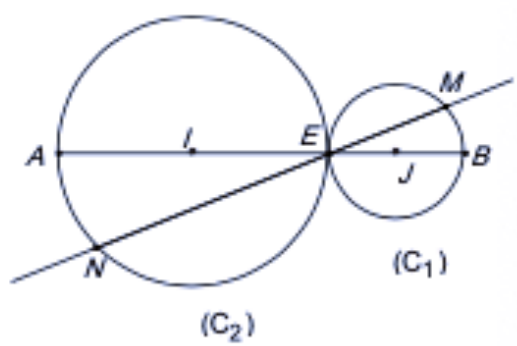
In an orthonormal system of axes x'Ox; y'Oy , where the unit of length is the centimeter, consider the line (d) with equation $y = -x - 4$ and the points $A(-1;-3)$,$B(-7;3)$ and C (3;1).

1) a. Verify that A and B are two points on (d).
   b. Plot the points A, B and C . Draw (d).
2) a. Calculate BC.
   b. Knowing that $AB = 6\sqrt{2}$ and $AC = 4\sqrt{2}$, prove that ABC is a right triangle.
3) Let J be the center of the circle circumscribed about the triangle ABC. Calculate the coordinates of J.
4) Prove that the line (d') with equation $y = x + 4$ is the perpendicular bisector of [AB].
5) a. Calculate the coordinates of the vector $\overrightarrow{BC}$.
   b. Let D be the translate of A under the translation with vector $\overrightarrow{BC}$. Calculate the coordinates of D.
6) Let A' be the symmetric of A with respect to J.
   a. Prove that ABA'C is a rectangle.
   b. Prove that C is the midpoint of [DA'].

VI- (4.5 points)

In the opposite figure :

- A,E and B are three collinear points
- AE = 8 and EB = 4
- (C_1) is the circle with diameter [EB] and center J
- (C_2) is the circle with diameter [EA] and center I
- M is a variable point on (C_1)
- The line (ME) intersects (C_2) at another point N.

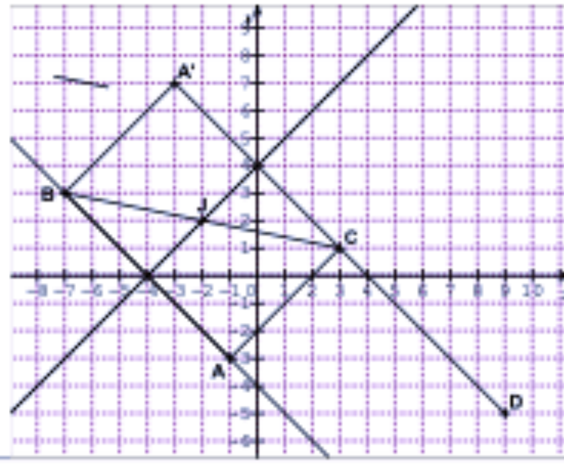
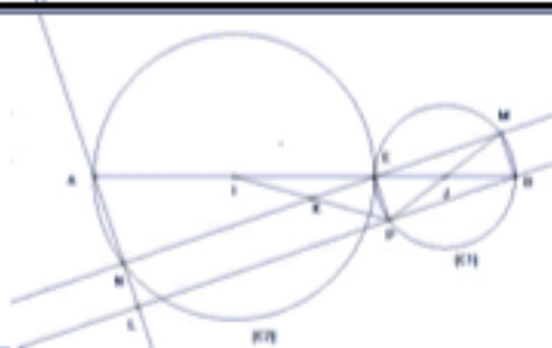


1) Copy this figure .
2) Show that the lines (MB) and (NA) are parallel.
3) Prove that the triangles ANE and BME are similar. Determine the ratio of similarity.
4) Let P be the point defined as $\overrightarrow{MP} = \overrightarrow{ME} + \overrightarrow{MB}$.
   a. Prove that the quadrilateral EPBM is a rectangle.
   b. Deduce that P is a point on (C_1).
5) Denote by K the intersection point of (ME) and (IP) .
   Prove that [MK] is a median in the triangle IMP.
6) The lines (BP) and (AN) intersect at L.
   As M moves on (C_1) , prove that L moves on a circle with diameter to be determined.

توزيع علامات مسابقة الرياضيات

2013 العادية

<table>
<tr><th colspan="2">Questions</th><th>Answers Keys</th><th>Grades</th></tr>
<tr><td rowspan="4">I</td><td>1.a</td><td>$A=\frac{5\sqrt{18}-2\sqrt{98}}{2\sqrt{3}\times\sqrt{24}-4\sqrt{2}}=\frac{15\sqrt{2}-14\sqrt{2}}{12\sqrt{2}-4\sqrt{2}}=\frac{1}{8}$</td><td>0.75</td></tr>
<tr><td>1.b</td><td>A=0.125=1.25×10^{-1}</td><td>0.25</td></tr>
<tr><td>2</td><td>$\tan^2\alpha=\frac{\sin^2\alpha}{\cos^2\alpha}$, $(1-\sin^2\alpha)=\cos^2\alpha$. Hence $\tan^2\alpha\,(1-\sin^2\alpha)=\sin^2\alpha$.</td><td>0.5</td></tr>
<tr><td>3</td><td>$x=\frac{(3+\sqrt{5})(3-\sqrt{5})}{\frac{7}{5}}=\frac{20}{7}$</td><td>0.5</td></tr>
<tr><td rowspan="3">II</td><td>1</td><td>$\overline{X}=\frac{0\times2+2\times7+4\times10+6\times9+7\times12}{40}=4.8$</td><td>0.75</td></tr>
<tr><td>2</td><td>
<table>
<tr><td>Number of read books</td><td>0</td><td>2</td><td>4</td><td>6</td><td>7</td><td>Total</td></tr>
<tr><td>Frequency</td><td>2</td><td>7</td><td>10</td><td>9</td><td>12</td><td>40</td></tr>
<tr><td>% Relative frequency</td><td>5</td><td>17.5</td><td>25</td><td>22.5</td><td>30</td><td>100</td></tr>
<tr><td>Central angle</td><td>18°</td><td>63°</td><td>90°</td><td>81°</td><td>108°</td><td>360°</td></tr>
</table>
</td><td>2</td></tr>
<tr><td>3</td><td>The number of students is 21</td><td>0.25</td></tr>
<tr><td rowspan="6">III</td><td>1.a</td><td>$3x^2-27$</td><td>0.5</td></tr>
<tr><td>1.b</td><td>$6\sqrt{2}-18$</td><td>0.5</td></tr>
<tr><td>2</td><td>(2x-3-x+6)(2x-3+x-6) $=3(x-3)(x+3)$</td><td>0.5</td></tr>
<tr><td>3</td><td>$B(x)=2(x-3)^2-(x^2-9)=(x-3)[2(x-3)-(x+3)$ = (x-3)(x-9)</td><td>0.5</td></tr>
<tr><td>4.a</td><td>F(x) is defined for $x\neq3$ and $x\neq9$</td><td>0.25</td></tr>
<tr><td>4.b</td><td>F(x)= $\frac{3(x+3)}{x-9}$; $\frac{3(x+3)}{x-9}$ = -1; then 4x = 0; x =0</td><td>0.75</td></tr>
<tr><td rowspan="2">IV</td><td>1</td><td>$\begin{cases}x-y=-6\\8x-5y=0\end{cases}$, ……., x=10, y=16</td><td>1</td></tr>
<tr><td>2</td><td>Refer toThales: $\frac{x}{y}=\frac{5}{8}$ and PK=6 :y-x=6 So,we get the given system.OP = y =6
.or……</td><td>1</td></tr>
<tr><td rowspan="2">V</td><td>1.a</td><td>The coordinates of A and B verify the equation of (d).</td><td>0.5</td></tr>
<tr><td></td><td></td><td></td></tr>
</table>

	1.b	Figure A, B, C and (d) 	0.75
	2.a	BC= $\sqrt{104}=2\sqrt{26}$	0.5
	2.b	$BC^2=AB^2+AC^2$; ABC is right.	0.5
	3	J midpoint of [BC] $x_J=\frac{-7+3}{2}=-2,\ y_J=\frac{3+1}{2}=2$; $J(-2;2)$.	0.5
	4	The perpendicular bisector of [AB] is perpendicular to [AB] at its midpoint (-4 ; 0) .Its slope =1 and its equation is y = x + 4.	0.75
	5.a	$\overrightarrow{BC}\ (10;-2)$	0.5
	5.b	$\overrightarrow{AD}=\overrightarrow{BC},\ D(9;-5)$	0.5
	6.a	ABA'C parallelogram + right angle: rectangle.	0.5
	6.b	A',, C and D are collinear . CA'=CD or……	0.5
VI	1		0.25
	2	(MB) and (NA) are perpendicular to (MN).	0.5
	3	$\widehat{BME}=\widehat{ENA}=90^0$ $\widehat{AEN}=\widehat{MEB}$, $\frac{EN}{EM}=\frac{EB}{EA}=\frac{1}{2}$	1
	4.a	EPBM is a parallelogram+ right angle: rectangle..	0.5
	4.b	BPE is right, [EB] diameter then P is a point on (C_1)	0.5
	5	In the triangle IMP, [MK] is a median	0.75
	6	(BP) is perpendicular to (MB) then perpendicular to (AN) so $\widehat{ALB}=90^\circ$ and L moves on the circle with diameter [AB].	1

دورة سنة 2013 الاستثنائية الاثنين 26 آب 2013	امتحانات الشهادة المتوسطة	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات

الاسم: الرقم:	مسابقة في مادة الرياضيات المدة: ساعتان	عدد المسائل : ستة

ملاحظة : يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I- (2.5points)

All steps of calculation must be shown in each exercise.

1) Given $A = 6\times10^2 + 10^2 + 4\times10^{-2} + 10 - 6$.

a. Write A in the form of a decimal number.

b. Write A in scientific notation .

c. Write A as the sum of an integer and a fraction less than 1, in its simplest form.

2) Show that the number $D = \dfrac{4}{2+\sqrt{3}} + \dfrac{2-\sqrt{3}}{2}$ is a natural number.

3) Given the two numbers $B = \dfrac{5}{8} + \dfrac{3}{8}\times\dfrac{4}{6} - \left(\dfrac{3}{2}-1\right)^2$ and $C = 2\sqrt{75} - 4\sqrt{27} + 4\sqrt{12}$.

a. Write B as a fraction in its simplest form.

b. Write C in the form $a\sqrt{3}$ where a is an integer.

II- (1.5points)

1) Solve the following system $\begin{cases} 2a - b = 4 \\ a + b = 5 \end{cases}$

2) Given the two polynomials $P(x) = (2a-b)x^2 + 5x - \dfrac{2}{3}$ and $Q(x) = 4x^2 + (a+b)x + c$.

Calculate a, b and c so that P(x) and Q(x) are identical.

III- (3.5points)

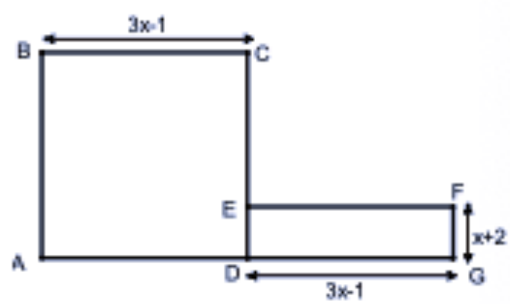
<u>Part A</u>

Given $E(x) = (3x-1)^2 - (3x-1)(x+2)$.

1) Expand and reduce E(x).
2) Calculate x such that E(x) = 3.
3) Factorize E(x).

<u>Part B</u>

In the next figure:



- x represents a length in *cm* so that $x > 0.5$
- ABCD is a square with side 3x-1.
- DEFG is a rectangle such that:

FG = x +2 and EF = 3x – 1.

1) Calculate, in terms of x, the area S of ABCD and the area S' of DEFG.
2) Solve the equation S-S' = 0.
3) Determine all the integers x so that:

$S - S' > 6x^2 - 5x - 12$.

IV- (5.5points)

In an orthonormal system of axes x'Ox;y'Oy, consider the points A(-2 ;-2) ,E(2 ;6) and B (6 ;-2).

1) a-Plot the points A,E and B .

   b-Verify that the equation of (AE) is $y= 2x+2$.

2) Determine the equation of (AB).

3) Verify that AE =BE .

4) Let K be the midpoint of the segment [AE].

   a-Calculate the coordinates of K.

   b-Let (d) be the perpendicular to (AE) at K. Determine the equation of (d).

5) The line (d) intersects the perpendicular bisector of [AB] at I.

   a- show that I is the center of circle circumscribed about the triangle ABE.

   b- Calculate the coordinates of I.

6) Let J be the symmetric of E with respect to I. Show that (AJ) is parallel to (d).

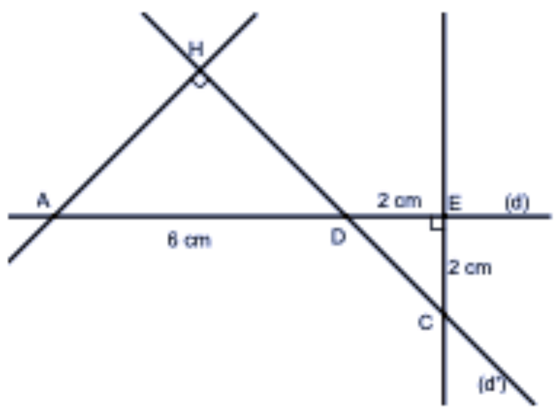
V- (2points)

Given a parallelogram ABCD with center O. The points E and F are such that $\overrightarrow{AE} = \overrightarrow{DA}$ and $\overrightarrow{CF} = \overrightarrow{OC}$.

1) Draw a figure.
2) Prove that $\overrightarrow{EB} = \overrightarrow{AC}$ and $\overrightarrow{OF} = \overrightarrow{AC}$.
3) The lines (EF) and (OB) intersect at K. Prove that K is the midpoint of the segment [EF].

VI- (5points)

In the next figure:



- (d) and (d') are two lines intersecting at D
- EDC is a right triangle at E
- $AD = 6cm$ and $DE = EC = 2cm$
- (AH) is perpendicular to (d').

1) Copy the figure.

2) What is the nature of the triangle ADH ?

   Calculate the exact lengths of [DH], [DC] and [AC].

3) Prove that the points A, H, E and C are on the same circle whose diameter to be determined.

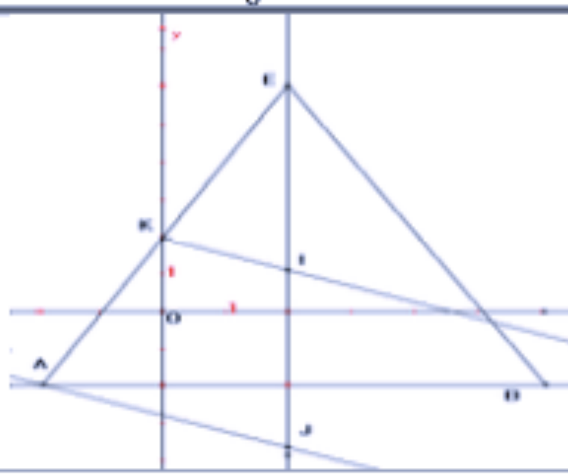
4) Let M be the orthogonal projection of D on (AC).

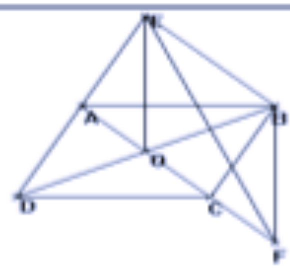
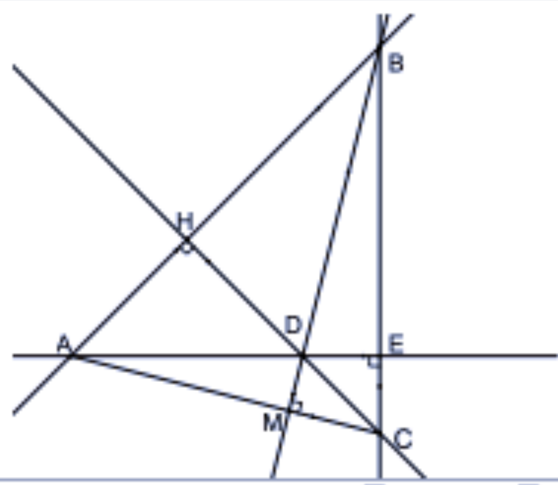
   Prove that the two triangles AHC and DMC are similar. Calculate the product $CM \times CA$.

5) Calculate $\tan \widehat{ACH}$. Deduce the value of the angle $\widehat{ACH}$ rounded to the nearest degree.

6) The lines (AH) and (CE) intersect at B, prove that (MD) passes through B.

دورة سنة 2013 الاستثنائية الاثنين 26 آب 2013	توزيع علامات مسابقة الرياضيات امتحانات الشهادة المتوسطة	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات

I		1.a	704,04	0.25
		1.b	$7,0404\times10^{2}$	0.25
		1.c	$704+\frac{1}{25}$	0.25
		2	$\frac{4}{2+\sqrt{3}}\times\frac{2}{2-\sqrt{3}}=8$	0.5
		3.a	$\frac{5}{8}+\frac{1}{4}-\frac{1}{4}=\frac{5}{8}$	0.5
		3.b	$10\sqrt{3}-12\sqrt{3}+8\sqrt{3}=6\sqrt{3}$	0.75
II		1	$\begin{cases}2a-b=4\\ a+b=5\end{cases}$ a=3, b=2	0,75
		2	a=3, b=2 et $c=-\frac{2}{3}$	0,75
III	A	1	$E(x)=9x^{2}-6x+1-(3x^{2}+5x-2)=6x^{2}-11x+3$	0.5
		2	x(6x -11) = 0 , alors x=0 ou $x=\frac{11}{6}$.	0.5
		3	E(x) = (3x-1)(3x-1-x-2) = (3x-1)(2x-3)	0.5
	B	1	$S=(3x-1)^{2}$ *et* $S'=(3x-1)(x+2)$	0.5
		2	(3x-1)(2x-3) = 0 alors $x=\frac{1}{3}$. $x=\frac{3}{2}$, $x=\frac{1}{3}$ inacceptable donc $x=\frac{3}{2}$.	0.75
		3	$6x^{2}-11x+3>6x^{2}-5x-12$. $-6x>-15$ donc $x<\frac{15}{6}$ alors x=1 ou x=2.	0.75
IV		1.a	Points A,B et E. 	0,5
		1.b	Les coordonnées de A et E vérifient l'équation.	0,5
		2	$y_A=y_B=-2$, Eq. de (AB) : y = -2.	0,5

IV	3	$AE = 4\sqrt{5}$ et $BE = 4\sqrt{5}$	0,75
	4a	K(0;2)	0,5
	4b	Equation de (d) : $y = -\frac{1}{2}x + 2$	0,75
	5.a	(d) est la médiatrice de [AE]donc I est le point d'intersection des médiatrices des côtés du triangle AEB alors c'est le centre du cercle circonscrit à ce triangle.	0,5
	5.b	I(2;1)	0,75
	6	Dans le triangle EAJ ,Ket I sont les milieux de [AE] et [JE] donc (AJ) parallèle à(d).Ou….	0,75
V	1		0,5
	2	$\overrightarrow{AE} = \overrightarrow{DA} = \overrightarrow{CB}$ alors AEBC est un parallélogramme. $\overrightarrow{EB} = \overrightarrow{AC}$; $\overrightarrow{AO} = \overrightarrow{OC} = \overrightarrow{CF}$ donc $\overrightarrow{OF} = \overrightarrow{AC}$	1
	3	$\overrightarrow{EB} = \overrightarrow{AC}$ et $\overrightarrow{OF} = \overrightarrow{AC}$ donc $\overrightarrow{EB} = \overrightarrow{OF}$ alors OBEF est un parallélogramme alors K milieu de [EF].	0,5
VI	1		0,25
	2	ADH est rectangle isocèle (angle de 45°), DH= $3\sqrt{2}$; DC= $2\sqrt{2}$ AC = $2\sqrt{17}$	1,75
	3	AHC et AEC sont deux triangles rectangles .A,H,E et C sont sur un même cercle de diamètre [AC]	0,5
	4	$\widehat{AHC} = \widehat{DMC} = 90°$ et $\widehat{HCA}$ angle commun. CM×CA=CD×CH= $2\sqrt{2} \times 5\sqrt{2}$ =20.	1
	5	$\tan \widehat{ACH} = \frac{AH}{HC} = \frac{3\sqrt{2}}{5\sqrt{2}} = \frac{3}{5}$ $\widehat{ACH} = \tan^{-1}\frac{3}{5} \approx 30,9 \approx 31°$	0,75
	6	Dans le triangle ABC, D est l'orthocentre donc (MD) est la troisième hauteur	0,75

وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات

الشهادة المتوسطة

دورة العام ٢٠١٤ العادية
الجمعة ١٣ حزيران ٢٠١٤

عدد المسائل : ستة

مسابقة في مادة الرياضيات
المدة ساعتان

الاسم:
الرقم:

ارشادات عامة :- يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة.

I- (2 points)

Given the following polynomial: $E(x) = (x-3)^2 + (x+7)(x-3)$.

1) Develop and reduce E(x).

2) Factorize E(x).

3) ABCD is a rectangle with AB = 2x and BC = x-1.
Calculate in terms of x the area S of the rectangle. Determine x such that S=12.

II- (2points)

The following table shows the thicknesses, in cm, of 40 books:

Thickness	2	2.5	3	3.25	3.5	4	5	Total
Frequency	2	4	6	8	12	6	2	
Relative frequency %			15					

1) Copy and complete the given table.

2) Calculate the average (mean) thickness of these 40 books.

3) Determine the percentage of books with a thickness between 2.75 and 3.75 cm.

III- (2.5 points)

1) Solve the following system: $\begin{cases} x + y = 120\,000 \\ 4x + 5y = 500\,000 \end{cases}$

2) A store offers 40% discount on the price of pants and 25% on the price of shirts.
The sum of the original prices of a pant and a shirt is 120 000 LL, while their sum is 75 000 LL after the discount.
Denote by x the original price of a pant and by y that of a shirt.

a. Express, in terms of x and y, the new prices after the discount.

b. Model the previous information into a system of two equations with two unknowns.

c. What is the original price of a pant and that of a shirt?

IV- (3.5 points)

1) Rationalize the denominators of the following fractions $\frac{6}{\sqrt{7}-1}$ and $\frac{6}{\sqrt{7}+1}$.

2) Consider a triangle ABC such that $AB = \frac{6}{\sqrt{7}-1}$, $AC = \frac{6}{\sqrt{7}+1}$ and $BC = 4$.

a. Calculate AB^2 and AC^2. Deduce that the triangle ABC is right at A.

b. Let M be the midpoint of [BC] and E a point on the ray (semi-line) [AM) such that $AE = \frac{10}{3}$. Calculate AM and ME.

3) Denote by F the orthogonal projection of C on (AE).
Show that the two triangles CAF and BCA are similar. Calculate CF.

V- (5 points)

In an orthonormal system with axes x'Ox; y'Oy, consider the line (d) with equation $y = -2x + 6$ and the points $E(6;4)$ and $F(0;1)$.

1) Plot the points E and F. Draw (d) and determine the coordinates of B, the intersection point of (d) and y'Oy.

2) Let (d') be the line passing through E and F. Show that $y = \frac{1}{2}x + 1$ is the equation of (d').

3) **a.** Verify that $P(2;2)$ is the intersection point of (d) and (d').

b. Calculate PB and PE.

c. Show that BPE is a right isosceles triangle.

4) Let M be the midpoint of [BE].

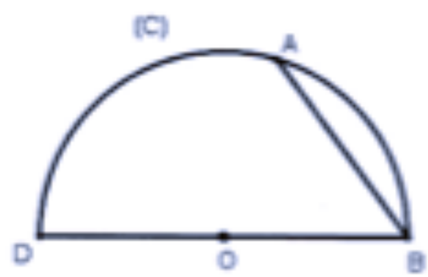
a. Calculate the coordinates of M.

b. Determine the equation of the bisector of angle $\widehat{BPE}$.

VI- (5 points)

In the figure to the right:

- (C) is a semicircle with center O and radius 5 cm,
- [BD] is the diameter of (C),
- A is a point on (C) such that AB= 6 cm.



1) **a.** What is the nature of triangle ABD? Justify.

b. Calculate AD.

2) The perpendicular through A to [BD] intersects it at H. Prove that triangles AHB and DAB are similar, and deduce that AH = 4.8 cm.

3) The tangent at D to (C) intersects (BA) at E.

a. In the two triangles ABD and BDE, write the ratios equal to $\tan \widehat{B}$ then calculate DE.

b. Determine the measure of $\widehat{B}$ rounded to the nearest degree.

4) Let P be the point on [BD] such that DP = 4 cm. The parallel through P to (AB) intersects [DE] at Q. Calculate DQ.

5) Denote by R the translate of P under the translation with vector $\overrightarrow{QE}$.

a. Show that R is a point on [BE].

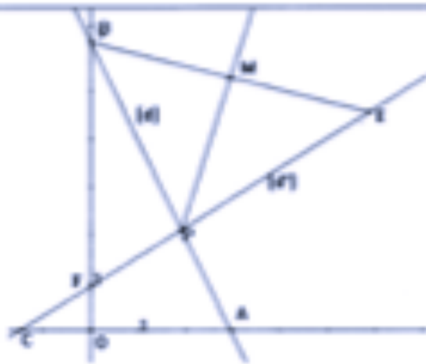
b. The line (PR) intersects [AD] at J. Prove that (BJ) is perpendicular to (DR).

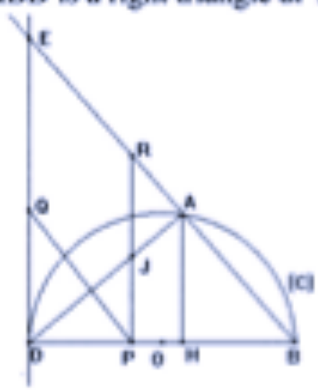
	Part of the Q	Solution	Grade
I	1	$E(x) = 2x^2 - 2x - 12$.	0.5
	2	$E(x) = 2(x-3)(x+2)$.	0.5
	3	Area of the rectangle is: $S = 2x(x-1) = 2x^2 - 2x$. If S=12 , then $2x^2 - 2x - 12 = 0$ E(x)= 0 so x =3 or (x = -2 rejected)	1

<table>
<tr><td rowspan="3">II</td><td>1</td><td>
<table>
<tr><td>Thickness</td><td>2</td><td>2,5</td><td>3</td><td>3,25</td><td>3,5</td><td>4</td><td>5</td><td>Total</td></tr>
<tr><td>Frequency</td><td>2</td><td>4</td><td>6</td><td>8</td><td>12</td><td>6</td><td>2</td><td>40</td></tr>
<tr><td>Relative frequency %</td><td>5</td><td>10</td><td>15</td><td>20</td><td>30</td><td>15</td><td>5</td><td>100</td></tr>
</table>
</td><td>1</td></tr>
<tr><td>2</td><td>$\bar{x} = \frac{4+10+18+26+42+24+10}{40} = \frac{134}{40} = 3.35\text{cm}$</td><td>0.75</td></tr>
<tr><td>3</td><td>Percentage of books having a thickness between 2.75 and 3.75 is 65.</td><td>0.25</td></tr>
</table>

III	1	x=100000 ; y = 20000	0.75
	2a	The price of the pant after discount is : $x - \frac{40x}{100} = 0.6x$ And the price of the shirt after discount is: $y - \frac{25y}{100} = 0.75y$.	0.75
	2b	$\begin{cases} x + y = 120\,000 \\ 0.6x + 0.75y = 75\,000 \end{cases}$	0.5
	2c	Price of the pant is 100 000LL and that of the shirt is 20000LL.	0.5

IV	1	$\frac{6}{\sqrt{7}-1} = \frac{6(\sqrt{7}+1)}{6} = \sqrt{7}+1$; $\frac{6}{\sqrt{7}+1} = \frac{6(\sqrt{7}-1)}{6} = \sqrt{7}-1$	0.75
	2a	. $AB^2 = 8 + 2\sqrt{7}$; $AC^2 = 8 - 2\sqrt{7}$; $AB^2 + AC^2 = 16 = BC^2$	0.75
	2b	$AM = \frac{1}{2}BC = 2$; $ME = AE - AM = \frac{10}{3} - 2 = \frac{4}{3}$	0.75
	3	The two right triangles ACF and CBA have $\hat{ACB} = \hat{CAF}$ (ACM is an isosceles triangle) $\frac{AC}{CB} = \frac{CF}{BA} = \frac{AF}{AC}$; $CF = \frac{BA \times AC}{BC} = \frac{7-1}{4} = \frac{3}{2}$	1.25

V	1	 Fig B(0;6)	1.25
	2	The coordinates of E and F verify the equation of (d') or..... The equation of (EF) is.....	0.5
	3a	P(2;2)	0.5
	3b	$PB = 2\sqrt{5} = PE = 2\sqrt{5}$.	1
	3c	PBE is a right isosceles triangle	0.75
	4.a	M (3;5)	0.5
	4b	[PM) is the bisector since it is a median in an isosceles triangle. Its equation is $y = 3x - 4$.	0.5

VI	1.a	ABD is a right triangle at A since........ 	0.5
	1.b	$AD^2 = DB^2 - AB^2 = 100 - 36 = 64$ so AD=8cm	0.5
	2	The two right triangles AHB and DAB have $\widehat{ABD}$ common angle, so they are similar. Then $AH \times BD = AB \times AD$ and $AH = 4.8$cm	1
	3a	$\tan \widehat{ABD} = \frac{AD}{AB} = \frac{4}{3}$ and $\tan \widehat{ABD} = \frac{DE}{BD}$ $\widehat{ABD} = \frac{AH}{BH}\ \frac{DE}{BD} = \frac{4}{3}; DE = \frac{40}{3}$ cm	1
	3b	$\widehat{ABD} = \tan^{-1}\frac{40}{3}$ □	0.5
	4	$\frac{DP}{BD} = \frac{DQ}{DE}; DQ = \frac{16}{3}$ cm.	0.5
	5.a	$\overline{PR} = \overline{QE}$ then PQER is a parallelogram. Hence (PQ) is parallel to (EB). Since (PQ) is parallel to (EB) then R is on EB.	0.5
	5.b	In triangle DBR, [PR] and [DA] are 2 altitudes, then J is the orthocenter of this triangle. Therefore ,(BJ) is the third altitude.	0.5

وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات

الشهادة المتوسطة

دورة العام 2015 العادية
الجمعة 5 حزيران 2015

عدد المسائل :ستة	مسابقة في مادة الرياضيات	الاسم:
	المدة ساعتان	الرقم:

ارشادات عامة :- يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الواردة في المسابقة.

I - (2 points)

In the following table, only one of the proposed answers to each question is correct.
Write the number of the question and its corresponding answer. Justify your choice.

Number	Question	Proposed answers		
		a	b	c
1	Let $P(x) = 3x^2 - 2x + 2\sqrt{3}$, then $P(\sqrt{3}) =$	9	0	$9 + 4\sqrt{3}$
2	The original price of an article is 5 200 L.L. After a discount of 15% , the new price will be:	5 980 L.L	780 L.L	4 420 L.L
3	x is the measure of an acute angle so that $\sin x = \frac{2}{5}$, then $\cos x =$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{\sqrt{21}}{5}$
4	If $2x - 3 > 5$, then:	$x + 4 > 0$	$-3x + 12 < 0$	$x < -4$

II - (2.5 points)

Consider the three numbers **A**, **B** and **C** so that:

$$A = \frac{8}{3} + 5 + (1 - \frac{2}{5}) \ ; \ B = \sqrt{2 - \frac{6}{5}} \times \sqrt{2 + \frac{6}{5}} \text{ and } C = \frac{2\sqrt{75} - \sqrt{48}}{5\sqrt{2} \times \sqrt{54} - 5\sqrt{27}}$$

In what follows, the steps of calculation must be shown.

1) **Show** that **A** is a natural number.
2) **Write B** in the form of a fraction in its simplest form.
3) **Prove** that **C** is decimal
4) **Prove** that $B + C = 2$.

III - (2 points)

1) **Solve** the following system: $\begin{cases} x + y = 35 \\ 2x - 3y = 0 \end{cases}$
2) **Find**, with justification, two natural numbers such that their sum is 35 and the double of one of them is the triple of the other.

IV- (3.5 points)

Given the following algebraic expression:

$E(x) = (3x-4)^2 - (3x-4)(x+2)$.

1) **a. Show** that $E(x) = 6x^2 - 26x + 24$

   b. Solve the equation $E(x) = 24$.

2) **Factorize** $E(x)$.

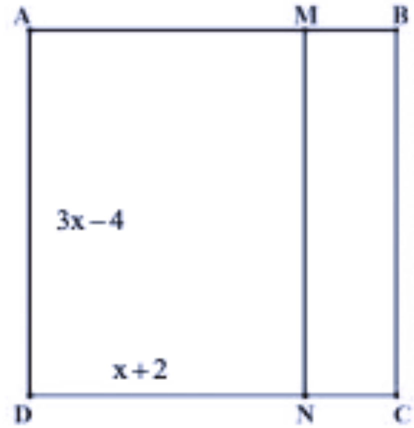
3) In the adjacent figure:

   ABCD is a square with side $3x-4$.

   AMND is a rectangle such that $DN = x+2$. **($x > 3$)**

   a. **Express**, in terms of x, the area **S** of the square **ABCD** and S' the area of the rectangle **MBCN**.

   b. **Determine x** so that $S = 4S'$.



V- (5 points)

In an orthonormal system of axes x'Ox ; y'Oy, consider the points $A(3;3)$, $B(0;-3)$ and $C(-6;0)$.

1) **Plot** the points A, B and C.
2) **Verify** that $y = 2x - 3$ is the equation of the line (AB) .
3) **Calculate** the slope of the line (BC).

   Deduce that (AB) and (BC) are perpendicular.
4) **Show** that ABC is a right isosceles triangle.
5) Let **D** be the point defined by $\overrightarrow{AD} = \overrightarrow{BC}$.

   a. **Verify** that the coordinates of **D** are $(-3;6)$.

   b. **Show** that the quadrilateral ABCD is a square.
6) Let **E** be the symmetric of D with respect to A and **(G)** the circle circumscribed about triangle **CDE**.

   a. **Calculate** the coordinates of E.

   b. **Calculate** the coordinates of I the center of circle (G).

   c. **Determine** the equation of the tangent at D to the circle (G).

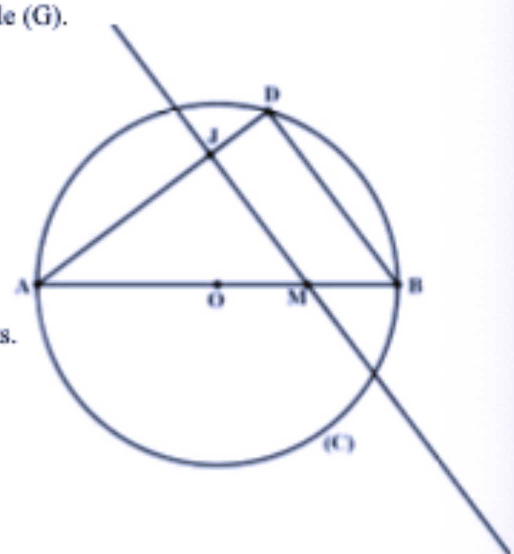
VI- (5 points)

In the adjacent figure, consider a circle **(C)** with center **O** and diameter **AB = 6 cm**.

Let **D** be a point on (C) such that **BD = 3.6 cm** .

Denote by **M** the midpoint of [OB].

The parallel through M to (BD) intersects [AD] at J.



1) **Copy** the figure, it will be completed in the following parts.
2) **Show** that ABD is a right triangle, and then **verify** that **AD = 4.8 cm**.
3) **Verify** that AJ = 3.6 cm and **calculate** JM.
4) The tangents to **(C)** at **A** and **D** intersect at **L**.

   The two lines **(AD)** and **(LO)** intersect at **F**.

   a. **Calculate** OF.

   b. **Prove** that the two triangles OFA and OAL are similar then **Calculate** AL.

   c. **Calculate**, rounded to the nearest degree, the measure of angle $\widehat{ALD}$.

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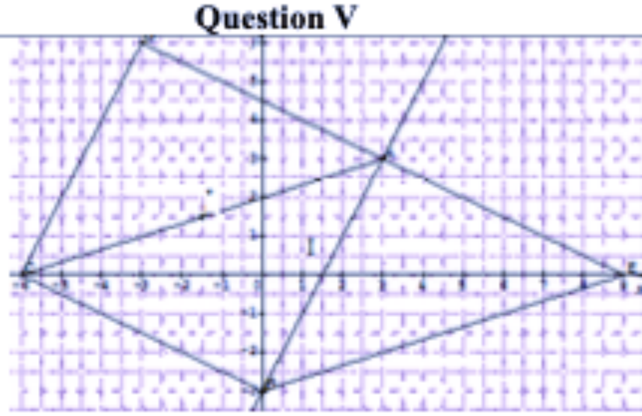
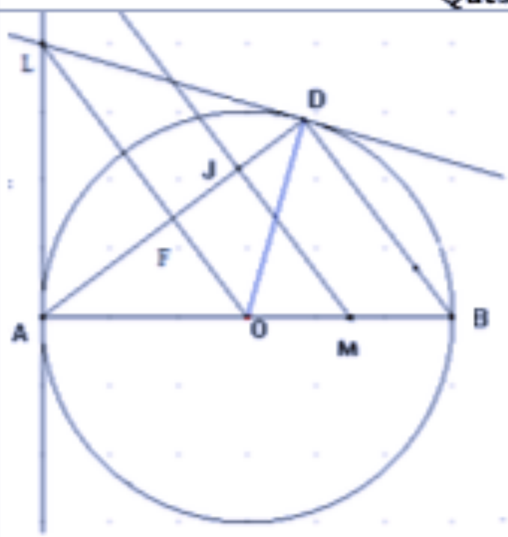
مشروع أسس التصحيح

Part of the ques.	Question I	Grade
1	$P(\sqrt{3})=9$ .The answeris(a)	0.5
2	The price will be 5 200 x 0.85=4420. So the answeris (c)	0.5
3	$\cos^2 x = 1-\frac{4}{25}=\frac{21}{25}$ .So the answeris(c)	0.5
4	If $2x-3>5$, then $x>4$ so $-3x+12<0$ .So the answeris(b)	0.5
	Question II	
1	$A=\frac{8}{3}+5+(\frac{3}{5})=\frac{8}{3}+\frac{25}{3}=11$	0.5
2	$B=\sqrt{4-\frac{36}{25}}=\sqrt{\frac{100-36}{25}}=\frac{8}{5}$	1
3	$C=\frac{10\sqrt{3}-4\sqrt{3}}{5\times2\times3\sqrt{3}-15\sqrt{3}}=\frac{6\sqrt{3}}{15\sqrt{3}}=\frac{2}{5}$ thus B+C = 2	1
	Question III	
1	x= 21 and y =14	1
2	x and y are two natural numbers, so the system is: $\begin{cases}x+y=35\\2x=3y\end{cases}$ hence x=21 and y = 14	1
	Question IV	
1.a	$E(x) = 6x^2-26x+24$	0.5
1.b	$E(x) = 24$ then x=0 or x= $\frac{13}{3}$	0.5
2	E(x) = (3x-4) (3x-4-x-2) = 2(3x-4)(x-3).	0.5
3.a	$S=(3x-4)^2$ and $S'=(3x-4)^2-(3x-4)(x+2)$	1
3.b	S= 4S' then x= $\frac{4}{3}$ rejected or $x=4$ acceptable .	1
	Question V	
1	 Figure: A, B andC	0.5
2	Equationof (AB) is y = 2x-3	0.5
3	$a_{(BC)}=\frac{-1}{2}$ then (AB) and (BC) are perpendicular (product of theirslopes = -1)	0.75

4.	(AB) perpendicularto (BC) , $AB = 3\sqrt{5}$; $BC = 3\sqrt{5}$ then ABC is right isosceles .	0.5
5.a	$\overrightarrow{AD} = \overrightarrow{BC}$ then $D(-3;6)$	0.5
5.b	$\overrightarrow{AD} = \overrightarrow{BC}$ thenABCD is a parm (BC) perpendicular to (AB)andAB = BC thenitis a square.	0. 5
6. a	E(9, 0).	0.5
6. b	$I(\frac{3}{2},0)$	0.5
6.c	$a_{(ID)} = -\frac{4}{3}$ so, the slope of the tangent $= \frac{3}{4}$ and the equation of the tangent is $y = \frac{3}{4}x + \frac{33}{4}$.	0.75
	Question VI	
1		0.5
2	ABD is right at D sinceitisinscribed in a (C) of diameter [AB] $AD^2 = 36 - 12.96 = 23.04$ hence AD = 4.8cm.	0.75
3	UsingThales', $\frac{AJ}{AD} = \frac{AM}{AB} = \frac{JM}{BD}$ then AJ=3.6cm and JM=2.7cm	1
4.a	F midpoint of [AD] and O midpoint of [AB] then OF= $\frac{1}{2}$ BD consequently OF= 1.8 or.…	0.5
4.b	$\hat{O}$ commonangle , $\hat{O}AL = \hat{O}FA = 90°$ $\frac{OF}{OA} = \frac{AF}{LA} = ...$ then $AL = 4$	1.25
5.a	$\tan \hat{O}LA = \frac{3}{4}$ then $\hat{O}LA = 37°$,so $\hat{A}LD = 74°$	1

وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات الرسمية	امتحانات الشهادة المتوسطة	دورة العام 2015 الاستثنائية الثلاثاء 11 آب 2015
عدد المسائل: خمسة	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

إرشادات عامة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الواردة في المسابقة.

I- (2 points)

Given the number $a = \dfrac{7+\sqrt{125}+\sqrt{20}}{14}$.

1) Write a in the form $x + y\sqrt{5}$ where x and y are two rational numbers.

2) Compare $a+1$ and a^2.

3) Verify that $a^3 = 2a+1$.

II - (4 points)

1) a. Verify that $x^2+4x+3=(x+2)^2-1$.

b. Factorize x^2+4x+3.

2) Given an isosceles triangle ABC with vertex A so that its area is equal to x^2+4x+3 and $BC = 2x+2$ $(x>0)$. Let [AH] be an altitude in this triangle.

a. Show that $AH = x+3$.

b. Calculate AB^2 in terms of x.

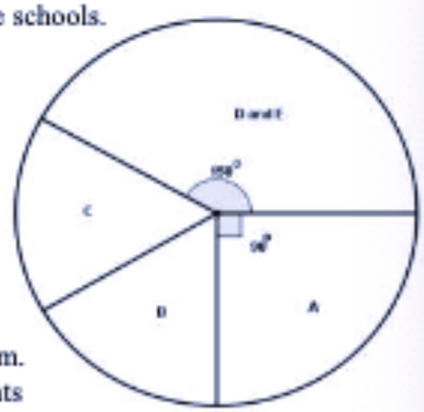
3) a. Find x such that the area of ABC is equal to 8. [Use **1)a.**]

b. For x = 1, calculate $\sin \widehat{ABC}$ and deduce, rounded to the nearest degree, the measure of angle $\widehat{ABC}$.

III - (4 points)

The Brevet students of five schools A, B, C, D and E sit for the official exam.
The adjacent circle graph represents the distribution of students in these schools.

- The total number of students is 240
- The angle that represents the students of D and E together is 150°
- The angle that represents the students of A is 90°
- The number of students of B is equal to that of C.



1) Verify that the number of students of A is 60.
2) Calculate the number of students of B and that of C.
3) Show that the number of students of D and E together is 100.
4) 20% of the students of A and 15% of the students of B failed, calculate the total number of students of A and B who passed the exam.
5) Three times the number of students of D minus the number of students of E is equal to 180.

a. Write a system of two equations with two variables to represent the number of students of D and E.

b. Solve the system and verify that the number of students of D is 70.

IV- (5points)

In an orthonormal system of axes x'Ox,y'Oy, consider the points A(0 ; 2) and B(- 4 ; 0) .

1) Plot the points A and B.

2) Show that $y = \frac{1}{2}x + 2$ is an equation of the line (AB).

3) Let [OH] be an altitude in triangle OAB.

a. Find an equation of line (OH).

b. Verify that the coordinates of H are $\left(-\frac{4}{5};\frac{8}{5}\right)$.

4) The parallel through B to y'Oy intersects (OH) at E.

a. Calculate the coordinates of point E.

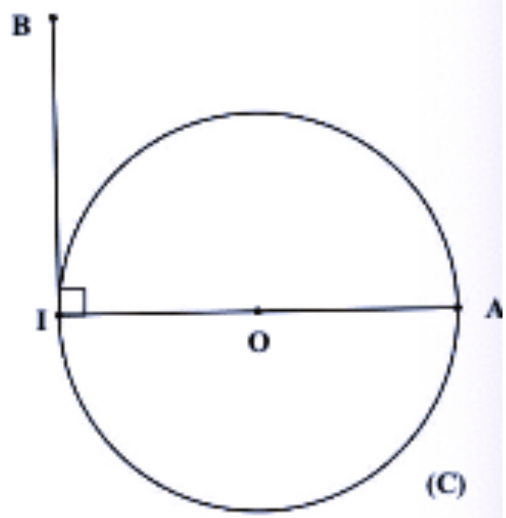
b. Calculate OE and HE.

5) Let (C) be the circle circumscribed about triangle OBE and (d) the tangent at O to (C).

The two lines (d) and (EA) intersect at F.

Prove that $\frac{EA}{EF} = \frac{4}{5}$.

V- (5 points)



In the adjacent figure:

- (C) is a circle with center O and diameter [IA] so that IA = 8
- B is a point on the tangent at I to (C) so that IB = 6.

1) Copy the figure that will be completed later.

2) Let (C') be the circle with diameter [IB].The two circles (C) and (C') intersect at I and another point E.

a. Prove that A, E and B are collinear.

b. Calculate AB.

3) a. Write in two different triangles the ratios equal to $\cos \widehat{IBA}$.

b. Show that BE = 3.6

c. Deduce the length AE, then calculate IE.

4) The tangent at B to (C') intersects (IE) at F.

a. Show that the two triangles EBF and EIB are similar.

b. Deduce the value of $EI \times EF$.

5) Let L be the translate of B under the translation with vector $\overrightarrow{IA}$.

Prove that the four points A, E, F and L are on the same circle with diameter to be determined.

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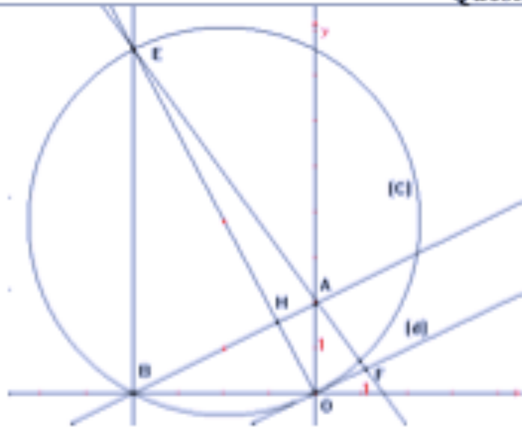
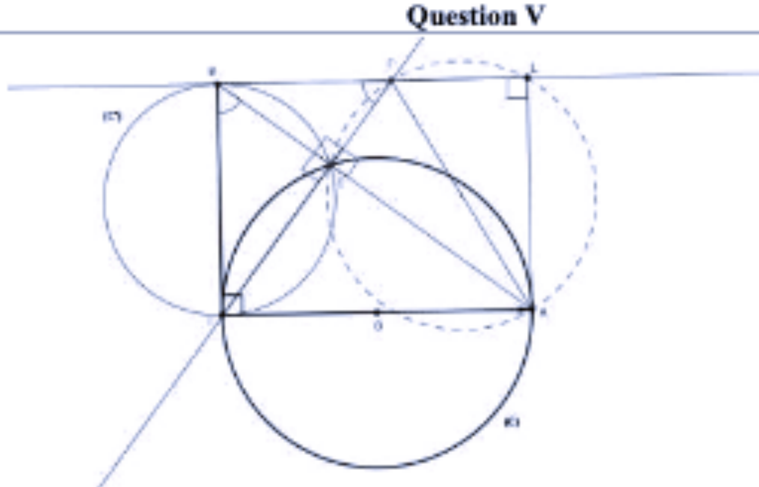
	Question I	
	Answers	Grade
1	$a=\frac{7+7\sqrt{5}}{14}=\frac{1+\sqrt{5}}{2}=\frac{1}{2}+\frac{1}{2}\sqrt{5}$ **(0.25) + (0.25)**	0.5
2	$a+1=\frac{3+\sqrt{5}}{2}$; $a^2=\frac{3+\sqrt{5}}{2}$ so $a+1=a^2$ **(0.25) + (0.5) + (0.25)**	1
3	$a^3=\frac{8+4\sqrt{5}}{4}=2+\sqrt{5}$; $2a+1=2+\sqrt{5}$ Or $a^3=a^2 . a=(a+1)a=a^2+a=a+1+a=2a+1$**(0.25) + (0.25)**	0.5
	Question II	
1.a	$(x+2)^2-1=x^2+4x+4-1=x^2+4x+3$	0.5
1.b	$x^2+4x+3=(x+1)(x+3)$	0.5
2.a	Area of $ABC=\frac{BC\times AH}{2}$;$x^2+4x+3=\frac{2(x+1)\times AH}{2}=,(x+1)(x+3)=\frac{2(x+1)\times AH}{2}$ so AH = x+3**(0.25) + (0.25) + (0.25)**	0.75
2.b	$AB^2=(x+3)^2+(x+1)^2=2x^2+8x+10$	0.5
3.a	$(x+2)^2-1=8;(x+2)^2=9, x+2=3$ ou $x+2=-3$ So x = 1 since x= -5 not accepted**(0.25) + (0.5) + (0.25)**	1
3.b	$\sin\hat{B}=\frac{AH}{AB}=\frac{2}{\sqrt{5}}=0.89$, so $\hat{B}\approx 63^\circ$, **(0.5) + (0.25)**	0.75
	Question III	
1	Number of students of $A=240\times\frac{90}{360}=60$	0.5
2	Number of students of $B=240\times\frac{60}{360}=40$, Number of students of $C=40$ **(0.25) + (0.25)**	0.5
3	Number of students of C and E $=240\times\frac{150}{360}=100$ or another method…	0.25
4	Number of students who failed in A and B $=60\times\frac{20}{100}+15\times\frac{40}{100}=18$ **(0.25) + (0.25)** Number of students who passed in A and B =100 – 18 = 82 students. **(0.5)**	1
5.a	$x+y=100$**(0.25)** $3x-y=180$.**(0.75)**	1
5.b	$4x=280$, $x=70$ and $y=30$**(0.5) + (0.25)**	0.75

	Question IV	
1	 Fig.	0.5
2	$y=\frac{1}{2}x+2$ is the equation of (AB)**slope (0.5) + b(0.25) or(verificationof a point (0.25)**	0.75
3.a	y = - 2x is the equation of (OH) **slope (0.5) + equation (0.25)**	0.75
3.b	$\frac{1}{2}x + 2 = -2x$ so $x = \frac{-4}{5}$ et $y = \frac{8}{5}$**(0.5) + (0.25)**	0.75
4.a	$x_E = x_B = -4$ *et* $y_E = -2x_E = -2(-4) = 8$ therefore$E(-4\,;8)$**(0.25) + (0.25)**	0.5
4.b	$OE = \sqrt{16+64} = 4\sqrt{5}\,;HE = \sqrt{\frac{256}{5}} = \frac{16\sqrt{5}}{5}$**(0.25) + (0.5)**	0.75
5	(d) // (AB) so : $\frac{EA}{EF} = \frac{EH}{EO}$(Thales') so : $\frac{EA}{EF} = \frac{\frac{16\sqrt{5}}{5}}{4\sqrt{5}} = \frac{4}{5}$//**(0.25) + ratio(0.5) + (0.25)**	1
	Question V	
1		0.25
2.a	IÊB = IÊA = 90° so IÊB+IÊA = 180° therefore the 3 points are collinear.	0.5
2.b	By applying Pythagorean AB^2=100 then AB=10.	0.5
3.a	$\cos\widehat{IBA} = \frac{IB}{AB}$in triangle IBA, and$\cos\widehat{IBA} = \frac{BE}{BI}$in triangle IBE.**(0.25) + (0.25)**	0.5
3.b	$\frac{IB}{AB} = \frac{BE}{BI}so\frac{6}{10} = \frac{BE}{6}$thenBE=3.6**(0.25) + (0.25)**	0.5
3.c	AE = AB – BE =6.4**(0.25)** By applying Pythagorean in triangle IAE we get IE^2= 23.04 therefore IE= 4.8**(0.25)**	0.5
4.a	$\widehat{BEF} = \widehat{BEI}$= 90°**(0.25)+(0.5)** $\widehat{BFE} = \widehat{IBE}$ having same complement $\widehat{EBF}$so the two triangles are similar.	0.75
4.b	$\frac{E}{E}$	0.5
5	Locating point L, AIBLis a rectangle,A, E, L and F are on the same circle of diameter [AF]**(0.25) + (0.25) + (0.25) + (0.25)**	1