وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات الرسمية	امتحانات الشهادة المتوسطة	دورة العام ٢٠١٦ العادية الثلاثاء ٣١ أيار ٢٠١٦
عدد المسائل: خمسة	مسابقة في مادة الرياضيات المدة ساعتان	الاسم: الرقم:

ارشادات عامة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة.

I - (2 points)

Consider the three numbers A, B and C:

$$A=\frac{9}{2}-\frac{9}{2}\times\frac{1}{3} \quad ; \quad B=\frac{10^{14}\times 2^{10}}{5\times 4\times 10^{12}\times 2^{9}} \quad ; \quad C=\left(2+\sqrt{5}\right)^2+\left(1-2\sqrt{5}\right)^2.$$

1) By writing all the steps of calculation, show that A, B and C are natural numbers.

2) Verify that $A\times B=C$.

II – (4 points)

Given the expression: $E(x)=(3x+1)(2x-1)-(3x+1)(x+1)$.

1) Show that $E(x)=(3x+1)(x-2)$.

2) Solve the equation $E(x) = 0$.

3) In the adjacent figure:

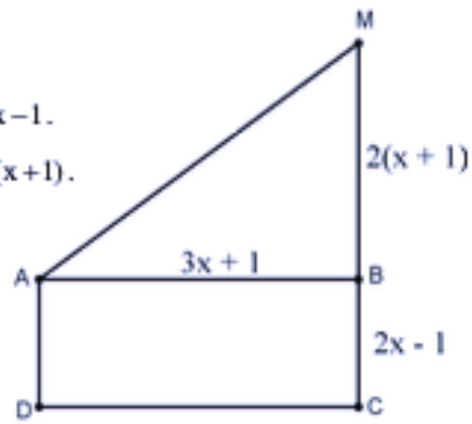
- x *is a length expressed in cm such that* $x > 1$.
- ABCD is a rectangle such that $AB=3x+1$ and $BC=2x-1$.
- ABM is a triangle, right angled at B, such that $MB=2(x+1)$.

Denote by S the area of ABCD and S' that of ABM.

a. Express S and S' in terms of x.

b. Verify that $S - S' = E(x)$.

c. Calculate x so that $S = S'$.



III – (3 points)

1) Solve the following system: $\begin{cases} 6x+4y=20\,000 \\ 2x+8y=15\,000 \end{cases}$

2) A bookshop offers 40% discount on the price of a copybook and 60% discount on that of a pencil.

The sum of the original prices of 2 copybooks and 8 pencils is 15 000 L.L.

The sum of prices, after discount, of one copybook and one pencil is 2 000 L.L.

a. Prove that the previous information can be modeled by the above system.

b. Find the price of a copybook and that of a pencil after the discount.

IV - (6 points)

In an orthonormal system of axes (x'Ox, y'Oy), consider the points A(–2; 0) and B(1; 3).

Let (d) be the line with equation $y = -x + 4$.

1) **a.** Plot the points A and B.

b. Verify, by calculation, that point B is on the line (d), then draw (d).

c. Determine the equation of line (AB) and verify that (AB) is perpendicular to (d).

d. The line (d) intersects x'Ox at E and y'Oy at F. Calculate the coordinates of points E and F.

2) Let (C) be the circle circumscribed about the triangle ABF.

a. Determine the coordinates of point I, the center of (C). Calculate the radius of (C).

b. Verify that O is a point on the circle (C).

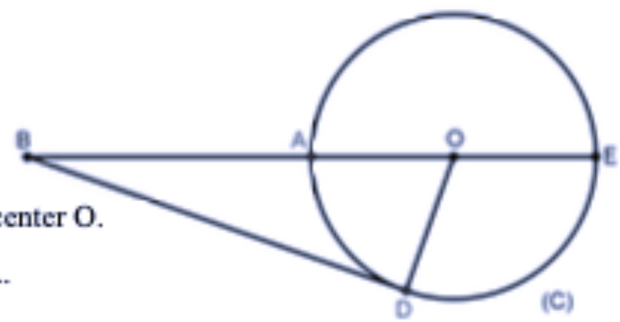
3) **a.** Calculate AB.

b. Calculate, rounded to the nearest degree, the measure of $\widehat{BAF}$.

V - (5 points)

In the adjacent figure:

- AE = 4 cm.
- (C) is the circle with diameter [AE] and center O.
- B is the symmetric of E with respect to A.
- (BD) is tangent to (C) at D.



1) Copy the figure.

2) Calculate BD.

3) The parallel through point A to (OD) intersects the line (BD) at M and (ED) at L.

a. Show that D is the midpoint of [EL].

b. Deduce that M is the centroid of triangle EBL.

4) **a.** Prove that the two triangles BDE and BAD are similar.

b. Calculate $\frac{DE}{DA}$.

5) Let F be the translate of A by the translation with vector $\overrightarrow{ED}$.

a. Prove that ADLF is a rectangle.

b. Prove that F is the midpoint of [BL].

c. Deduce that the points E, M and F are collinear.

Question I

	Answers	note
1	$A = \frac{9}{2} - \frac{9}{2} \times \frac{1}{3} = \frac{9}{2} - \frac{3}{2} = \frac{6}{2} = 3,$ ¼ + ¼ $B = \frac{10^{14} \times 2^{10}}{5 \times 4 \times 10^{12} \times 2^{9}} = \frac{10^2 \times 2}{20} = \frac{200}{20} = 10$ ¼ + ¼ $C = \left(2+\sqrt{5}\right)^2 + \left(1-2\sqrt{5}\right)^2 = 4 + 4\sqrt{5} + 5 + 1 - 4\sqrt{5} + 20 = 30$ ¼ + ¼ +¼	1¾
2	$A \times B = 3 \times 10 = 30$ $c = 30$ Then : $A \times B = C$	¼

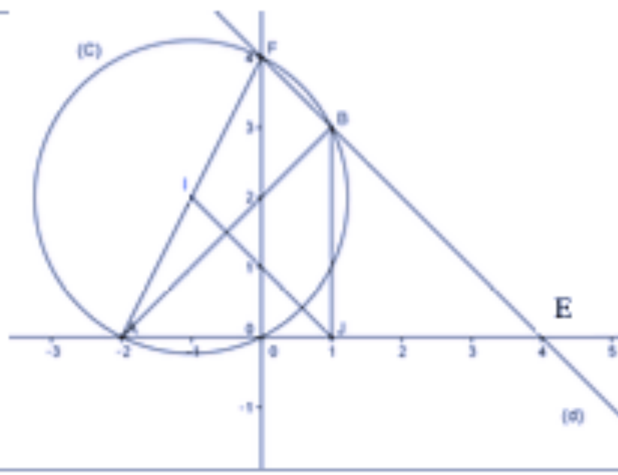
Question II

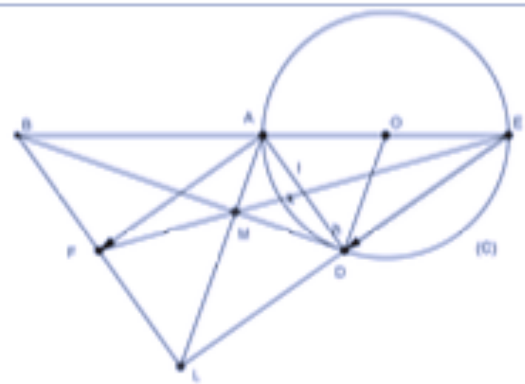
1	$E(x) = (3x+1)(2x-1-x-1) = (3x+1)(x-2).$ ½ + ¼	¾
2	$(3x+1)(x-2) = 0$ Then $x = \frac{-1}{3}$ or $x = 2$ ¼ +¼	½
3.a	$S = (3x+1)(2x-1)$ $S' = (3x+1)(x+1)$ ½ + ¾	1¼
3.b	$S - S' = (3x+1)(2x-1) - (3x+1)(x+1) = E(x)$	½
3.c	$S = S'$ then $S - S' = 0$; $E(x) = 0$ $x = -1/3$ (rejected) $x = 2$ (accepted) ¼ + ¼ +¼ + ¼	1

Question III

1	$x = 25000$; $y = 1250$ ¾ + ¼	1
2.a	1st equation: $2x + 8y = 15000$ ¼ 2nd equation: $(1-0.4)x + (1-0.6)y = 2000$ then $6x + 4y = 20000$ ¾	1
2.b	The price of a copybook = 2500 ×(0.6) = 1500 L.L ½ The price of a pencil = 1250× (0.4) = 500 L.L ½	1

Question IV

1.a		½
1.b	$x_B = 1$ $- x_B + 4 = 3 = y_B$ ¼ + ½ (For drawing the line)	¾
1.c	Equation of (AB) : $y = x + 2$ ½ + ½ slope (AB) = 1 and slope (d) = -1 then slope(AB) × slope(d) = -1 ¼ So (AB) is perpendicular to (d)	1¼
1.d	E(4;0) and F(0 ; 4) ¼ + ¼	½

2.a	$\widehat{ABF} = 90°$ (ABF is inscribed in a semicircle of diameter [AF]) ¼ I midpoint of [AF] then I(-1; 2) ½ R = $\frac{AF}{2} = \sqrt{5}$ or AI = IB = IF = R = $\sqrt{5}$ ½	1¼
2.b	OI = $\sqrt{5}$ or $\widehat{AOF} = 90°$ then O is a point of the circle.	½
3.a	AB = $\sqrt{18} = 3\sqrt{2}$	½
3.b	$\cos\widehat{BAF} = \frac{AB}{AF} = \frac{3\sqrt{2}}{2\sqrt{5}}$ ¼ Then $\widehat{BAF} = cos^{-1}\left(\frac{3\sqrt{2}}{2\sqrt{5}}\right)$=18,43°≈ 18° ¼ + ¼	¾
	Question V	
1		½
2	The triangle ABD is right at D Then by Pythagoras theorem $BD^2 = OB^2 - OD^2$ BD = $\sqrt{32} = 4\sqrt{2}$	½
3.a	In the triangle ALE : (AL) // (OD) and O midpoint of [AE] Then by the converse of the midpoint theorem, D midpoint of [EL].	½
3.b	In the triangle BEL : [LA] and [BD] are two medians intersect at M, then M centroid.	½
4.a	The 2 triangles BDE and BAD are similar since : $\hat{B}$ is a common angle ½ $\widehat{ADB} = \widehat{AED} = \frac{\widehat{AD}}{2}$ ½	1
4.b	The ratio of similarity : $\frac{BE}{BD} = \frac{DE}{DA} = \frac{BD}{BA}$ ¼ $\frac{DE}{DA} = \frac{BD}{BA} = \frac{4\sqrt{2}}{4} = \sqrt{2}$ ¼	½
5.a	$\overrightarrow{AF} = \overrightarrow{ED} = \overrightarrow{DL}$ then AFLD is a parallelogram ¼ $\widehat{ADL} = 90°$ then AFLD is a rectangle ¼	½
5.b	AD = FL (Opposite sides in a rectangle) AD = $\frac{BL}{2}$ (Midpoint theorem) ¼ Then BL = 2 FL , B, F and L are collinear ((AD) // (BL) and (AD) // (FL)) ¼ Then F midpoint of [BL]	½
5.c	[EF] 3rd median in the triangle EBL then E, M and F are collinear.	½

وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات الرسمية

امتحانات الشهادة المتوسطة

دورة العام ٢٠١٦ الاستثنائية
الجمعة ٢٩ تموز ٢٠١٦

عدد المسائل: خمسة	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

ارشادات عامة :- يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
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I– (علامتان)

لنعتبر الاعداد التالية A و B و C:

$A = \frac{1}{3} + \frac{7}{6} \div \frac{5}{3}$ ؛ $B = \frac{5 \times 10^{-2} \times 7 \times 10^{5}}{2 \times 10^{7}}$ وَ $C = \sqrt{45} - 4\sqrt{5} + 3\sqrt{125}$.

بيّن كل تفاصيل العمليات الحسابية التالية:

١) أحسب A واكتب الجواب على شكل كسرفي أبسط صورة.
٢) أحسب B واكتب الجواب على الصورة العلمية.
٣) أكتب C على الشكل $a\sqrt{5}$ حيث أن a هو عدد طبيعي.

II– (٣ علامات)

السؤلان التاليان ١) و ٢) مستقلان.

١) حل ّالمتباينة التالية ومثّل الحلّ على خط الاعداد:

$$4x - \frac{3}{2} \leq \frac{5}{2}x + 3$$

٢) صندوق يحتوي على 400 كرة زجاجية موزّعة كما يلي:

- 30 % من هذه الكرات هي حمراء.
- 108 كرات هي خضراء.
- باقي الكرات هي بيضاء.

أ- أحسب النسبة المئوية للكرات الخضراء.
ب- أحسب عدد الكرات البيضاء.

III– (٣ علامات)

نعطي: $E(x) = 5(x - 1)(x + 2) - (x + 2)^2 + 3(x + 5)$.

١) بيّن أن $E(x) = 4x^2 + 4x + 1$.
٢) حلّ المعادلة $E(x) = 1$.
٣) لنعتبر $H(x) = 9x^2 - (2x + 1)^2$.
أ- بيّن أن $H(x) = (5x + 1)(x - 1)$.
ب- حل المعادلة $H(x) = 0$.

IV- (٦ علامات)

في المستوي الإحداثي $(x'Ox , y'Oy)$ نأخذ المستقيم (d) ذو المعادلة $y = -2x + 3$ وكذلك النقاط $A(0; -2)$، $E(6; 1)$ وَ $G(0; 3)$.

١) ضع النقاط A، E وَ G في المستوي الإحداثي.

٢) تحقق ان النقطة G تقع على المستقيم (d)، ثم ارسم (d).

٣) أ- برهن أن $y = \frac{1}{2}x - 2$ هي معادلة المستقيم (AE).

ب- برهن أن المستقيمين (d) وَ (AE) متعامدان.

ج- تحقق حسابياً أن $B(2; -1)$ هي نقطة التقاء المستقيمين (d) وَ (AE).

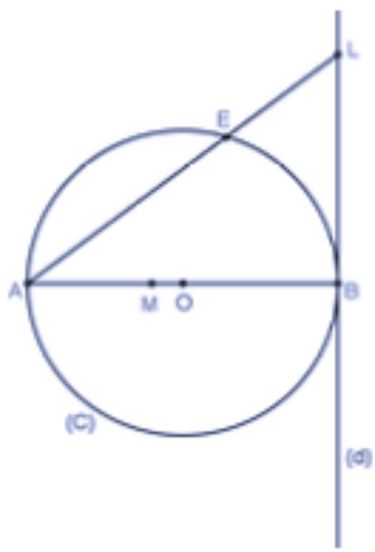
د- برهن ان المثلث GBE قائم الزاوية ومتساوي الساقين.

٤) لنرمز بالنقطة M الى انسحاب النقطة E في الانسحاب ذو المتجه $\overrightarrow{BG}$.

أ- برهن أن الرباعي BEMG هو مربع.

ب- أحسب BM.

V- (٦ علامات)



في الرسم المقابل:

- (C) هي دائرة مركزها O وقطرها $AB = 10$ cm
- المستقيم (d) هو مماس الدائرة (C) في النقطة B
- لتكن L نقطة على (d) حيث أن $BL = 7.5$ cm
- لتكن M نقطة على [AB] حيث أن $AM = 4$cm.

١) انسخ الصورة.

٢) أحسب طول AL.

٣) أحسب $\cos \widehat{BAL}$.

٤) يتقاطع المستقيم (AL) مع الدائرة (C) بالنقطة E.

أ- برهن أن المثلثين ABL وَ BEL متشابهين. أكتب نسبة التشابه.

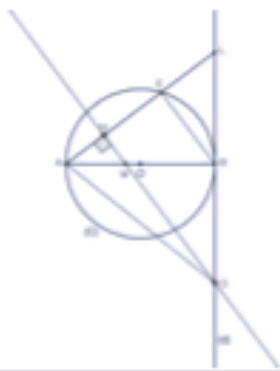
ب- أستنتج أن $EL = 4.5$ cm.

٥) المستقيم المار بالنقطة M والمتعامد مع (AL) يتقاطع مع [AL] في النقطة N، كما يتقاطع مع المستقيم (d) في النقطة G.

أ- استخدم $\cos \widehat{BAL}$ في المثلث MAN لتتأكد أن $AN = 3.2$ cm.

ب- برهن أن: $\frac{EB}{NG} = \frac{15}{31}$.

Part	Correction	Note
	Question I	
1	$A = \frac{1}{3}+\frac{7}{6}+\frac{5}{3}=\frac{1}{3}+\frac{7}{6}\times\frac{3}{5}=\frac{1}{3}+\frac{7}{10}=\frac{31}{30}$. **0.25 +0.25**	**0.50**
2	$B = = \frac{5\times10^{-2}\times7\times10^{5}}{2\times10^{7}}=\frac{35\times10^{3}}{2\times10^{7}}=\frac{35\times10^{-4}}{2}=17.5\times10^{-4}=1.75\times10^{-3}$ **0.25 +0.25+0.25**	**0.75**
3	$C = \sqrt{45}-4\sqrt{5}+3\sqrt{125}=3\sqrt{5}-4\sqrt{5}+15\sqrt{5}=14\sqrt{5}$. **0.5 +0.25**	**0.75**
	Question II	
1	$4x-\frac{3}{2}\leq\frac{5}{2}x+3$; $\frac{8x}{2}-\frac{3}{2}\leq\frac{5x}{2}+\frac{6}{2}$; $8x-3\leq5x+6$; $8x-5x\leq3+6$; $3x\leq9$; $x\leq3$. **0.25 +0.25+0.25 +0.25** O 3	**1** **0.5**
2.a	The percentage of green balls is : $\frac{108}{400}\times100=27\%$	**0.5**
2.b	The percentage of white balls is: $100-(27+30)=100-57=43\%$ The number of white balls is: $\frac{43\times400}{100}=172$	**0.5** **0.5**
	Question III	
1	$E(x) = 5(x^2+x-2)-(x^2+4x+4)+3x+15=4x^2+4x+1$. **- 0.25** (MISTAKE)	**1**
2	$E(x)=1$ so $4x^2+4x=0$. where $4x(x+1)=0$; $x=0$ or $x=-1$. **0.25 +0.25**	**0.5**
3.a	$H(x)=9x^2-(2x+1)^2=(3x+2x+1)(3x-2x-1)=(x-1)(5x+1)$.). **0.5 +0.5**	**1**
3.b	$H(x)=0$; $(x-1)(5x+1)=0$. So $x=1$ or $x=-\frac{1}{5}$	**0.5**
	Question IV	
1		**0.5**
2	$y_G=-2x_G+3$ then $3=-2(0)+3$ $3=3$ so G is a point on (d) Two points are enough to draw a line : x: 0, 1; y: 3, 1	**0.5** **0.25**
3.a	$y_E=\frac{1}{2}x_E-2$ $1=\frac{1}{2}(6)-2$ $1=1$ so E is a point on (AE). $y_A=\frac{1}{2}x_A-2$ $-2=\frac{1}{2}(0)-2$ $-2=-2$ so A is a point on (AE).	**0.5** **0.5**
3.b	$a_{(d)}\times a_{(AE)}=-2\times\frac{1}{2}=-1$ So (d) $\perp$ (AE).	**0.5**

3.c	$y_B = \frac{1}{2}x_B - 2$ $-1 = \frac{1}{2}(2) - 2$ $-1 = -1$ so B is a point on (AE). $y_B = -2x_B + 3$ $-1 = -2(2) + 3$ $-1 = -1$ so B is a point on (d).	0.5 0.25
3.d	$\widehat{GBE} = 90^\circ$ (since (d) $\perp$ (AE)) $BG = 2\sqrt{5}$. $BE = 2\sqrt{5}$. So GBE is a right isosceles triangle of vertex B.	0.25 0.25 0.25 0.25
4.a	We have: $\overrightarrow{BG} = \overrightarrow{EM}$ so BEMG is a parallelogram And since : $\widehat{GBE} = 90^\circ$ then it is a rectangle and: BG = BE so it is a rhombus.therefore: BEMG is a square.	0.25 0.25 0.25
4.b	$BM = GE = \sqrt{(6-0)^2 + (1-3)^2} = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$ u (diagonals in a square are equal)	0.5
	Question V	
1		0.5
2	Using pythagorean : $AL^2 = AB^2 + BL^2 = 100 + 7.5^2 = \frac{625}{4}$ so: AL = 12.5 cm	0.5 0.25
3	$\cos BAL = \frac{AB}{AL} = \frac{10}{12.5} = \frac{4}{5}$ **0.5+0.25**	0.5
4.a	The two triangles ABL and LBE have: • $\widehat{L}$ (common angle) • $\widehat{B} = \widehat{E} = 90^0$ (ABE is an inscribed triangle in a semi-circle) So they are similar. $S^{ABL}_{BEL}; \frac{AB}{BE} = \frac{BL}{EL} = \frac{AL}{BL} = \frac{12.5}{7.5} = \frac{5}{3}$	0.5 0.5 0.5
4.b	Consider $\frac{BL}{EL} = \frac{5}{3}$; $\frac{7.5}{EL} = \frac{5}{3}$; $EL = 4.5\,cm$	0.25 0.25
5.a	$\cos BAL = \frac{4}{5}$ and $\cos BAL = \cos NAM = \frac{AN}{AM} = \frac{4}{5}$ so $AN = 3.2cm$	0.25 0.25 0.5
5.b	$(EB)//(NG)$ Using Thales' $\frac{LE}{LN} = \frac{EB}{NG} = \frac{4.5}{9.3} = \frac{15}{31}$ (LN = 12.5 – 3.2 = 9.3 cm)	0.25 0.5 0.25

وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات الرسمية	امتحانات الشهادة المتوسطة	دورة العام ٢٠١٧ العادية الثلاثاء ٦ حزيران ٢٠١٧
عدد المسائل: خمس	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

إرشادات عامة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
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I – (3 points)

The questions 1) and 2) are independent.
All the steps of calculation must be shown.

1) Given $A = \dfrac{2-\dfrac{1}{3}}{2+\dfrac{1}{3}}$ and $B = \dfrac{24\times10^3\times5\times10^6}{8\times\left(10^3\right)^3}$.

a. Calculate A and give the result as a fraction in its simplest form.
b. Show that B is a natural number.

2) Given $C = \dfrac{\sqrt{45}-\sqrt{180}+9}{3+\sqrt{5}\times\sqrt{35}-5\sqrt{7}}$ and $D = \left(1-\sqrt{5}\right)^2$.

a. Write C in the form $n-\sqrt{5}$ where n is a natural number.
b. Calculate D, then verify that $D = 2\times C$.

II – (3 points)

Given $A(x) = (2x-3)^2 + (x-5)(3-2x)$.

1) Factorize $A(x)$.

2) Let $B(x) = 2x^2 - 5x + 3$.
Verify that $B(x) = (2x-3)(x-1)$.

3) Let $F(x) = \dfrac{(2x-3)(x+2)}{B(x)}$.

a. For what values of x, is $F(x)$ defined ?
b. Simplify $F(x)$.
c. Does the equation $F(x) = 7$ have a solution? Justify.

III – (3 points)

1) Solve the following system: $\begin{cases} x+y=35 \\ 9x+8y=300. \end{cases}$

2) The number of students (girls and boys) of a certain class is 35.
When 10% of the girls and 20% of the boys leave this class to participate in a sportive activity, the number of remaining students is then 30.

a. Denote by x the number of girls and by y that of boys of this class.
Write a system of two equations with two unknowns to model the text above.

b. Find the number of girls and that of boys in this class.

IV – (5.5 points)

In an orthonormal system of axes x'Ox and y'Oy , consider the points $A(-2;2)$, $B(0;-2)$, $C(5;3)$ and $I(-1;0)$. Let (d) be the line with equation $y=\frac{1}{2}x+\frac{1}{2}$.

1) **a.** Plot the points A, B, C and I.

b. Verify that C and I are two points on the line (d). Draw (d).

2) Prove that I is the midpoint of [AB].

3) **a.** Find the equation of the line (AB).

b. Prove that (AB) is perpendicular to the line (d).

c. Show that the triangle ABC is isosceles.

4) Consider the point $F(7;-1)$.

Show that F is the translate of C under the translation with vector $\overrightarrow{AB}$.

5) Denote by E the point on the line (AB) so that $x_E=1$.

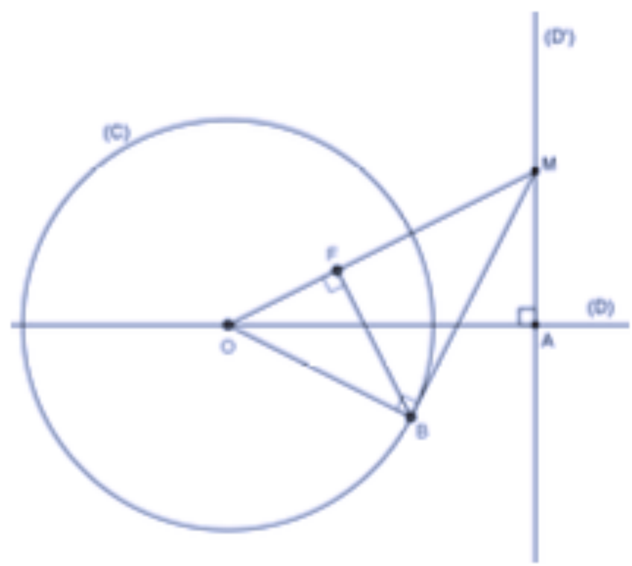
a. Show that $y_E=-4$.

b. Prove that the quadrilateral CIEF is a rectangle.

V – (5.5 points)

In the adjacent figure :

- (D) and (D') are two perpendicular lines at A
- O is a point on (D) so that OA = 6
- (C) is a circle with center O and radius 4
- M is a point on the line (D') so that AM = 3
- (MB) is a tangent through M to the circle (C)
- [BF] is an altitude in the triangle OBM.



1) Copy the figure that will be completed in the remaining parts of the problem.

2) Show that $OM=3\sqrt{5}$.

3) **a.** Show that the two triangles OFB and OBM are similar.

b. Deduce that $OF\times OM=16$.

c. Calculate OF.

4) The two segments [BF] and [OA] intersect at I.

a. Write in the two triangles FOI and MOA the ratios equal to $cos\,MOA$.

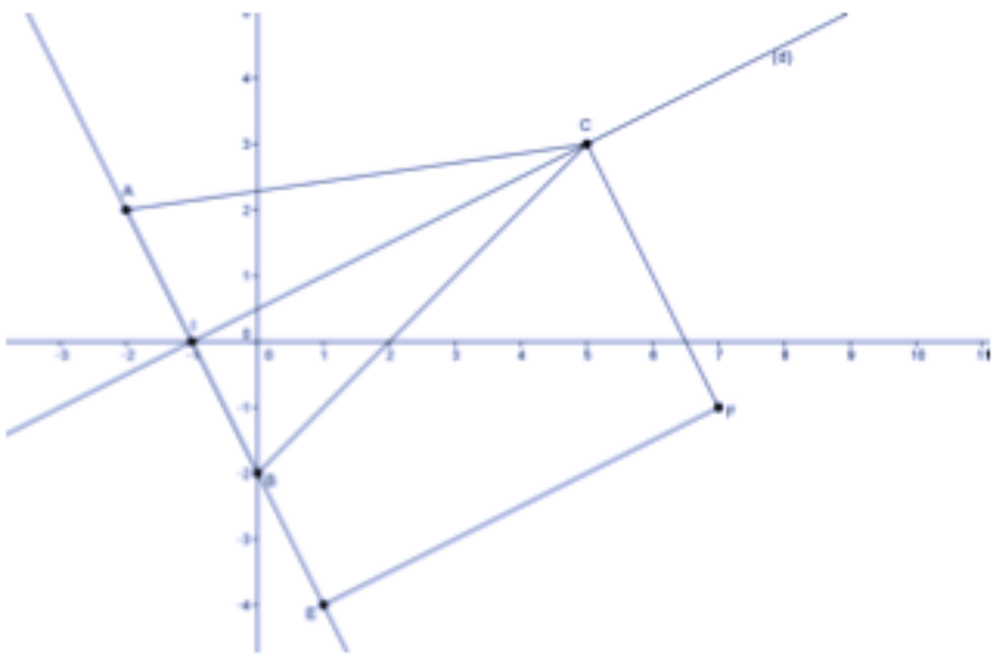
b. Deduce that $OI\times OA=16$.

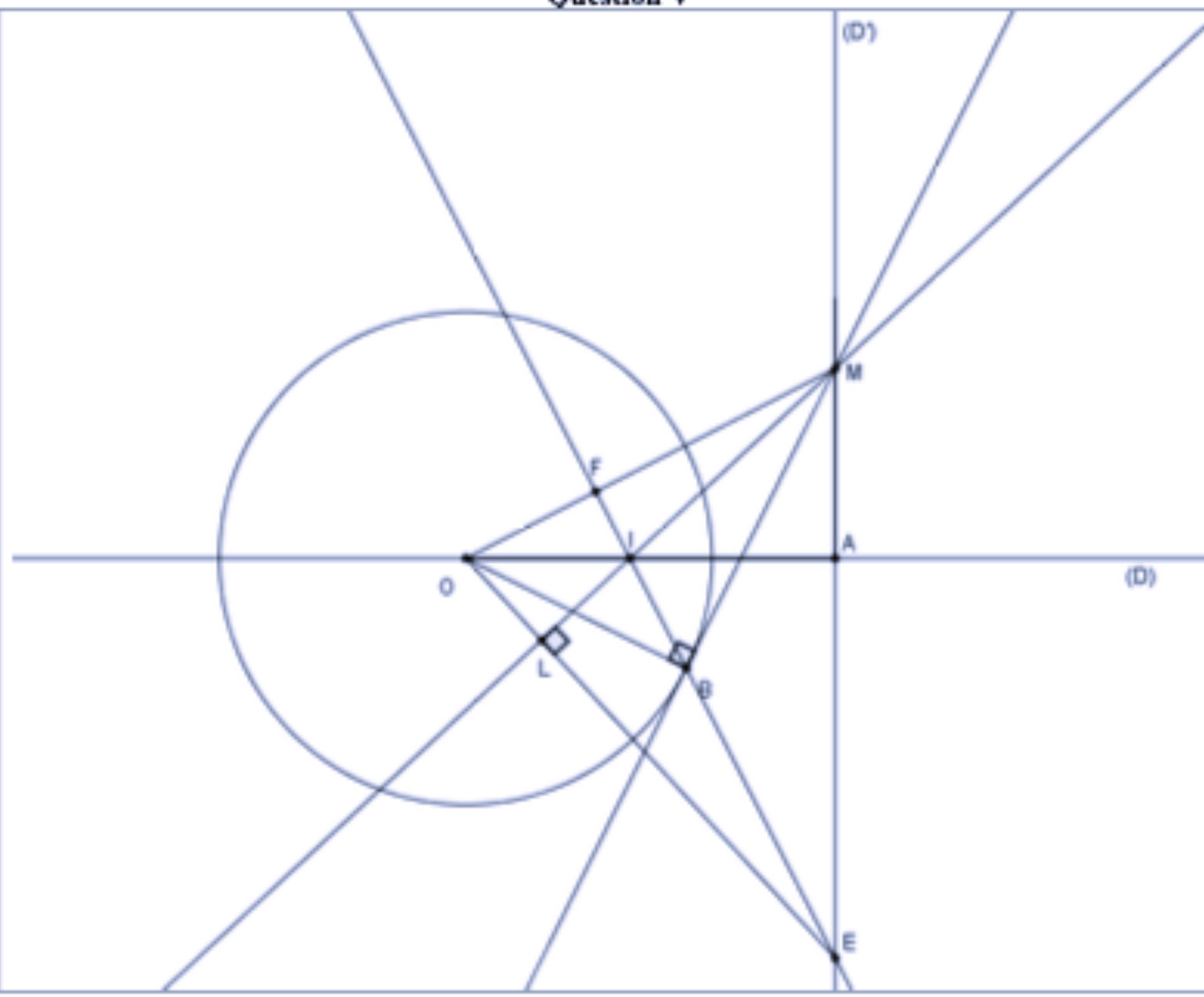
c. Calculate OI.

5) The line (FB) intersects (D') at E.

Show that (MI) is perpendicular to (OE).

Part of the ques.	Answer Key	Grade
	Question I	
1a	$A=\dfrac{2-\frac{1}{3}}{2+\frac{1}{3}}=\dfrac{\frac{6}{3}-\frac{1}{3}}{\frac{6}{3}+\frac{1}{3}}=\dfrac{\frac{5}{3}}{\frac{7}{3}}=\dfrac{5}{7}$ **0.25 + 0.25 + 0.25**	0.75
1b	$B=\dfrac{24\times10^{3}\times5\times10^{6}}{8\times\left(10^{3}\right)^{3}}=\dfrac{3\times5\times10^{9}}{10^{9}}=15$ is a natural number. **0.25 + 0.25 + 0.25**	0.75
2a	$C=\dfrac{\sqrt{45}-\sqrt{180}+9}{3+\sqrt{5}\times\sqrt{35}-5\sqrt{7}}=\dfrac{3\sqrt{5}-6\sqrt{5}+9}{3+5\sqrt{7}-5\sqrt{7}}=\dfrac{-3\sqrt{5}+9}{3}=-\sqrt{5}+3=3-\sqrt{5}$ with $n=3$ (natural number) **0.25 + 0.25 + 0.25**	0.75
2b	$D=\left(1-\sqrt{5}\right)^{2}=1-2\sqrt{5}+5=6-2\sqrt{5}$ **0.5** $2\times C=2\left(3-\sqrt{5}\right)=6-2\sqrt{5}=D$ **0.25**	0.75
	Question II	
1	$A(x)=(2x-3)^{2}+(x-5)(3-2x)$ **0.25** (*change sign*) $A(x)=(2x-3)\left[(2x-3)-(x-5)\right]$ **0.5** (common factor) $A(x)=(2x-3)(2x-3-x+5)$ $A(x)=(2x-3)(x+2)$ **0.25**	1
2	$B(x)=(2x-3)(x-1)$ $B(x)=2x^{2}-2x-3x+3$ **0.25** $B(x)=2x^{2}-5x+3$ **0.25**	0.5
3a	$F(x)=\dfrac{A(x)}{B(x)}=\dfrac{(2x-3)(x+2)}{(2x-3)(x-1)}$ $F(x)$ is defined if $x\neq\dfrac{3}{2}$ and $x\neq1$ **0.25 + 0.25**	0.5
3b	$F(x)=\dfrac{x+2}{x-1}$ **0.25**	0.25
3c	$F(x)=7$ $\dfrac{x+2}{x-1}=7$ gives $x=\dfrac{9}{6}=\dfrac{3}{2}$ (rejected since $F(x)$ is not defined). $F(x)=7$ does not admit any solution. **0.25 + 0.25 + 0.25**	0.75

	Question III	
1	$\begin{cases} x+y=35 \\ 9x+8y=300 \end{cases}$ $x=20\ ;\ y=15$ **0.5 + 0.5**	**1**
2a	$\begin{cases} x+y=35 \\ 0.9x+0.8y=30 \end{cases}$ **0.25** **0.75**	**1**
2b	$\times 10 \begin{cases} x+y=35 \\ 0.9x+0.8y=30 \end{cases}$ **0.5** $\begin{cases} x+y=35 \\ 9x+8y=300 \end{cases}$ **0.25** $x=20\ ;\ y=15$ **0.25** The number of girls is 20 and that of boys is 15.	**1**
	Question IV	
1a		**0.5**
1b	$y_C = \frac{1}{2}x_C + \frac{1}{2}$ $3 = \frac{1}{2}(5) + \frac{1}{2}$ **0.25** $3 = 3$ $y_I = \frac{1}{2}x_I + \frac{1}{2}$ $0 = \frac{1}{2}(-1) + \frac{1}{2}$ **0.25** $0 = 0$	**0.5**

2	$x_I = \frac{x_A + x_B}{2}$ $-1 = \frac{-2+0}{2}$ $-1 = -1$ $Y_I = \frac{Y_A + Y_B}{2}$ $0 = \frac{2-2}{2}$ $0 = 0$	**0.5**
3a	$a_{(AB)} = -2$; $(AB): y = -2x - 2$ **0.5 + 0.25**	**0.75**
3b	$a_{(AB)} \times a_{(d)} = -1$	**0.5**
3c	(CI) ⊥ (AB) at its midpoint I then ABC is an isosceles triangle of vertex C.	**0.75**
4	$\overrightarrow{CF}(2;-4) = \overrightarrow{AB}(2;-4)$ then F is the translate of C under the translation with vector $\overrightarrow{AB}$ **0.25 + 0.5**	**0.75**
5a	$y_E = -2x_E - 2 = -2(1) - 2 = -4$	**0.5**
5b	$\overrightarrow{CF}(2;-4) = \overrightarrow{IE}(2;-4)$ then CIEF is a parallelogram **0.5** and $\widehat{CIF} = 90^0$ then it is a rectangle. **0.25**	**0.75**
	Question V	
1	 	**0.5**

2	OMA is a right triangle at A. $OM^2 = OA^2 + AM^2 = 36 + 9 = 45$ (Pythagorean theorem) **0.25** $OM = \sqrt{45} = 3\sqrt{5}$ **0.5**	**0.75**
3a	The two triangles OFB and OBM have : $\widehat{OFB} = \widehat{MBO} = 90^0$ **0.5** $\widehat{MOB} = \widehat{FOB}$ (common angle) **0.5** Therefore they are similar	**1**
3b	$\frac{OF}{OB} = \frac{OB}{OM}$; $OF \times OM = OB^2 = 4^2 = 16$ **0.25 + 0.25**	**0.5**
3c	$OF \times OM = 16$ $OF \times 3\sqrt{5} = 16$ $OF = \frac{16}{3\sqrt{5}} = \frac{16\sqrt{5}}{15}$	**0.5**
4a	$\cos \widehat{MOA} = \frac{OA}{OM}$ **0.5** $\cos \widehat{FOI} = \frac{OF}{OI}$ **0.25**	**0.75**
4b	$\frac{OA}{OM} = \frac{OF}{OI}$ gives $OI \times OA = OF \times OM = 16$	**0.5**
4c	$16 = OI \times OA.$ $16 = 6OI$ $OI = \frac{8}{3}$	**0.25**
5	In the triangle OME we have : [OA] is the first altitude. **0.5** [EF] is the second altitude. [OA] and [EF] intersect at I which is the orthocenter of this triangle. **0.25** [ML] passes through I, then it is the third altitude. Then (MI) $\perp$ (OE)	**0.75**

I – (2 points)

In the following table, only one of the proposed answers to each question is correct.

Write the number of the question and its corresponding answer. Justify your choice.

	Questions	Answers		
		a	b	c
1	$3\sqrt{2}-\sqrt{50}+\sqrt{8}=$	$10\sqrt{2}$	0	$-30\sqrt{2}$
2	$\frac{1}{\sqrt{5}-2}=$	$\sqrt{5}+2$	$\frac{\sqrt{5}+2}{3}$	-1
3	ABCD is a parallelogram, then $\overrightarrow{AB}+\overrightarrow{DA}=$	$\overrightarrow{BC}$	$\overrightarrow{CA}$	$\overrightarrow{DB}$
4	After an increase of 15%, the price of an article becomes 23 000 L.L. The original price of this article is:	17 000 L.L.	20 000 L.L.	19 550 L.L.

II – (3.5 points)

Given $A(x)=(x-3)^2-(x-3)(2x-7)$.

1) Prove that $A(x)=(x-3)(4-x)$.

2) Let $B(x)=(16-x^2)+A(x)$.

Factorize $B(x)$.

3) Let $F(x)=\dfrac{A(x)}{(4-x)(2x+1)}$.

a. For what values of x, is $F(x)$ defined?

b. Simplify $F(x)$, then solve the equation $F(x)=\dfrac{2}{3}$.

c. Does the equation $F(x)=x$ have a solution? Justify.

III – (3 points)

1) Solve the following system: $\begin{cases} 5x+2y=12\,000 \\ x+2y=8\,000. \end{cases}$

2) A restaurant sells 10 green salads and 4 vegetarian pizzas for 24 000 L.L.

The same restaurant sells 6 green salads and 12 vegetarian pizzas for 48 000 L.L.

Show that this text is modeled by the system given in question **1)**.

3) Nadine orders 8 green salads and 6 vegetarian pizzas, how much will she pay?

IV – (5.5 points)

In an orthonormal system of axes (x'ox, y'oy), consider the points $A(4;2)$, $B(-1;2)$ and $E(1;3)$.

Let (d) be the line with equation $y = 3x$.

1) **a.** Plot A, B and E.

b. Verify that E is a point on the line (d). Draw (d).

2) **a.** Calculate OB and show that OA = 2OB.

b. Show that OAB is a right angled triangle.

3) **a.** Determine the coordinates of the point L, the symmetric of O with respect to B.

b. Verify that E is the midpoint of [AL].

4) Let (d') be the line passing through A and perpendicular to (OA).

Show that the equation of (d') is $y = -2x + 10$.

5) Let F be the point with coordinates $(2;6)$.

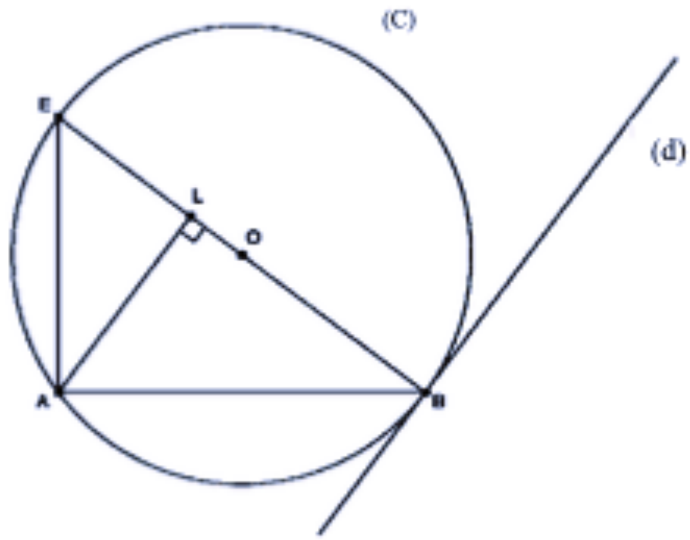
a. Verify that F is the intersection point of (d) and (d').

b. Prove that the quadrilateral OAFL is a square.

V – (6 points)

In the adjacent figure:

- (C) is a circle with center O, radius 5 and diameter [EB]
- A is a point on (C) so that AE = 6
- (d) is the tangent at B to (C)
- [AL] is an altitude in the triangle ABE.



1) Copy the figure that will be completed in the remaining parts of the problem.

2) **a.** Calculate AB.

b. Verify that $\sin \widehat{AEB} = \frac{4}{5}$.

3) The parallel through L to (AB) intersects [EA] at M and the line (d) at F.

a. Prove that the two triangles EML and FBL are similar.

b. Calculate, rounded to the nearest degree, the measure of angle $\widehat{BFL}$.

4) Prove that the points E, M, B and F are on the same circle whose center I should be determined.

5) Prove that the quadrilateral ALFB is a parallelogram.

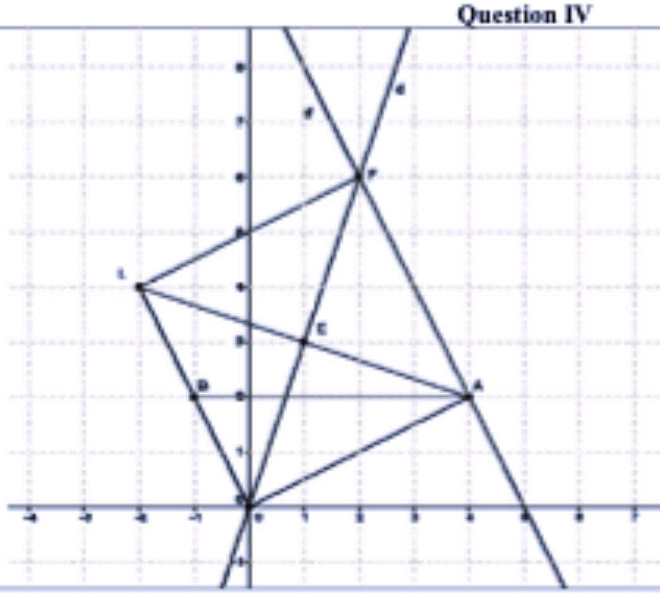
6) The diagonals [AF] and [BL] of the parallelogram ALFB intersect at J.

Prove that (IJ) is perpendicular to (AB).

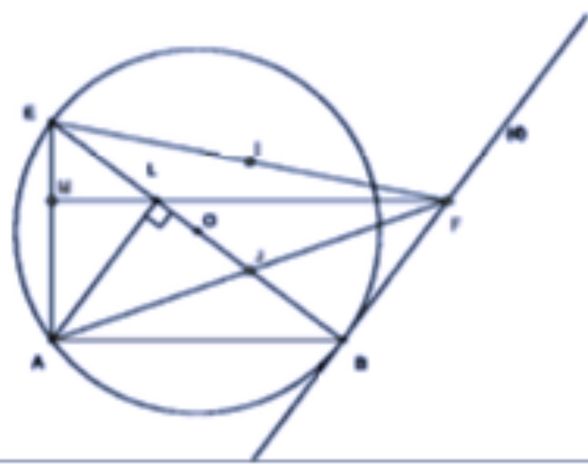
	Correction	
Part	**Question I**	**Note**
1	$3\sqrt{2}-\sqrt{50}+\sqrt{8}=0$ **b)**	0.5
2	$\frac{1}{\sqrt{5}-2}=\frac{1}{\sqrt{5}-2}\times\frac{(\sqrt{5}+2)}{(\sqrt{5}+2)}=\sqrt{5}+2$ **a)**	0.5
3	$\overline{DA}+\overline{AB}=\overline{DB}$ **c)**	0.5
4	$23000\div1.15=20000$ **b)**	0.5

Part	**Question II**	**Note**
1	$A(x)=(x-3)^2-(x-3)(2x-7)=(x-3)(x-3-2x+7)=(x-3)(4-x)$ 0.25+0.25	0.5
2	$B(x)=(16-x^2)+A(x)=(16-x^2)+(x-3)(4-x)$ 0.25 + 0.25 $B(x)=(4-x)(4+x)+(x-3)(4-x)=(4-x)(2x+1)$ 0.25 + 0.25	1
3	**a.** F(x) is defined for $x\neq4$ et $x\neq-\frac{1}{2}$ 0.25 + 0.25	0.5
	b. $\frac{(x-3)(4-x)}{(4-x)(2x+1)}=\frac{(x-3)}{(2x+1)}$; $3x-9=4x+2$ alors $x=-11$ 0.25+0.5	0.75
	c. $\frac{x-3}{2x+1}=x$ then $(2x+1)x=(x-3)$ and $x^2=-\frac{3}{2}$ No solution 0.25+0.25+0.25	0.75

Part	**Question III**	**Note**
1	x = 1 000 and y = 3 500 0.5 + 0.5	1
2	$\begin{cases}10x+4y=24\,000(\div2)\\6x+12y=48\,000(\div6)\end{cases}$ (0.5+0.25) (0.25+0.25)	1.25
3	The price of a green salad is 1 000 L.L. and the price of a vegetarian pizza is 3 500L.L. 0.25 so Nadine will pay 8(1000) +6(3500) = 29 000 L.L 0.5	0.75

Part	**Question IV**	**Note**
1	 One point (0.25) Two points (0.5)	0. 5
	b. $y_E=3x_E$ then E is a point on the line (d) 0.25+0.25	0.5

2	**a.** $OB=\sqrt{5}$ and $OA=2\sqrt{5}$ then OA=2OB 0.25+0.25+0.25	0.75
	b. AB=5 and by Pythagorean theorem $AB^2=OA^2+OB^2$ 0.25+0.25	0.5
3	**a.** $x_B=\frac{x_L+x_O}{2}=-1$ and $y_B=\frac{y_L+y_O}{2}=2$ then L(-2;4)	0.5
	b. $x_E=\frac{x_A+x_L}{2}=1$ and $y_E=\frac{y_A+y_L}{2}=3$ verified	0.5
4	$a_{(OA)}=\frac{1}{2}$ and $a_{(OA)}.a_{(d')}=-1$ then $a_{(d')}=-2$; (d'):y = - 2 x +10 0.25+0.5+0.25	1
5	**a.** F belongs to (d) since $y_F=3x_F$ and F belongs to (d') since $y_F=-2x_F+10$ 0.25+0.25	0.5
	b. E midpoint of [LA] and E miodpoint of [OF] then OAFL is a parallelogram. OL=2OB=OA and $\hat{O}=90^0$ then OAFL is a square. 0.25+0.25+0.25	0.75

Part	Question V	Note
1		0. 5
2	**a.** Pythagorean Theorem : $AB^2=EB^2-AE^2=100-36$ so $AB=8$	0.5
	b. $\sin \hat{AEB}=\frac{AB}{EB}=\frac{8}{10}=\frac{4}{5}$	0.5
3	**a.** $\hat{ELM}=\hat{BLF}$ (vertically opposite angles) 0.5 $\hat{EML}=\hat{LBF}=90^0$ 0.5 So the two triangles are similar	1
	b. $\sin \hat{BFL}=\sin \hat{AEB}=0.8$ so $\hat{BFL}\approx 53^0$ 0.5+0.5	1
4	The two triangles EMF and EBF are two right triangles facing the same hypotenuse [EF] so the four points E,M,B and F are on the same circle of center I the midpoint of [EF]. 0.25+0.25+0.25	0.75
5	(AL) ⊥ (EB) and (BF) ⊥ (EB) so (AL)//(BF) 0.5+0.25 And we have (LF)//(AB) then ALFB is a parallellogram.	0.75
6	I midpoint of [EF] and J midpoint of [AF] so (IJ)//(EA) and (EA) ⊥ (AB) so (IJ) ⊥ (AB) 0.5+0.5	1

عدد المسائل: خمس	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

إرشادات عامة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الواردة في المسابقة.

I - (2 points)

In the table below, only one of the proposed answers to each question is correct.

Write down the number of the question and give, **with justification**, its corresponding answer.

N°	Questions	Proposed answers		
		a	b	c
1	$\frac{1}{3}-\frac{1}{3}\times\frac{6}{7}=$	0	$\frac{1}{21}$	$\frac{6}{7}$
2	$\left(3+\sqrt{5}\right)^2-14=$	$9+\sqrt{5}$	0	$6\sqrt{5}$
3	The five grades of a student over 20 are: 10 ; 12 ; 13 ; 16 and 19. The average grade is:	13	14	14.5
4	<table><tr><td>x</td><td>$\sqrt{2}$</td></tr><tr><td>$\sqrt{2}$</td><td>4</td></tr></table> The table above is a table of proportionality for x =	4	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$

II - (3.5 points)

Given $A(x)=(3x-2)^2-(2x-1)(3x-2)$ and $B(x)=9x^2-4$.

1) **a.** Verify that $A(x)=(3x-2)(x-1)$.

b. Solve the equation $A(x)=0$.

2) Factorize $B(x)$.

3) Let $F(x)=\dfrac{(3x-2)(3x+2)}{A(x)}$.

a. For what values of x, is F(x) defined?

b. Simplify F(x).

c. Does the equation $F(x)=-12$ admit a solution? Justify.

III - (3.5 points)

1) Solve the following system: $\begin{cases}2x+5y=50\,000\\2x+3y=38\,000\end{cases}$

2) In a museum, 2 adults and 5 kids buy tickets and pay 50 000 LL;
4 adults and 6 kids pay 76 000 LL.

a. Prove that the previous information is modeled by the system given in question **1)**.

b. Find the price of the ticket of an adult and that of a kid.

3) For a group of 30 kids and 4 adults, the director of the museum decided to offer a reduction of 25% on the total amount paid for the tickets. Calculate then the amount paid.

IV - (5.5 points)

In an orthonormal system of axes x'Ox and y'Oy, consider the points $A(-1;0)$ and $B(1;4)$.

Let (d) be the line with equation $y = 2x + 2$.

1) **a.** Verify that A and B are two points on line (d).

b. Plot the points A and B then draw (d).

2) Let I be the point of intersection of (d) with y'Oy.

a. Calculate the coordinates of I.

b. Verify that I is the midpoint of [AB].

3) Let (d') be the perpendicular bisector of [AB].Verify that the equation of (d') is $y = -\frac{1}{2}x + 2$.

4) Consider the point $M(4;0)$. Show that triangle MAB is isosceles of vertex M.

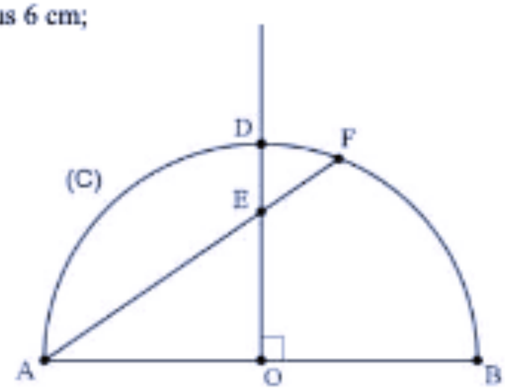
5) Let K be the translate of B under the translation with vector $\overrightarrow{MA}$.

Prove that quadrilateral MBKA is a rhombus.

V- (5.5 points)

In the adjacent figure:

- (C) is a semicircle of diameter [AB], with center O and radius 6 cm;
- The perpendicular bisector of [AB] intersects (C) at D;
- E is a point on segment [OD] so that $OE = 4$ cm;
- (AE) intersects (C) at F.



1) Reproduce the figure.

2) Verify that $AE = 2\sqrt{13}$ cm.

3) **a.** Prove that AFB is a right triangle at F.

b. Prove that the two triangles AOE and AFB are similar.

c. Deduce the value of $AE \times AF$.

4) The line (BF) intersects line (OD) at K and the line (BE) intersects line (AK) at I.

a. Prove that line (BE) is perpendicular to line (AK).

b. Deduce that I is a point on (C).

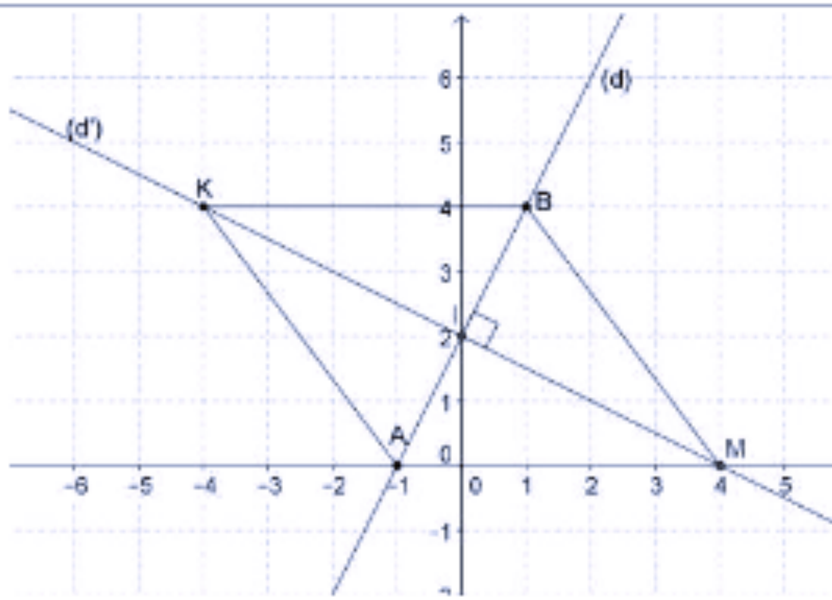
5) The tangent to (C) at A intersects (BE) at S.

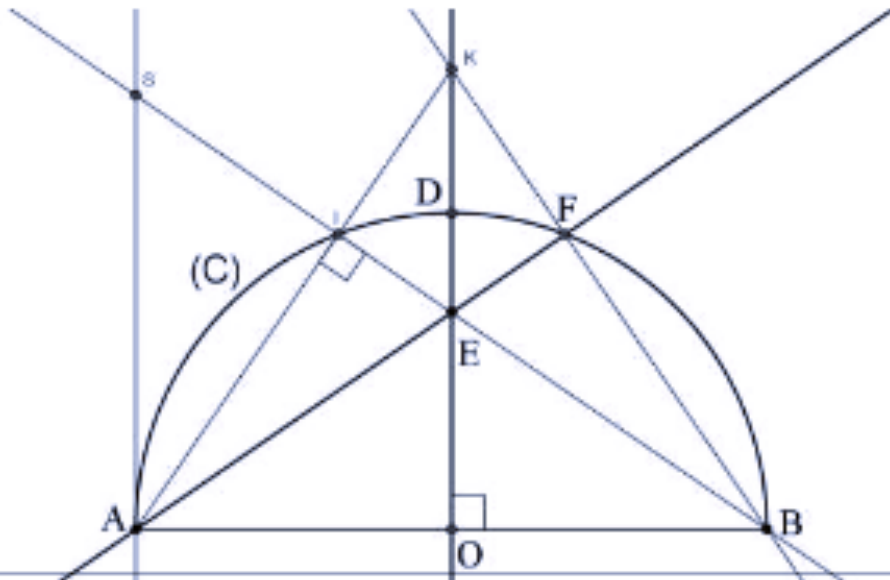
a. Show that E is the midpoint of [BS].

b. Verify that $BS = 4\sqrt{13}$ cm.

عدد المسائل: خمس | مسابقة في مادة الرياضيات | مشروع اسس التصحيح - انكليزي

Part	Answer Key		Notes
	Question I		
1	$\frac{1}{3}-\frac{1}{3}\times\frac{6}{7}=\frac{1}{3}-\frac{2}{7}=\frac{7-6}{21}=\frac{1}{21}$ (b)	0.25 + 0.25	0.5
2	$\left(3+\sqrt{5}\right)^2-14=9+6\sqrt{5}+5-14=6\sqrt{5}$ (c)	0.25 + 0.25	0.5
3	The average grade $=\frac{10+12+13+16+19}{5}=14$ (b)	0.25 + 0.25	0.5
4	$\frac{x}{\sqrt{2}}=\frac{\sqrt{2}}{4}$ so : $4x=2$; $x=\frac{1}{2}$ (b)	0.25 + 0.25	0.5
	Question II		
1	$A(x)=(3x-2)^2-(2x-1)(3x-2)$ $A(x)=(3x-2)[(3x-2)-(2x-1)]$ $A(x)=(3x-2)(3x-2-2x+1)$ $A(x)=(3x-2)(x-1)$	 0.25 0.25 0.25	0.75
2a	$A(x)=0$ $(3x-2)(x-1)=0$ gives $3x-2=0$ or $x-1=0$ $x=\frac{2}{3}$ or $x=1$	 0.25 0.25	0.5
2b	$B(x)=9x^2-4=(3x-2)(3x+2)$		0.5
3a	$F(x)=\frac{B(x)}{A(x)}=\frac{(3x-2)(3x+2)}{(3x-2)(x-1)}$; $F(x)$ is defined when $x\neq\frac{2}{3}$ and $x\neq1$	0.25 + 0.25	0.5
3b	$F(x)=\frac{3x+2}{x-1}$		0.25
3c	$F(x)=-12$ $\frac{3x+2}{x-1}=-12$ gives $x=\frac{10}{15}=\frac{2}{3}$ (rejected). $F(x)=-12$ does not admit a solution.	0.25 + 0.5 + 0.25	1
	Question III		
1	$\begin{cases}2x+5y=50000\\2x+3y=38000\end{cases}$ $x=10\ 000$; $y=6\ 000$		1
2a	Let x be the price of the ticket of an adult And y the price of the ticket of a kid $\begin{cases}2x+5y=50000\\4x+6y=76000\end{cases}$ $\div 2$ implies $\begin{cases}2x+5y=50000\\2x+3y=38000\end{cases}$	0.25 0.25 0.25 + 0.25	1
2b	Using question 1) ; $x=10\ 000$; $y=6\ 000$. The price of the ticket of an adult is 10 000LL and that of a kid is 6 000 LL.	0.25 + 0.25	0.5
3	$30\times 6\ 000+4\times 10\ 000=220\ 000$ LL $220\ 000\times 0.75=165\ 000$ LL After the reduction, the amount paid is 165 000 LL.	0.5 0.5	1

Question IV		
1a	$y_A = 2x_A + 2$ and $y_B = 2x_B + 2$ $4 = 2(1) + 2$ $\quad$ $0 = 2(-1) + 2$ $\quad$ **0.25 + 0.25** $4 = 4$ $\quad$ $0 = 0$	**0.5**
1b	 **0.25** **0.25** **0.25**	**0.75**
2a	$I \in y'oy$ then $x_I = 0$ **0.25** $I \in (d)$ then $y_I = 2x_I + 2 = 2(0) + 2 = 2$ **0.5** So $I(0; 2)$	**0.75**
2b	$x_I = \frac{x_A + x_B}{2}$ $0 = \frac{-1+1}{2}$ **0.25** $0 = 0$ $y_I = \frac{y_A + y_B}{2}$ $2 = \frac{0+4}{2}$ **0.25** $2 = 2$ then I is the midpoint of [AB].	**0.5**
3	(d') is the perpendicular bisector of [AB] then (d') is perpendicular at the midpoint of [AB]. $(d') \perp (d)$ then $a_{(d)} \times a_{(d')} = -1$ so $a_{(d')} = -\frac{1}{2}$ **0.25 + 0.25** $I \in (d')$ then $y_I = -\frac{1}{2}x_I + b$ so b = 2. (d'): $y = -\frac{1}{2}x + 2$ **0.25 + 0.25**	**1**
4	$M \in (d')$ since $y_M = -\frac{1}{2}x_M + 2$ so MA = MB since any point on the perpendicular bisector is equidistant from the extremities of this segment **0.5 + 0.5** Or MA = 5 and MB = 5….	**1**
5	$\overrightarrow{BK} = \overrightarrow{MA}$ (definition of translation) so MBKA is a parallelogram. **0.5** Moreover: MA = MB then MBKA is a rhombus. **0.5**	**1**

Question V			
1			0.5
2	OEA is a right triangle at O. $AE^2 = OA^2 + OE^2 = 36 + 16 = 52$ (Pythagorean) $AE = \sqrt{52} = 2\sqrt{13}$ cm	0.25 0.25 0.25	0.75
3a	AFB is a right triangle at F since it is inscribed in a semicircle of diameter [AB]		0.5
3b	In the two triangles AOE and AFB: $\widehat{AOE} = \widehat{AFB} = 90^0$ $\widehat{OAE} = \widehat{FAB}$ (common angle) Therefore they are similar.	 0.5 0.25	0.75
3c	$\frac{OA}{AF} = \frac{AE}{AB} = \frac{OE}{FB}$ (ratio of similitude) $AE \times AF = AO \times AB = 6 \times 12 = 72$	0.25 0.25	0.5
4a	In triangle AKB we have: [AF] is the first height. [KO] is the second height. [AF] and [KO] intersect at E, the orthocenter of this triangle. [BS] passes through E, so it is the third height. Therefore: (BE) $\perp$ (AK)	 0.25 0.25 0.25	0.75
4b	AIB is a right triangle at I so it is inscribed in the semicircle of diameter [AB]. Hence I is a point on (C).		0.5
5a	In triangle ASB, we have: O is the midpoint of [AB] (OE) // (AS) (Two lines perpendicular to the same line) Hence E is the midpoint of [BS] (Converse of midsegment theorem)	 0.25 0.25 0.25	0.75
5b	$AE = \frac{BS}{2}$ (Median relative to the hypotenuse) Then: BS = 2 AE = $4\sqrt{13}$ cm.	0.25 0.25	0.5