Lycée Des Arts

Nom/ Name: ……………………………..…………………

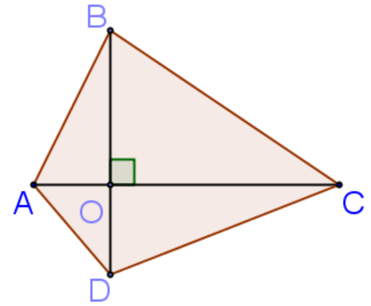
Classe/Class: **Grade 8** Section:…………. Date: …………..……….

Examen de/ Exam in: **Math** **Midterm**

1st exercise: (6 ½ pts)

In the figure below, $ABCD$ is a quadrilateral with perpendicular diagonals that intersect at O, such that

$$AC = 6cm,\ AO = \frac{2^{11}+4^5}{2^{11}-4^5},\ DO = \frac{\frac{2}{10}+\frac{2}{5}}{\frac{2}{3}\times\frac{9}{10}-\frac{2}{5}} \text{ and } BD = \frac{0.24\times 9}{5^{-2}\times 36\times 2^{-2}}.$$



1) a) Prove that AO is a natural number to be determined. (¾ pt)

   b) Verify that: $OD = \dfrac{BD}{2}$? (1½ pts)

   c) Deduce the relative position of O with respect to $ABCD$.(¾ pt)

   d) Show that the quadrilateral $ABCD$ is a square. (¾ pt)

2) The parallel to (AB) through O cuts $[AD]$ at F.

   a) Reproduce and complete the figure. (¼ pt)

   b) Show that $OF = \dfrac{AD}{2}$ (1 pt)

   c) Prove that the area of triangle AOD is $4.5cm^2$. Deduce the area of $ABCD$.(1 ½ pts)

2nd exercise: (6¼ pts)

Part A: Consider the numbers: $n = \dfrac{\sqrt{18}\times\sqrt{20}}{\sqrt{45}\times\sqrt{2}}$ and $u = \sqrt{3x-8}$

1) a) Prove that $n = 2$.(¾ pt)

   b) Can you calculate u for $x = n$? Justify your answer. (½ pt)

   c) Determine the values of x so that u exists. (¾ pt)

2) Calculate x for $u = n$.(¾ pt)

Part B: Let $m = 2\sqrt{75} - 3\sqrt{48} + 2\sqrt{12} + \sqrt{1}$

1) a) Write m in the form $a + b\sqrt{3}$ where a & b are natural integers to be determined. (¾ pt)

   b) Use the calculator to find an approximate value of m to the nearest 0.001by excess. (½ pt)

2) a) Calculate m^2 then develop: $(\sqrt{3}+2)^2$. (¾ pt)

   b) If $t = m^2 - (\sqrt{3}+2)^2 + 3$, prove that $t = 9$. (¾ pt)

   c) Calculate the measure of the side of a square $ABCD$ knowing that its **area equals** t. (¾ pt)

3rd exercise: (6½ pts)

Consider the following algebraic expressions:

$$P(x) = (3x-5)^2 + (1+x)(5-3x) \quad \text{and} \quad Q(x) = (x^2-9) - (3x-9)(x-5)$$

1) a) Show that $P(x) = 2(3x-5)(x-3)$.(¾ pt)

   b) Solve in the set of natural numbers the equation: $P(x) = 0$.(¾ pt)

2) a) Factorize $Q(x)$.(¾ pt)

b) Write $Q(x)$ in the form $ax^2 + bx + c$ where $a, b \& c$ are integers to be determined. (¾ pt)

c) Solve $Q(x) = -54$.(¾ pt)

d) Is $x = 9$ a root of $Q(x)$? Justify. (½ pt)

3) Let $F(x) = \dfrac{P(x)}{Q(x)}$

a) For what values of x does $Q(x)$ vanish? Deduce the domain of definition of F. (1 pt)

b) Simplify $F(x)$.(½ pt)

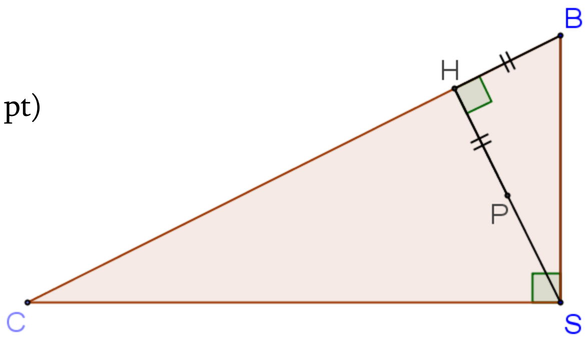
c) Calculate $F(\sqrt{2})$ and give the answer in simplest form. (¾ pt)

__4th exercise: (5 ¾ pts)__

The adjacent figure is a triangle right BSC at S , and $[SH]$ is the height relative to the hypotenuse $[BC]$.

P is a point on $[SH]$, such that $HB = HP$.

(***Draw an enlarged figure at the center of the answer sheet***) (¼ pt)



1) Draw the parallel through P to (SC) that cuts $[BC]$ & $[BS]$ at $E \& F$ respectively.

   a) Prove that the triangle BEF is right at F .(½ pt)

   b) Verify that: $S\hat{B}C = E\hat{P}H$. (¾ pt)

   c) Deduce that triangles $BHS \& HEP$ are congruent. (Write the homologous elements) (¾ pt)

   d) What is the nature of triangle HSE? (½ pt)

2) Let I be the point of intersection of the straight lines (BP) and (SE), and K be the orthogonal projection of E on $[SC]$.

   a) Prove that (BI) is perpendicular to (SE) .(¾ pt)

   b) Determine the nature of quadrilateral $SFEK$. Justify. (¾ pt)

3) Designate by T the intersection point of :

   the parallel through E ***to*** (SH)***, and the parallel through*** S ***to*** (HC).

   a) Prove that $HETS$ is a square. (¾ pt)

   b) Deduce that $HETS \& SFEK$ have the same center of symmetry. (¾ pt)

Good Work