Name / Nom:
Class / Classe: Gr8

## يمنع استعمال الآلة الحاسبة

## Exercise I:(13pts)

## The following parts are independent:

## Part A: ( $3^{1} / 2 \mathrm{pts}$ )

In the adjacent figures we have:

- $\boldsymbol{A B C}$ is a right triangle at $\boldsymbol{A}$ such that:

$$
\boldsymbol{A B}=2(x+1) \text { and } A C=x+1
$$

- $\boldsymbol{D E F G}$ is a rectangle of dimensions

$$
\begin{aligned}
& 4(x-3) \text { and } x-3 ; \\
& (x>3 \text { and the unit of length is cm })
\end{aligned}
$$



Designate by $A_{t}$ the area of the triangle $A B C$ and by $A_{2}$ the area of the rectangle $D E F G$.
Calculate $\boldsymbol{x}$ when $A_{1}=A_{2}$.

## Part B( $5^{1 / 2}$ pts)

ABC is any triangle such that:

- $\boldsymbol{A B}=\frac{2 \frac{1}{4}+\frac{3}{2}}{1+\frac{1}{1+\frac{1}{2}}} \times 2^{2} \mathrm{~cm}$. (where $2 \frac{1}{4}$ is a mixed number.)
- $I$ is any point on $[A B]$ such that:
- $\boldsymbol{A I}=\frac{16^{2}+24^{3}}{128 \times 22}-\frac{1}{2} \mathrm{~cm}$.

1) Given $x=2 \frac{1}{4}+\frac{3}{2} \quad$ and $y=1+\frac{1}{1+\frac{1}{2}}$.
a) Prove that $x=\frac{15}{4}$ and $y=\frac{5}{3}$.( $\left.11 / 2 \mathrm{pts}\right)$
b) Calculate $\frac{x}{y}$. Deduce $\boldsymbol{A B}$. (1pt)
2) Show that $A I=\frac{A B}{2}=\frac{9}{2}$. (1pt)
3) The parallel drawn from I to ( $B C$ ) cuts $[A C]$ at $J$. What is the relative position of point $J$ with respect to segment [AC]? Justify. (2pts)

## $\underline{\text { Part C: }}$ (4pts)

$A B C$ is any triangle such that:

- $M$ is the midpoint of segment $[B C]$.
- $\mathbf{B C}=(2 x+1)^{2}-\frac{1}{2} x(8 x-1)+\frac{3}{2} x+3$
- $\mathbf{A M}=x(-9 x+3)+(3 x-1)(3 x+1)+3$
( $x>-\frac{2}{3}$ and the unit of length is in cm )

1) Show that $B C$ is the double of $A M$. $(21 / 2 \mathrm{pts})$
2) Deduce the nature of the triangle $A B C$. ( $11 / 2 \mathrm{pts}$ )


## Exercise II: ( $151 / 4 \mathrm{pts}$ )

1) Given the two polynomials:

$$
Q(x)=(3 x-6)(x+1)-x^{2}+4 x-4 \text { and } P(x)=(m-2) x^{2}-8+3(2-x)(x-1)
$$

a) Develop and reduce $\boldsymbol{Q}(\boldsymbol{x})$ then deduce its degree. (1 $1 / 4 \mathrm{pts}$ )
b) What does $\boldsymbol{Q}(\boldsymbol{x})$ represent? and for what values of $x$ is it defined?. (1pt)
c) Is $x=-1$ a root of $Q(x)$ ? Justify. (1pt)
d) Write $Q(x)$ in form of product of two factors. (1pt)
e) Define a root of a polynomial then deduce the roots of $\boldsymbol{Q}(\boldsymbol{x}) \cdot\left(1^{11 / 2 p t s}\right)$
2) Calculate $\boldsymbol{m}$ such that $\mathbf{X}$ is a root of $\boldsymbol{P}(\boldsymbol{x})$. (1pt)
3) In what follows we consider that $\underline{m=4}$
a) Prove that $P(x)$ can be written in the form $a x^{2}+b x+c$, where $a, b \& c$ are integers to be determined. ( $11 / 2 \mathrm{pt}$ )
b) Solve $\boldsymbol{P}(\boldsymbol{x})=-14(1 \mathrm{pt})$
c) Calculate $P\left(-\frac{1}{2}\right)(1 \mathrm{pt})$
d) Show that $\boldsymbol{P}(\boldsymbol{x})=(x-2)(-x+7)(1 \mathrm{pts})$
4) Let $F(x)=\frac{(x-2)(-x+7)}{(x-2)(2 x+5)}$
a) What does $F(x)$ represent? Justify. ( $3 / 4 \mathrm{pt}$ )
b) Given the following table:

| $\boldsymbol{x}$ | $\boldsymbol{F}(\boldsymbol{x})$ |
| :---: | :---: |
| 2 | $\ldots \ldots \ldots$ |
| 3 | $\ldots \ldots \ldots$ |
| $\frac{-5}{2}$ | $\ldots \ldots \ldots$ |

Complete the following table showing your justification for each answer. What conclusion can you conclude? ( $21 / 4 \mathrm{pts}$ )
c) Simplify $\boldsymbol{F}(\boldsymbol{x})$ then solve $\boldsymbol{F}(\boldsymbol{x})=\frac{1}{2}$. $(1 \mathrm{pt})$

## Exercise III: ( 11 3/4pts)

$A B C$ is an isosceles triangle at A such that:

$$
\boldsymbol{A} \boldsymbol{B}=\frac{1.05 \times 10^{2} \times 20}{420} \mathrm{~cm} \quad \text { and } \quad \boldsymbol{B} \boldsymbol{C}=\frac{28}{75} \times \frac{45}{21}-5^{-1}+3 \frac{2}{5} \mathrm{~cm}
$$

1) Show that $A B=5 \mathrm{~cm}$. ( $11 / 2 \mathrm{pts}$ )
2) a) Verify that $B C$ is a power of 2 . $(13 / 4 \mathrm{pts})$
b) Calculate the perimeter of the triangle $A B C$ then give the answer in scientific notation. ( $11 / 4 \mathrm{pts}$ )
3) a) Trace the triangle $A B C$ such that $A B=5 \mathrm{~cm}$ and $B C=4 \mathrm{~cm} .(1 / 4 \mathrm{pt})$
b) $[B M]$ and $[C N]$ are two heights relative to $[A C]$ and $[A B]$ respectively.

By using two congruent triangles, show that $B N=C M$. ( $11 / 2 \mathrm{pts}$ )
c) What is the nature of the triangle $A M N$ ?Justify ( $1 \mathrm{pt)}$
4) Show that $M N B C$ is an isosceles trapezoid. ( $3 / 4 \mathrm{pt}$ )
5) a) The parallel (Bx) from $B$ to $(N C)$ cuts the perpendicular that issued from $C$ to $(N C)$ at $K$. What is the nature of the quadrilateral $N B K C$ ? ( $11 / 2 \mathrm{pts}$ )
b) Deduce the nature of the triangle $M C K$ ? $(3 / 4 \mathrm{pt})$
6) let $O$ be the center of the rectangle $B N C K$, and $F$ be the orthogonal projection of $O$ on $[\boldsymbol{N C}]$. Show that $\boldsymbol{O F}=\frac{\boldsymbol{M C}}{\mathbf{2}} .\left(1^{11 / 2 \mathrm{pts})}\right.$

