I- (4 points)

In the space referred to a direct orthonormal system $(O;\vec{i},\vec{j},\vec{k})$, consider the points A($a$; 2 ; 0), B(2 ; 1 ; 6), and C(1 ; –2 ; 24), where a is a real number. In the table below, only one of the proposed answers to each question is correct. Write down the number of each question and give, **with justification**, the answer corresponding to it.

Questions		Proposed Answers		
		A	**B**	**C**
1)	A, B, and C are collinear when a =	-1	$\frac{7}{3}$	$\frac{5}{3}$
2)	O, A, B and C are coplanar when a =	0	-8	8
3)	If $\vec{u} = \overrightarrow{AB} - 2\overrightarrow{AC}$, then a =	$\vec{i}$	$x_{\vec{u}}$	$\lVert\vec{u}\rVert$

II- (10 points)

Part A:

Let g be a function defined, over IR, by: $g(x) = x^3 + ax^2 - 3x + b$, where a and b are two real numbers. Let (G) be the representative curve of g, **Find** a and b, knowing that (G) has A (1; 1) as a minimum.

Part B:

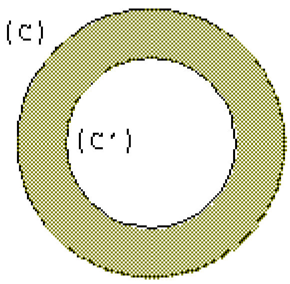
Let f be a function defined, over IR, by: $f(x) = x^3 - 3x + 3$. Let (C) be the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$.

1) **Study** the variations of f.
2) **Prove** that the point I (0; 3) is a center of symmetry of (C).
3) **Write** the equation of the straight line (T), the tangent to (C) at I.
4) **Draw** (T) and (C).
5) **Show** that the equation f(x) = 0 has a unique root α such that $-3 < \alpha < -2$.
6) Let h be a function defined by: $h(x) = |x|^3 - 3|x| + 3$.
   a- **Prove** that h is an even function.
   b- **Draw** (H), the curve of h, in the previous system $(O;\vec{i},\vec{j})$.

III - (3.5 points)

Consider, in an orthonormal system $(O;\vec{i},\vec{j})$, the circle (C): $x^2 + y^2 - 2x + 4y = 0$.

1) **Find** the center and the radius of (C).
2) In the adjacent figure, (C') is another circle that is **concentric** with (C). **Find** the equation of (C'), knowing that the area of the shaded part is 3π (unit)^2.



IV - (2.5 points)

Consider the function f defined by $f(x) = \dfrac{|x+4|-3x}{3x-9-|x+4|}$.

1) **Calculate** $\lim\limits_{x\to -4} f(x)$.
2) **Is f** continuous at -4? Justify.

V-(7 points)

Let (U_n) be a sequence defined by: $U_0 = 1$, $U_1 = 3$ and $U_{n+2} = \dfrac{1}{2}a^2U_{n+1} + (a-3)U_n$ for every n belongs to IN and a is a real.

Let (V_n) be the sequence, such that $V_n = U_{n+1} - U_n$.

1) Suppose that a = 2.
   a- **Show** that the sequence (V_n) is constant.
   b- **Deduce** the nature of the sequence (U_n)?
   c- **Write**, in terms of n, (U_n) and $S_n = \sum_{i=0}^{n} U_i$ in terms of n.
2) Suppose that a = –4.
   a- **Show** that the (V_n) is a geometric sequence. **Determine** its ratio and its first term.
   b- **Write** (V_n) in terms of n, **then calculate** $L_n = \sum_{i=0}^{n} V_i$ in terms of n.

VI- (8 points)

Consider the equation (E): $x^2 - 2(m-1)x + 3 - 2m = 0$, where x is an unknown and m is a real parameter.

1) **Find** m so that (E) admits two distinct real roots.
2) **Find** the values of m when $x^2 - 2(m-1)x + 3 - 2m$ is a perfect square.
3) Let x_1 and x_2 be distinct roots of (E) such that $x_1.x_2 = 3-2\sqrt{2}$ and $x_1 + x_2 = 2\sqrt{2}-2$.
   a- **Prove** that $m = \sqrt{2}$, then solve (E) when $m = \sqrt{2}$.
   b- Let a and b be two angles such that $a + b = \dfrac{\pi}{4}$ and $\tan(a) \times \tan(b) = 3-2\sqrt{2}$.
      i. **Calculate** tan(a + b), then **deduce** that $\tan(a) + \tan(b) = 2\sqrt{2}-2$.
      ii. **Determine** the values of tan(a) and tan(b).

VII- (5 points)

Remark: The parts of this question are independent.

1) **Show** that: $\cos^2 x - \cos\left(x+\dfrac{\pi}{3}\right)\times\cos\left(x-\dfrac{\pi}{3}\right) = \dfrac{3}{4}$.
2) **Prove** the identity: $\dfrac{2\sin x - \sin 2x}{2\sin x + \sin 2x} = \tan^2\left(\dfrac{x}{2}\right)$
3) **Calculate** $(\vec{a},\vec{c})$ when $(\vec{a},\vec{b}) = \dfrac{\pi}{3} + 2k\pi$ and $(\vec{c},\vec{b}) = -\dfrac{\pi}{4} + 2k\pi$.