In his name

The Islamic Institution For
Education & Teaching
Al-Mahdi Schools

Mid Year Exam

Math Department
February 2009

Subject: Mathematics

Grade 11 S

Duration: 150 minutes

The plane, when needed, is referred to an orthonormal system $(O;\vec{i};\vec{j})$

I. (3 pts)

Each of the following questions has exactly one correct answer. Write down the number of the question then indicate, with **justification**, the correct answer.

	Questions	Answers A	Answers B	Answers C
1)	$\lim\limits_{x\to-\infty}\frac{-x}{x-\sqrt{1+x^2}}=$	$-\frac{1}{2}$	$\frac{1}{2}$	-1
2)	The curve representing $f(x)=x^2+4x$ admits as an axis of symmetry the line:	$y=-2$	$x=-2$	$x=-4$
3)	Given $f(x)=2\sin 3x$ & $g(x)=2\sin(3x+6)$. The curve of g is the image of the curve of f by translation of vector:	$\vec{V}(-6;0)$	$\vec{V}(6;0)$	$\vec{V}(-2;0)$

II. (3 pts)

1. Consider the quadratic equation (E) : $(m+1)x^2-(m-2)x+1-m=o$

Determine m so that (E) admits 2 distinct real roots.

2. Consider the equation (F): $(\cos a)x^2-(2\sin a)x+\cos a=0$, where a is a real number.

a. Prove that : If (F) admits one double root in R , then $\cos 2a=0$.

b. Let S and P be the sum and the product of the roots x_1 and x_2 of (F), when they exist.
Write $\tan 2a$ in terms of S and P.

III. (5 pts) ***<u>Remark</u>: The parts of this question are independent.***

1. Given f(x) = $\frac{-2x^2}{\sqrt{5x+2}}$ $\left(x>\frac{-2}{5}\right)$ and g(x) = $(\cos 5x-\sin x)^3$

Find $f'(x)$ and $g'(x)$.

2. Let f be a function defined by: $\begin{cases}\frac{1}{2} & \text{for } x=0\\ a\frac{\sin 2x}{x} & \text{for } x\neq 0\end{cases}$

Find "a" so that f is continuous at 0.

3.

a. Show that $\tan(\frac{\pi}{4}-x)=\frac{\cos x-\sin x}{\cos x+\sin x}$

b. Deduce that : $\frac{\cos 2x}{1+\sin 2x}=\tan(\frac{\pi}{4}-x)$

IV. (3 ½ pts)

Given a circle (C): $x^2 - 2x + y^2 = 2$ and the points A(2 , $\sqrt{2}$) and N(x , $\sqrt{2}$)

1. Determine the center W and the radius R of the circle (C).
2. Show that A belongs to (C) .
3.
   a. Show that $(\mathbf{NW})^2 - \mathbf{R}^2 = x^2 - 2x$
   b. Deduce the values of x so that N is exterior point of (C)
4. Solve in R the following system :

$$\begin{cases} \dfrac{x-1}{-x^2-2x-1} \geq 0 \\ x^2 - 2x > 0 \end{cases}$$

V. (3 ½ pts)

Consider the sequence (Un) defined by $U_0 = 0$ and $U_{n+1} = \dfrac{2U_n + 1}{U_n + 2}$ ($U_n \neq -2$ for all n $\in$ N)

1. Calculate U_1, U_2 & U_3.
2. Consider the sequence (V_n) defined by $V_n = \dfrac{U_n - 1}{U_n + 1}$
   a. Show that (V_n) is a geometric sequence whose common ratio and first term are to be determined.
   b. Find, in terms of n, V_n and U_n.
   c. Determine, in terms of n, $\sum_{i=0}^{n} V_i$.

VI. (6 pts)

A. Consider the function f defined over IR; by: f(x) = $ax^4 + bx^2$ and let (C) be the representative curve of f.

1. Determine a & b so that(C) admits A (1; 1) as maximum.

B. Consider in this part a=-1 & b = 2

   a. Study the variations of f.
   b. Prove that $-x^4 + 2x^2 = 0$ admits 3 different roots.
   c. Draw the representative curve (C) of f in an orthonormal reference $(O;\vec{i},\vec{j})$.

2. Let g(x) = $\left| -x^4 + 2x^2 \right|$
   b. Construct the graphical representation of the function g.
   c. Study, graphically ,the differentiability of g at the point of abscissa $\sqrt{2}$