The Islamic Institution for Education & Teaching Al-Mahdi Schools

Mid-Year Exam

Math Department
February 2010

Subject: Mathematics

Grade: 11th Scientific

Duration: 150 minutes

I. (2 points)

In the table below, one of the answers is correct. ***Write the letter corresponding to the correct Answer and justify.***

	Questions	A	B	C	D
		Answers			
1)	Knowing that: $\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$, then $\cos\frac{\pi}{8}=$	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$-\frac{\sqrt{2+\sqrt{2}}}{2}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$
2)	In an orthonormal system consider the equation of circle (C): $2x^2 + 2y^2 + 4x - 8y + 4 = 0$. The center Ω and the radius r of (C) are:	$\Omega(1;-2)$ $r=3$	$\Omega(1;-2)$ $r=\sqrt{3}$	$\Omega(-1;2)$ $r=3$	$\Omega(-1;2)$ $r=\sqrt{3}$
3)	$\lim\limits_{x\to-\infty}\frac{2x+3}{\sqrt{9x^2-1}}=$	$\frac{2}{3}$	0	$-\frac{2}{3}$	$-\infty$

II. (5.5 Points)

1. Calculate the derivative of the following functions:

a) $f(x)=\frac{3}{x^2+1}+\sqrt{x^2-1}$ defined on $]-\infty\,;-1[\,\cup\,]-1\,;+\infty[$

b) $f(x)=\sin^2(x^2)$ defined on IR.

2. Determine the domain of the following function $f(x)=\sqrt{\frac{2x^2+3x-5}{x^2+x-2}}$.

3. Given the function f defined on IR by :

$$f(x)=\begin{cases}\frac{x^2-4x+3}{x^2-3x+2} & si\ x<1\\ k^2+k & si\ x=1\\ \frac{1-x}{1-\sqrt{x}} & si\ x>1\end{cases}\qquad ;\qquad \text{k is a real constant}$$

a-Calculate $\lim\limits_{\substack{x\to1\\x<1}} f(x)$ and $\lim\limits_{\substack{x\to1\\x>1}} f(x)$.

b- Find k so that f is continuous at $x = 1$

III. (3 points)

Consider the second degree equation in x (E): $(m-1)x^2 - 2(m-2)x + m - 7 = 0$; $m \in IR$.

1. Calculate m knowing that one of the roots is 4, and deduce the other root.
2. Calculate m so that (E) admits two distinct roots.
3. Let $F = \frac{(x_1-1)(x_2-1)}{x_1^2 + x_2^2}$,with x_1 and x_2 are the roots of (E) when they exist, without calculate x_1 and x_2 verify that : $F = \frac{2(1-m)}{m^2+1}$.
4. On an axis x'ox place the points $M_1(x_1)$ and $M_2(x_2)$. Calculate m so that M_1 and M_2 are symmetric with respect to the point I of abscissa $\frac{-3}{2}$.

IV. (3 points)

1. Prove that : $\sin^2 3a.\cos^2 a - \cos^2 3a.\sin^2 a = 2\cos 2a.\sin^2 2a$.
2. Simplify: $E = \frac{\sin^2 3a.\cos^2 a - \cos^2 3a.\sin^2 a}{\cos^3 2a + \cos^2 2a}$.
3. Verify that: $\frac{E}{2\tan a} = \frac{2\tan a}{1-\tan^2 a}$.

V. (6 points)

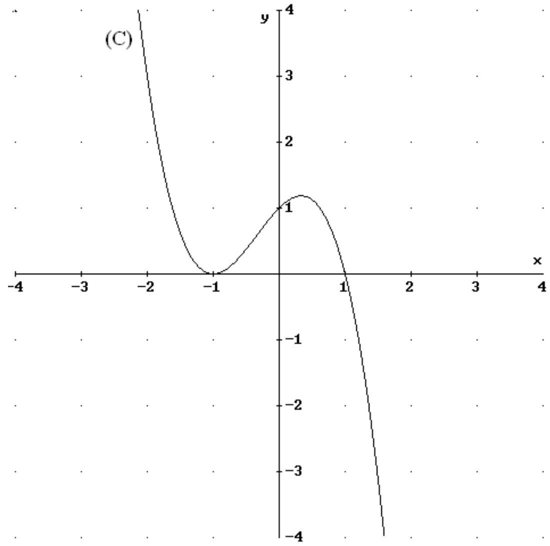
Define the sequence u_n by: $\begin{cases} u_0 = 1 \\ u_{n+1} = \frac{1}{2}u_n + 2n - 1 \end{cases}$

1. Prove that $u_1 = -\frac{1}{2}$, then calculate u_2 and u_3. Is the sequence (u_n) arithmetic or geometric?
2. Let $v_n = u_n - 4n + 10$. Calculate v_0, v_1, v_2, v_3.
3. Prove that the sequence (v_n) is geometric; calculate its common ratio r and first term v_0.
4. Deduce v_n as a function of n.
5. Prove that an expression of u_n as a function of n is given by: $u_n = 11 \times \left(\frac{1}{2}\right)^n + 4n - 10$.
6. $T_n = v_0 + v_1 + v_2 + \cdots + v_n$. Give an expression of T_n as a function of n.

VI. (2.75 points)

Let f be a function defined on IR. Let (G) be the representative curve of f in an orthonormal system $\left(O;\vec{i},\vec{j}\right)$.

$f(0) = -2$, $f(1) = 0$, $f(-1) = -4$, $\lim\limits_{x\to-\infty} f(x) = -\infty$, et $\lim\limits_{x\to+\infty} f(x) = -\infty$. The figure at the right (C) is the curve of the function f', the derivative of f. using (C) :



1. Determine: $\lim\limits_{x\to 0} \dfrac{f(x)-f(0)}{x}$.
2. Write the equation of tangent (T) to (G) at the point of abscissa 0.
3. Draw the table of variation of f.
4. Deduce the sign of f on IR.

VII. (7.75 points)

Let g be a function defined on IR by $g(x) = x^4 - 6x^2 + 8x + 20$, designate by (C) its representative curve in an orthonormal system $\left(O;\vec{i};\vec{j}\right)$.

1. Determine the limits of g at the open boundaries of its domain.
2. Show that $x^3 - 3x + 2 = (x-1)^2(x+2)$
3. Calculate $g'(x)$, then deduce it's sign.
4. Draw the table of variations of g.
5. Prove that the equation $g(x) = 0$, admits on IR two roots α and β, and verify that $-3 < \alpha < -2$ and that $-2 < \beta < -1$.
6. Prove that g admits two points of inflection, to be determined.
7. Trace (C).
8. Let $h(x) = g\left(|x|\right)$.

   Explain how to deduce the graphical representation (C ') of h *from* (C) and draw (C ') in the system $\left(O;\vec{i};\vec{j}\right)$.

GOOD WORK