Subject: Mathematics | **Mark : ------ /**

Grade 11(S) | **Duration: 150 minutes**

I. *(points)*

One, among the following answers, is true. Choose it after you justify your answer.

	Question	A	B	C
1	*a* & *b* are two real numbers such that : $\begin{cases} 3a+3b=-8 \\ ab=-1 \end{cases}$ then *a* & *b* are the roots of the quadratic equation:	$x^2+\frac{8}{3}x+1=0$	$3x^2+8x-3=0$	$x^2-\frac{8}{3}x-1=0$
2	*If* $h(x)=\frac{3}{\sqrt{x^2+1}}$ then *h '(x)*=	$\frac{-3}{(x^2+1)\sqrt{x^2+1}}$	$\frac{-3x}{(x^2+1)\sqrt{x^2+1}}$	$\frac{-6x}{(x^2+1)\sqrt{x^2+1}}$
3	The value of *m* for which $f(x)=mx^3+(m-6)x+7=0$ admits an extremum at $x_0=\frac{1}{3}$ is:	$\frac{7}{6}$	$\frac{1}{2}$	$\frac{9}{2}$

II. *(points)*

Consider the second degree equation $(E): x^2+2mx+m^2+m+5=0$ with *m* is a real parameter.

1. Calculate Δ or Δ' and discuss according to the values of *m* the existence of the roots of Equation (E).
2. Without calculating the roots x_1 & x_2 ,determine *m* so that the roots of the equation (E) Verify the relation $(x_1+1)(x_2+1)=12$

III.

1. **Calculate the following limits:**
   a. $\lim_{x\to 1}\frac{x^2+2x-3}{3x^2-2x-1}=$
   b. $\lim_{\alpha\to 0}\frac{\sqrt{1+\sin\alpha}-1}{\alpha}=$
   c. $\lim_{x\to 0}\frac{\pi(\cos^2 x-\cos x)}{\cos^2 x-3\cos x+2}=$

2. **Given** $f(x)=\begin{cases} \frac{x^2-a^2}{x+2} & \text{if } x\le 0 \\ x+\frac{a}{2}-1 & \text{if } x>0 \end{cases}$

   Calculate *a* so that *f* is continuous at $x_0 = 0$.

IV. (points)

Consider the sequences (U_n) and (V_n) defined by $U_0 = 1$, $U_{n+1} = \frac{U_n - 3}{2}$ and $V_n = U_n + 3$.

1. Calculate V_0 , V_1 and V_2.
2. Prove that (V_n) is a geometric sequence whose common ratio is to be determined.
3. Express V_n and U_n in terms of n.
4. Calculate $S_n = \sum_{i=0}^{n} V_i$ then deduce $S'_n = U_1 + U_2 + ... + U_n$.

V. (points)

Given the circle (C) of equation: $x^2 - 2x + y^2 - 4y - 4 = 0$

1. Determine the center I and the radius R of circle (C).
2. Does line of equation (y=-x) cuts circle (C) ?If yes , determine the coordinates of the points of intersection .

VI. (points)

1. Let x be a real number such that $x \in]\pi; \frac{3\pi}{2}[$ with *sinx* = -0.8.

a. Calculate *cosx* and *sin*2*x.*

b. Find *tan2x.*

2. Given $A \in]\frac{\pi}{2}, \pi[$ and (*tan*A. *tan*B) = -1 and (A + B) = $\frac{5\pi}{6}$.

a. Show that *tan(A+B)*= $-\frac{\sqrt{3}}{3}$

b. Show that *tan* A + *tan* B= $-2\frac{\sqrt{3}}{3}$

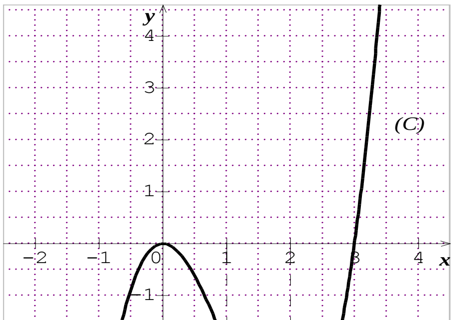
3. Given a **direct triangle ABC**.

a. Show that sin(2A)+sin(2B)+sin(2C)= 4 sin A .sin B . sin C

b. Deduce that : $\frac{\cos A}{\sin B.\sin C} + \frac{\cos B}{\sin A.\sin C} + \frac{\cos C}{\sin A.\sin B} = 2$

VII. (points)

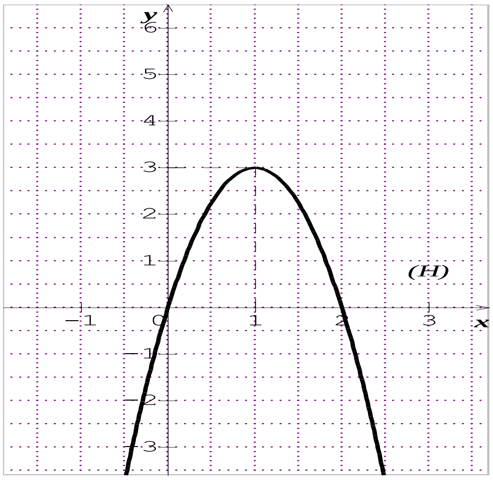
Consider the curve (C) of a function f

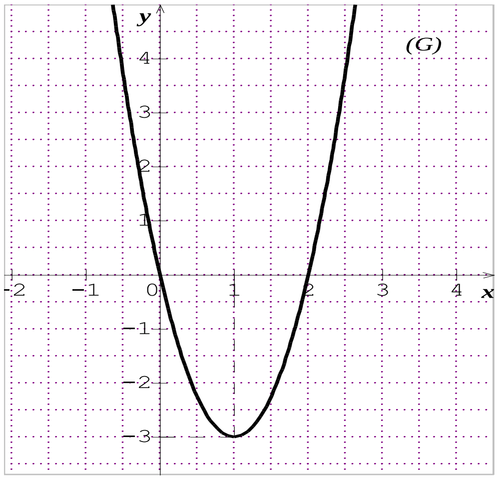


Note: (C) cuts $x'Ox$ at 0 and at 3

(C) admits a horizontal tangent at $x = 0$ and at $x = 2$.

1. Set up the table of variations of f .
2. Solve graphically: $f(x) <0$; $f(x)\geq 0$; $f'(x) =0$.
3. Discuss graphically according to the values of m the number of roots of the equation $f(x)= m$.
4. One of the graphs (H) or (G) below represents the Derivative function f' of f .
   a. Determine the graph of f'. Justify your answer.
   b. Write the equation of the tangent to (C) at $x= 1$.





5. Assume that f is defined by: $f(x) = ax^3+bx^2+cx$.
   Use (C) to calculate a, b and c .

VIII. <u>(points)</u>

Consider the function f defined over IR by $f(x) = \frac{1}{4}x^4 - \frac{3}{2}x^2 + 2x + 2$

and let (C) be its representative curve.

1. Determine the limits of f at $-\infty$ and $+\infty$.
2. Verify that $f'(x) = x^3 - 3x + 2 = (x+2)(x-1)^2$
3. Set up the table of variation of f.
4. Deduce that $f(x) =0$ admits two distinct roots α & β .
5. Show that f admits two inflection points to be determined.
6. Given that $\alpha \in]-3;-2.5[$ & $\beta \in]-1;-0.5[$;
   Draw (C) in an orthonormal system.
7. Deduce (C') the graph of g where $g(x) = |f(x)|$.

GOOD WORK!