The Islamic Institution For Education & Teaching Al-Mahdi Schools		Mathematics Department Feb 2014
Subject : Mathematics	Mid-year Exam	عدد المسائل: 7
Grade11(Sc)		Duration:150 min

ملاحظة: يسمح باستخدام آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات

يستطيع التلميذ الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة)

Exercise I (4 points)

In the following table, only one of the answers to each question is correct.

Write the number of each question and give, **with justification**, the **answers** that correspond to it.

N°	Questions	Answers		
		a	**b**	**c**
1	Let f be a function such that f(2 – x) + 4 = - f (x) (C_f) admits a center of symmetry :	w(1 ;2)	w(1 ;-2)	w(2 ;-2)
2	$\lim\limits_{x \to 1}\left(\frac{\sqrt{x+3}-2}{x^2-1}\right)=$	$\frac{1}{8}$	$-\frac{1}{8}$	$\frac{1}{4}$
3	The equation $2013x^4+x^2-2014=0$ admits	2 solutions	4 solutions	No solution
4	If $f(x) = \cos^2 x - \sin^2 x$ then $f'(x) =$	2sin2x	sin2x	–2sin2x

Exercise II (5 points)

Given $T(x) = x^2 - 2(2m + 1)x + 2m + 3$ where m is a real parameter. x' and x" are the roots of the equation T(x) = 0 when they exist.

1. Show that the discriminant of T(x) = 0 is $\Delta = 16m^2 + 8m - 8$.
2. Discuss according to the values of m, the number of roots of the equation T(x) = 0.
3. Calculate m so that T(x) > 0 for all values of x.
4. ABC is a right triangle at A such that AB = |x'| and AC = |x"|; Calculate m so that BC = $\sqrt{2}$.

Exercise III(2 points)

Let f be the function defined in IR by: $f(x) = \begin{cases} x^2 - a & \text{for } x \le 2 \\ x - \frac{a}{3} & \text{for } x > 2 \end{cases}$

1. Determine the real "a" for f is continuous at x = 2.
2. Take a=3.Study the differentiability of f at the point x=2.Give the graphical interpretation.

Exercise IV (6 points)

Consider the sequence (U_n) defined by : $U_0 = 2$ $U_{n+1} = \frac{1}{2}U_n + 3$.

1. Calculate U_1 and U_2 then verify that (U_n) is neither arithmetic nor geometric.
2. let $V_n = U_n - 6$
   a. Show that (V_n) is a geometric sequence whose common ratio and first term are to be determined.
   b. Calculate Vn and Un in terms of n.
3. Calculate the sum $S_n = V_0 + V_1 + ... + V_n$ in terms of n and deduce $S'_n = U_0 + U_1 + ... + U_n$.

Exercise V (7 points)

1) x and y are 2 acute angles such that : $\tan x = \frac{1}{2}$ and $\tan y = \frac{1}{3}$. Calculate tan(x+ y) and deduce (x + y).

2) Show that $\dfrac{\left(\cos\frac{\pi}{8}+\sin\frac{\pi}{8}\right)^2}{\cos^2\frac{\pi}{8}-\sin^2\frac{\pi}{8}} = \sqrt{2}+1$.

3) Given $\sin x = \frac{1}{4}(\sqrt{5}-1)$ and $0<x<\frac{\pi}{2}$

a) Calculate cos 2x and sin 2x.

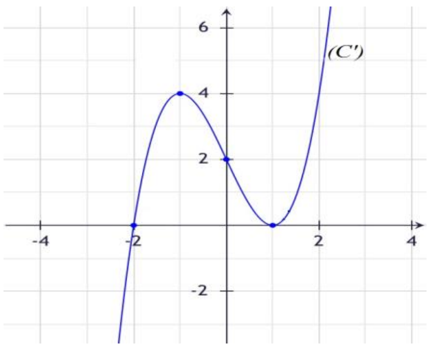
b) Verify that cos 4x = sin x.

Exercise VI(12 points)

Part A :

Let g be the function defined on IR by $g(x) = ax^3 + bx + c$; a, b, and c are non-zero real numbers.

(C') is the representative curve of g. **use the adjacent graph (C').**



1. Show that a= 1 , b = -3 and c = 2.
2. Show that g admits a point of inflexion whose coordinates are to be determined.
3. Construct the table of variation of g.
4. Solve: $g(x) = 0$ and g(x) >0.

Part B :

Consider the function f defined on IR by $f(x) = \frac{x^4}{4} - \frac{3}{2}x^2 + 2x + 2$ let (C) be its representative curve.

1. Calculate the limits at the open boundaries of the domain of definition.
2. Calculate $f'(x)$ show that $f'(x)$ and g(x) have the same sign.
3. Construct the table of variations of f.
4. Show that f has two points of inflexion to be determined.
5. Show that the equation $f(x) = 0$ has two roots α and β such that $-0.8<\alpha<-0.6$ and $-2.9<\beta<-2.7$.
6. Trace (C).
7. Let h(x) = $|f(x)|$ and let (C_h) be its representative curve.

   Trace (C_h) with justification.

Exercise VII (4 points)

(C) and (C') are two circles of centers I and I 'and radii R and R' such that:

(C) : $x^2 + y^2 - 4x - 8y - 5 = 0$ and (C') : $x^2 + y^2 - 8x - 18y - 3 = 0$.

1. Find the coordinates of I and I', and calculate R and R'.
2. Prove that the point A (–2; 1) belongs to (C) and find an equation of the tangent (L) at A to (C).
3. Given (d): y = 2x + 5. Study the intersection between (d) and (C').

Good work