IN HIS NAME

The Islamic Institution For Education & Teaching Al-Mahdi Schools		Math Department February 2015
Subject: Mathematics		**Mark**: 30 points
Class: Grade 11 - Scientific	**Mid-Year Exam**	**Duration**: 150 minutes

I- (6 points)

Remark: The three parts of this question are independent.

1) Determine the derivative function of each of the following functions.

a) $g(x) = \left(\frac{2x}{x-1}\right)^2$.

b) $h(x) = \frac{1}{\sqrt{2x-3}}$.

2) Consider the function f defined, on ℝ, by: $f(x) = \frac{3\cos x - 1}{x^2+2}$.

a) Show that: $\frac{-4}{x^2+2} \le f(x) \le \frac{2}{x^2+2}$.

b) Deduce $\lim\limits_{x\to+\infty} f(x)$.

3) Given the function f defined, on ℝ, by: $f(x) = \begin{cases} \frac{x^2-5x+4}{18x-72} & \text{for } x < 4 \\ b-1 & \text{for } x = 4 \\ \frac{3-\sqrt{x+5}}{4-x} & \text{for } x > 4 \end{cases}$, where b is a real number.

a) Prove that $\lim\limits_{x\to 4^-} f(x) = \lim\limits_{x\to 4^+} f(x) = \frac{1}{6}$.

b) Find the value of b so that f is continuous at x = 4.

II- (2.5 points)

Consider the second degree equation (E): $x^2 - 2(m-1)x + m^2 + 2 = 0$, where m is a real parameter.

1) Determine m so that "–3" is a root of the equation (E).

2) Determine the set of values of m so that the equation (E) admits two distinct real roots x' and x''.

3) Determine the set of values of m if $\frac{m}{x'} + \frac{m}{x''} \ge 0$.

III- (4 points)

Let (U_n) be a sequence defined, for every natural number n, by: $\begin{cases} U_0 = 1 \\ U_{n+1} = U_n + \frac{1}{2^n} \end{cases}$.

1) Calculate U_1 and U_2.

2) Show that the sequence (U_n) is neither arithmetic nor geometric.

3) Calculate $U_{n+1} - U_n$. Deduce that the sequence (U_n) is strictly increasing.

4) Consider the sequence (V_n) defined by $V_n = U_{n+1} - U_n$.

a) Verify that (V_n) is a geometric sequence whose common ratio r and first term V_0 are to be determined.

b) Express V_n in terms of n, then deduce the value of V_{10}.

c) Calculate, in terms of n, the sum $S = V_0 + V_1 + V_2 + \dots\dots + V_n$.

IV- (3 points)

Consider, in an orthonormal system of axes $(O;\ \vec{i},\vec{j})$, the circle (C) of equation: $x^2 + y^2 - 4x - 6y = 0$.

1) Determine the center I and the radius R of the circle (C).
2) Let B(1, –2). Determine the distance BI, then deduce the position of point B with respect to (C).
3) Given the line $(d_m): mx - y - m - 2 = 0$, where m is a real parameter.
   a) Verify that (d_m) passes through B.
   b) For which values of m is (d_m) tangent to (C)?
   c) Deduce the equations of the tangent lines to (C) that pass through B.

V- (5 points)

Remark: The four parts of this question are independent.

1) Calculate, without using the calculator, $\cos\left(\frac{\pi}{12}\right)\sin\left(\frac{5\pi}{12}\right) + \sin\left(\frac{\pi}{12}\right)\cos\left(\frac{5\pi}{12}\right)$.
2) Show that $\tan(2x) - \tan(x) = \frac{\tan(x)}{\cos(2x)}$.
3) Given: $\sin(x) = \frac{\sqrt{5}-1}{4}$, where $0 < x < \frac{\pi}{2}$. Calculate, without using the calculator, cos(2x), then verify that cos(4x) = sin(x)
4) Let ABCD be a direct square of center O. Give a measure of each of the following oriented angles: $(\overrightarrow{AB},\overrightarrow{BC})$, $(\overrightarrow{AD},\overrightarrow{CB})$, and $(\overrightarrow{OA},\overrightarrow{BC})$.

VI- (2 points)

Let f be a function defined by: $f(x) = \frac{3x^2 + ax + b}{x^2 + 1}$, where a and b are two real numbers. Let (C) be the representative curve of f in an orthonormal system $(O;\ \vec{i},\vec{j})$.

A and B are two points such that A(0, 3) and B(–1, 1).

1) Write $f'(x)$ in terms of a and b.
2) Find the values of a and b, knowing that the straight-line (AB) is tangent to (C) at A such that $A \in (C)$.

VII- (7.5 points)

Let f be a function defined, on $\mathbb{R}$, by: $f(x) = x^3 - 6x^2 + 9x + 1$. Let (C) be the representative curve of f in an orthonormal system $(O;\ \vec{i},\vec{j})$.

1) Determine the limits of f(x) at the boundaries of its domain of definition.
2) Set up the table of variations of f.
3) Show that the equation f(x) = 0 has a unique root α. Verify that $-0.2 < \alpha < -0.1$.
4) Write an equation of (T), the tangent to (C) at the point of abscissa 0.
5) Prove that (C) has an inflection point I of abscissa 2. Find the coordinates of I.
6) Draw (T) and (C).
7) Solve, graphically, the inequality f(x) > 0.
8) Let g(x) = f(| x |).
   a) Verify that g is an even function.
   b) Deduce the construction of (G), the representative curve of g, in the same previous system $(O;\ \vec{i},\vec{j})$.

GOOD WORK