IN HIS NAME

The Islamic Institution for Education & Teaching
Al-Mahdi Schools
Class: Grade 11 (Scientific)
Name: ______________

Mathematics Department
Scholastic Year: 2015-2016
Date: / 02 / 2016
Duration: 150 minutes
Mark: 40 points

Mid-Year Exam

I- (4 points)

In the table below, only one of the proposed answers is correct. Write the number of each question and give, with justification, its correct answer.

	Questions	Proposed Answers A	B	C
1)	We suppose that the equation: $x^2-(m^2+1)x-m^2-5=0$ admits, in $\mathbb{R}$, two distinct roots. The roots	are positive	are negative	have opposite signs
2)	The equation $x^6-5x^3+4=0$ admits, in $\mathbb{R}$:	2 roots	4 roots	6 roots
3)	$\lim\limits_{x\to 1}\dfrac{x^2+x-2}{2x^2-3x+1}=$	0	$+\infty$	3
4)	Let f be a function defined by: $f(x)=(m+1)x^2+2mx-3$, where m is a real parameter. $f(1)$ and $f(2)$ have opposite signs when $m\in$	$\mathbb{R}$	$\left]-\frac{1}{8};\frac{2}{3}\right[$	$\left]-\infty;-\frac{1}{8}\right[\cup\left]\frac{2}{3};+\infty\right[$
5)	Let f be a function defined by: $f(x)=\sin^2(x^2)$. Then $f'(x)=$	$2x\sin(2x^2)$	$2\cos(x^2)$	$4x\cos(x^2)$

II- (6 points)

1) Consider the equation: $x^2+2(m+2)x-m=0$, where m is a real parameter.
   a) Determine the set of values of m so that the equation admits two real roots x' and x".
   b) Determine m so that the two roots x' and x" verify the relation: $2x'+2x''-x'x''+8=m^2$.
2) Consider the function f defined by: $f(x)=x-\sqrt{x^2+2(m+2)x-m}$.
   Determine the set of values of m so that f is defined on $\mathbb{R}$.

III- (4 points)

Consider the function f defined, on $\mathbb{R}$, by: $f(x)=\begin{cases}\dfrac{2x+2}{-x+2} & \text{if } x<0\\ x+2-\cos x & \text{if } x\geq 0\end{cases}$

1) Prove, for $x\geq 0$, that : $x+1\leq f(x)\leq x+3$ and deduce $\lim\limits_{x\to+\infty}f(x)$.
2) Prove that f is continuous at the point with abscissa 0.

IV- (5 points)

Consider the sequence (U_n) defined by: $U_0=4$ and $U_{n+1}=-\frac{1}{3}U_n+4$, where $n\in\mathbb{N}$.

1) Calculate U_1 and U_2.
2) Verify that (U_n) is neither arithmetic nor geometric.
3) Consider the sequence (V_n) defined by: $V_n=U_n-3$ for every $n\in\mathbb{N}$.
   a) Show that (V_n) is a geometric sequence. Find its common ratio r and first term V_0.
   b) Express V_n in terms of n. Deduce the value of U_n in terms of n.
   c) Let $S_n=V_0+V_1+\dots+V_n$ and $T_n=U_0+U_1+\dots+U_n$, where $n\in\mathbb{N}$.
   Express S_n in terms of n, then deduce T_n in terms of n.

V- (3 points)

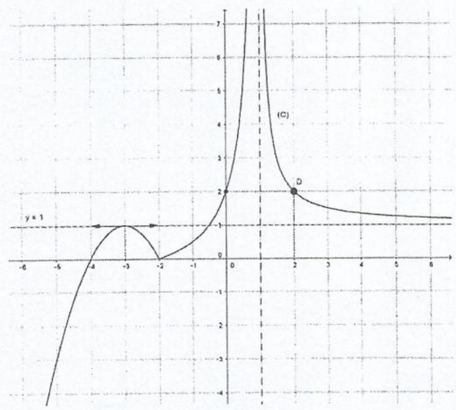
In an orthonormal system $(O; \vec{i}, \vec{j})$, consider the circle (C) with equation: $x^2 + y^2 - 4x + 2y = 0$.

1) Determine the coordinates of the center I of (C) and calculate its radius R.
2) **a)** Verify that the point A(3 ; 1) is a point on (C).
   b) Write an equation of (T), the tangent to (C) at A.
3) Consider the straight line (d) of equation: $y = x + b$, where b is a real number. Determine the values of b so that (d) is tangent to (C).

VI- (4 points)

In the adjacent figure, the curve (C) is the representative curve of a function f in an orthonormal system $(O; \vec{i}, \vec{j})$.

- The tangent to (C) at point E(0 ; 2) cuts x'Ox in (–0.5 ; 0).
- The tangent (T) to (C) at point D (2 ; 2) is parallel to the straight-line of equation $y = -x$.



1) Determine, with justification, $f'(-3)$, $f'(2)$, and $f'(0)$.
2) Study, **graphically**, the continuity and the differentiability of f at the point with abscissa –2.
3) Write an equation of the tangent (T).
4) Let h be a function defined by: $h(x) = \frac{1}{f(x)}$.
   a) Determine the domain of definition h.
   b) Calculate $h'(0)$.

VII- (9 points)

Let f be a function defined, on $\mathbb{R}$, by: $f(x) = ax^3 + bx - 2$, where a and b are two real numbers.
Designate by (C) its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

Part A

Calculate a and b so that the tangent to (C) at point A (2 ; –4) is parallel to straight-line (d) of equation: $y = -9x$.

Part B

Suppose, in this part, that $a = -1$ and $b = 3$ ($f(x) = -x^3 + 3x - 2$).

1) Calculate $\lim\limits_{x \to -\infty} f(x)$ and $\lim\limits_{x \to +\infty} f(x)$.
2) Calculate $f'(x)$ and set up the table of variations of f.
   a) Verify that f could be written in the form $f(x) = (-x - 2)(x - 1)^2$.
   b) Calculate the coordinates of the common points between (C) and the axes of coordinates.
3) Prove that the point I(0 ; –2) is a center of symmetry of (C).
4) Calculate the coordinates of the common points between (C) and the straight-line (L) of equation $y = -x - 2$.
5) Draw (C) and (L).
6) Solve, graphically, the inequality: $f(x) > -x - 2$.

VIII- (5 points)

The three parts of this question are independent.

1) Given: $\cos\left(\frac{\pi}{5}\right) = \frac{1+\sqrt{5}}{4}$. Calculate, without using the calculator, the exact value of $\cos\left(\frac{2\pi}{5}\right)$.
2) **a)** Prove, for every $x \in \left]0; \frac{\pi}{2}\right[$, that $\tan x = \frac{1-\cos(2x)}{\sin(2x)}$.
   b) Deduce the exact value of $\tan\left(\frac{\pi}{12}\right)$.
3) Prove, for every real number x, that: $\sin\left(\frac{\pi}{3} + x\right) \times \sin\left(\frac{\pi}{3} - x\right) = \frac{3}{4} - \sin^2 x$.

Good work