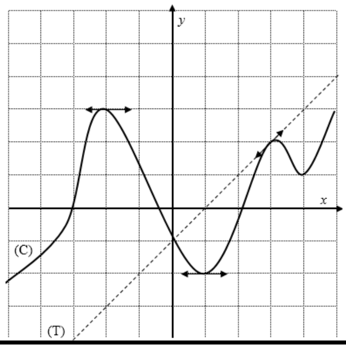
IN HIS NAME

The Islamic Institution for Education & Teaching Al-Mahdi Schools		**Mathematics Department** **Scholastic Year**: 2016-2017 **Date**: / 02 / 2017
Class: Grade 11 (Scientific)		**Duration**: 150 minutes
Name: ____________________	**Mid-Year Exam**	**Mark**: 30 points

I- (3 points)

In the table below, only one of the proposed answers is correct. Write the number of each question and give, **with justification**, its correct answer.

	Questions	Proposed Answers A	B	C
1)	If g is a function such that: $1-\frac{5}{x} \le g(x) \le \frac{4x+1}{4x-3}$ then, $\lim\limits_{x\to+\infty} g(x) =$	1	0	$+\infty$
2)	If $x^4 - 3x^2 - 4 = 0$, then $x^6 - 63 =$	-62	1	-62 or 1
3)	Let (C) be the representative curve of a function f (T) is a tangent to (C) at x = 3. So, $f(-2)+f'(1)+f'(3)=$ 	1	3	4
4)	If f is a function defined by: $f(x)=\begin{cases}\frac{\sin x}{x} \text{ for } x \neq 0\\ 0 \text{ for } x=0\end{cases}$, then f is differentiable on	$\mathbb{R}$	$\mathbb{R}^*$	$\mathbb{R}-\{1\}$

II- (2 points)

Consider the equation of second degree (E): $2x^2 - 5x - k = 0$, where k is a real number.

1) For which value of k does the equation (E) admit one double root in $\mathbb{R}$?
2) Calculate k so that -1 is a root of (E). Deduce the other root.
3) In this part, let $k = \sqrt{3}$ and let x_1 and x_2 be the roots of (E). Without finding x_1 and x_2, form an equation of second degree in y that admits y_1 and y_2 as solutions, where $y_1 = \frac{x_1}{x_2}$ and $y_2 = \frac{x_2}{x_1}$.

III- (4 points)

Consider the sequence (U_n) defined by: $U_1 = 1$ and $U_{n+1} = \frac{3n}{n+1}U_n$, where $n \in \mathbb{N}^*$.

1) Calculate U_2 and U_3.
2) Verify that (U_n) is neither arithmetic nor geometric.
3) Consider the sequence (V_n) defined by: $V_n = nU_n$, for every $n \in \mathbb{N}^*$.
   a) Show that (V_n) is a geometric sequence. Find its common ratio r and first term V_1.
   b) Express V_n in terms of n. Deduce the value of U_n in terms of n.
   c) Let $S_n = V_1 + \ldots + V_n$ where $n \in \mathbb{N}^*$. Express S_n in terms of n.

IV- (4 points)

Remark: The three parts of this question are independent.

1) Given: $a + b = \frac{\pi}{3}$. Prove that: $\frac{\sin a - \sin b}{\cos b - \cos a} = \sqrt{3}$.
2) Knowing that $\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$, write $E = \sin x + \sqrt{3}\cos x$ in the form of $a\sin(bx + c)$, where a, b, and c are three real numbers to be determined.
3) Prove that: $\sin 8x = 8\sin x\cos x\cos 2x\cos 4x$.

V- (4 points)

Remark: The two parts of this question are independent.

1) Find the derivative of the function g defined by: $g(x) = \sqrt{\frac{x^2+1}{x-1}}$.

2) Consider the function f defined, on $\mathbb{R}$, by: $f(x) = \begin{cases} \frac{2-\sqrt{4-x}}{x} & \text{for } x < 0 \\ \frac{x^2-3x+2}{x^2-4} & \text{for } 0 \le x < 2 \\ \frac{ax-3}{x+2} & \text{for } x \ge 2 \end{cases}$, where a is a real number.

a) Is f continuous at x = 0? Justify your answer.

b) Determine a so that f is continuous at x = 2.

VI- (13 points)

The table at right is the table of variations of a function f defined, on $\mathbb{R}$, by: $f(x) = ax^3 + bx^2 + c$, where a , b, and c are three real numbers.

x	$-\infty$		0		1		$+\infty$
$f'(x)$		$+$	0	$-$	0	$+$	
$f(x)$	$-\infty$	$\nearrow$	1	$\searrow$	0	$\nearrow$	$+\infty$

Part A

1) Show that a = 2, b = –3, and c = 1.

2) Calculate $f\left(-\frac{1}{2}\right)$. Deduce the sign of f(x).

3) Write an equation of the tangent (T) to the curve of f at $x = -\frac{1}{2}$.

Part B

Let F be the function defined by $F(x) = \left(f(x)\right)^3$.

1) Find the domain of definition of F and justify your answer.

2) Find $F'(x)$ in terms of f(x) and $f'(x)$.

3) Study the variations of F.

Part C

Let g be the function defined, on $\mathbb{R}$, by: $g(x) = -2x^4 + 4x^3 - 4x$.

Designate by (C) the representative curve of g in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) Calculate $\lim_{x\to-\infty} g(x)$ and $\lim_{x\to+\infty} g(x)$.

2) Verify that $g'(x) = -4f(x)$.

3) Set up the table of variations of g.

4) Show, without using the calculator, that the equation g(x) = 0 admits two roots. Verify that one of the roots is α such that: $-0.9 < \alpha < -0.8$.

5) Prove that g admits two points of inflection whose coordinates are to be determined.

6) Draw (C).

7) Let h be the function defined by: $h(x) = -2x^4 + 4|x|^3 - 4|x|$

a) Prove that h is an even function.

b) Deduce the construction of (H), the curve of h, then draw (H) in the previous system $(O; \vec{i}, \vec{j})$.

GOOD WORK