IN HIS NAME

The Islamic Institution for Education & Teaching Al-Mahdi Schools		**Mathematics Department** **Scholastic Year**: 2017-2018 **Date**: / 02 / 2018
Class: Grade 11 (Scientific)		**Duration**: 150 minutes
Name: ____________________	**Mid-Year Exam**	**Mark**: 30 points

ملاحظة: يسمح باستخدام آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
يستطيع التلميذ الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (4 points)

In the table below, only one of the proposed answers is correct. Write the number of each question and give, **with justification**, its correct answer.

Nº	Questions	Proposed Answers		
		A	**B**	**C**
1)	$x^2 - 4x + 4 > 0$ for $x \in$	$\mathbb{R}$	$]0, +\infty[$	$]-\infty, 2[\cup]2, +\infty[$
2)	$\lim\limits_{x \to -3} \dfrac{x^3 + 6x^2 + 9x}{(x+3)(x-2)} =$	-3	0	$+\infty$
3)	If f is a function defined by $f(x) = \dfrac{x^2 - 2x + 3}{x - 1}$, then $f'(3) =$	0.5	1	3
4)	$\lim\limits_{x \to 0} \dfrac{\sin 2x}{x} =$	1	2	0

II- (5 points)

Remark: The three parts of this question are independent.

1) Solve, in $\mathbb{R}$, the system: $\begin{cases} x^2 + 4x + 3 > 0 \\ x^2 + x - 2 < 0 \end{cases}$.

2)

a) Solve, in $\mathbb{R}$, the equation $8x^4 + 2x^2 - 1 = 0$.

b) Deduce the solution(s) of the equation $8\left(\dfrac{1}{1+y}\right)^2 + \dfrac{2}{1+y} - 1 = 0$.

3) Let $k(x) = ax^2 + bx + c$, where a and c are two strictly positive real numbers and b is a strictly negative real number. Suppose that $k(x)$ has two real roots x_1 and x_2.

a) Let $x_1 = 2$. Find x_2 in terms of a and b.

b) Study the sign of $\dfrac{1}{x_1} + \dfrac{1}{x_2}$.

III- (5 points)

Let (U_n) be the sequence defined by $U_0 = 3$ and $U_{n+1} = \frac{1}{3}U_n - 4$, for every $n \in \mathbb{N}$.

1) Calculate U_1 and U_2.

2) Deduce that the sequence (U_n) is neither arithmetic nor geometric.

3) Let (V_n) be the sequence defined by $V_n = U_n + 6$, for every $n \in \mathbb{N}$.

a) Prove that the sequence (V_n) is a geometric sequence whose common ratio and first term are to be determined.

b) Find V_n in terms of n, then deduce that $U_n = \dfrac{1}{3^{n-2}} - 6$.

4) Prove that the sequence (U_n) is decreasing.

5) Calculate the sum $S = V_0 + V_1 + \cdots + V_n$ in terms of n.

IV- (5 points)

Remark: The three parts of this question are independent.

1) Let f be a function defined, on ℝ, by (x) = $\begin{cases} \frac{x^2-1}{2x-2} & \text{if } x < 0 \\ \frac{x^2+x-1}{-2} & \text{if } x \geq 0 \end{cases}$.

   Study the differentiability of f at $x = 0$.

2) Given $0 < x < \frac{\pi}{2}$ and $-\frac{\pi}{2} < y < 0$ such that $\tan x = 2$ and $\tan y = \frac{8-5\sqrt{3}}{11}$.

   a) Knowing that $\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$, verify that $\tan(x+y) = \sqrt{3}$.

   b) Deduce the value of x + y.

3) Let $a \in \mathbb{R}$ such that $\sin a + \cos a = \frac{7}{5}$.

   a) Show that $\sin a \times \cos a = \frac{12}{25}$.

   b) Calculate sina and cosa, knowing that sina > cosa.

V- (8 points)

Consider the function f defined, on ℝ, by: $f(x) = -2x^3 - 3x^2 + 4$.

Designate by (C) the representative curve of f in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) Calculate $\lim\limits_{x\to-\infty} f(x)$ and $\lim\limits_{x\to+\infty} f(x)$.
2) Calculate $f'(x)$ and set up the table of variations of f.
3) Show that the equation f(x) = 0 has a unique root α and show that $\alpha \in]0.5\ ;\ 1[$.
4)
   a) Show that I(–0.5 ; 3.5) is a center of symmetry of (C).

   b) Write an equation of the line (T), the tangent to (C) at I.
5) Draw (C) and (T).
6) Study graphically, in terms of α, the sign of f(x).
7) Find, graphically, the number of roots of the equation $-2x^3 - 3x^2 + 0.999 = 0$. Justify your answer.
8) Consider the function h defined, on ℝ, by: $h(x) = [f(x)]^2$.

   a) Determine the limits of h at $-\infty$ and at $+\infty$.

   b) Determine h'(x), then complete the following table of variations.

x	$-\infty$		-1		0		α		$+\infty$
h'(x)							0	+	
h(x)		↘	9						

VI- (3 points)

Let ABC be a right isosceles triangle such that AB = AC = 1 cm.

(Δ) is a line perpendicular to plane (ABC) at A and S is a point on (Δ) such that SA = AB.

1) Show that the line (AB) is perpendicular to plane (SAC).
2) Let H be the midpoint of segment [SC].

   a) Show that the plane (ABH) is the mediator plane of segment [SC].

   b) Prove that the two planes (SBC) and (ABH) are perpendicular.