The Islamic Institution for Education & Teaching Al-Mahdi Schools Class: Grade 11 (Scientific)



Mathematics Department Scholastic Year: 2017-2018 Date: / 02 / 2018 Duration: 150 minutes Mark: 30 points

Name:

Mid-Year Exam

ملاحظة: يسمح باستخدام آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات. يستطيع التلميذ الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (4 points)

In the table below, only one of the proposed answers is correct. Write the number of each question and give, **with justification**, its correct answer.

Nº	Questions	Proposed Answers		
		Α	В	С
1)	$x^2 - 4x + 4 > 0$ for $x \in$	$\mathbb R$]0, +∞[]−∞, 2[∪]2, +∞[
2)	$\lim_{x \to -3} \frac{x^3 + 6x^2 + 9x}{(x+3)(x-2)} =$	-3	0	+∞
3)	If f is a function defined by $f(x) = \frac{x^2 - 2x + 3}{x - 1}$, then f '(3) =	0.5	1	3
4)	$\lim_{x \to 0} \frac{\sin 2x}{x} =$	1	2	0

II- (5 points)

Remark: The three parts of this question are independent.

- 1) Solve, in \mathbb{R} , the system: $\begin{cases} x^2 + 4x + 3 > 0 \\ x^2 + x 2 < 0 \end{cases}$
- 2)

a) Solve, in \mathbb{R} , the equation $8x^4 + 2x^2 - 1 = 0$.

b) Deduce the solution(s) of the equation $8\left(\frac{1}{1+y}\right)^2 + \frac{2}{1+y} - 1 = 0.$

- 3) Let $k(x) = ax^2 + bx + c$, where a and c are two strictly positive real numbers and b is a strictly negative real number. Suppose that k(x) has two real roots x_1 and x_2 .
 - **a**) Let $x_1 = 2$. Find x_2 in terms of a and b.

b) Study the sign of
$$\frac{1}{x_1} + \frac{1}{x_2}$$
.

III- (5 points)

Let (U_n) be the sequence defined by $U_0 = 3$ and $U_{n+1} = \frac{1}{3}U_n - 4$, for every $n \in \mathbb{N}$.

- 1) Calculate U_1 and U_2 .
- 2) Deduce that the sequence (U_n) is neither arithmetic nor geometric.
- 3) Let (V_n) be the sequence defined by $V_n = U_n + 6$, for every $n \in \mathbb{N}$.
 - a) Prove that the sequence (V_n) is a geometric sequence whose common ratio and first term are to be determined.

b) Find V_n in terms of n, then deduce that $U_n = \frac{1}{3^{n-2}} - 6$.

- 4) Prove that the sequence (U_n) is decreasing.
- 5) Calculate the sum $S = V_0 + V_1 + \dots + V_n$ in terms of n.

1) Let f be a function defined, on \mathbb{R} , by (x) = $\begin{cases} \frac{x^2 - 1}{2x - 2} & \text{if } x < 0\\ \frac{x^2 + x - 1}{-2} & \text{if } x \ge 0 \end{cases}$ Study the differentiability of f at x = 0.

2) Given $0 < x < \frac{\pi}{2}$ and $-\frac{\pi}{2} < y < 0$ such that $\tan x = 2$ and $\tan y = \frac{8-5\sqrt{3}}{11}$.

- **a**) Knowing that $\tan(x + y) = \frac{\tan x + \tan y}{1 \tan x \tan y}$, verify that $\tan(x + y) = \sqrt{3}$.
- **b)** Deduce the value of x + y.
- 3) Let $a \in \mathbb{R}$ such that $\sin a + \cos a = \frac{7}{5}$.

a) Show that sina
$$\times \cos a = \frac{12}{2}$$

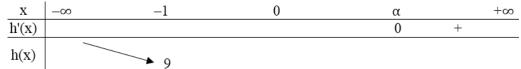
b) Calculate sina and cosa, knowing that sina > cosa.

V- (8 points)

Consider the function f defined, on \mathbb{R} , by: $f(x) = -2x^3 - 3x^2 + 4$.

Designate by (C) the representative curve of f in an orthonormal system (0; \vec{i}, \vec{j}).

- 1) Calculate $\lim_{x \to \infty} f(x)$ and $\lim_{x \to \infty} f(x)$.
- 2) Calculate f'(x) and set up the table of variations of f.
- 3) Show that the equation f(x) = 0 has a unique root α and show that $\alpha \in [0.5; 1[$.
- **4**)
- a) Show that I(-0.5; 3.5) is a center of symmetry of (C).
- **b**) Write an equation of the line (T), the tangent to (C) at I.
- **5**) Draw (C) and (T).
- 6) Study graphically, in terms of α , the sign of f(x).
- 7) Find, graphically, the number of roots of the equation $-2x^3 3x^2 + 0.999 = 0$. Justify your answer.
- 8) Consider the function h defined, on \mathbb{R} , by: $h(x) = [f(x)]^2$.
 - a) Determine the limits of h at $-\infty$ and at $+\infty$.
 - **b**) Determine h'(x), then complete the following table of variations.



VI- (3 points)

Let ABC be a right isosceles triangle such that AB = AC = 1 cm.

- (Δ) is a line perpendicular to plane (ABC) at A
- and S is a point on (Δ) such that SA = AB.
 - 1) Show that the line (AB) is perpendicular to plane (SAC).
 - 2) Let H be the midpoint of segment [SC].
 - a) Show that the plane (ABH) is the mediator plane of segment [SC].
 - **b**) Prove that the two planes (SBC) and (ABH) are perpendicular.

