IN HIS NAME

The Islamic Institution for Education & Teaching Al-Mahdi Schools		**Mathematics Department** **Scholastic Year**: 2019-2020 **Date**: January 2020
Class: Grade 11 (Scientific)		**Duration**: 150 minutes
Name: ___________________	**Mid-Year Exam**	**Mark**: 30 points

ملاحظة: يسمح باستخدام آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.

يستطيع التلميذ الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (4 points)

In the table below, only one of the proposed answers is correct. Write the number of each question and give, **with justification**, its correct answer.

Nº	Questions	Proposed Answers		
		A	B	C
1)	If f is a function defined by $\frac{x^2-3x+2}{4x^2-x+7} < f(x) < -\frac{6-3x^2}{12x^2-5}$, then $\lim\limits_{x\to+\infty} f(x)$	equal to 4	doesn't exist	equal to $\frac{1}{4}$
2)	Consider the sequence (U_n) defined by: $U_0 = -1$ and $U_{n+1} = \frac{nU_n+4}{n+1}$ for all $n \in \mathbb{N}$. The sequence (V_n) defined by $V_n = n.U_n$ is an arithmetic sequence of common difference d =	-4	1	4
3)	$\lim\limits_{x\to 2} \frac{\sqrt{2x^2+1}-3}{x^2-2x} =$	$\frac{3}{2}$	$\frac{2}{3}$	$\frac{4}{9}$
4)	$\frac{2x^2-3x+1}{-x+3} > 0$ for $x \in$	$]\frac{1}{2};1[\cup]3;+\infty[$	$]-\infty;\frac{1}{2}[\cup]1;3[$	$]-\infty;-3[\cup]\frac{1}{2};1[$

II- (3 points)

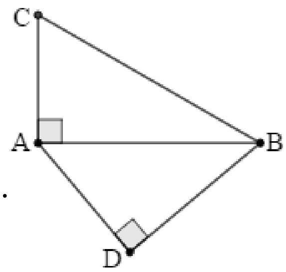
Consider the equation (E): $x^2 - 9x + 20 = 0$. Let x_1 and x_2 be the roots of (E).
Answer the following questions **without calculating x_1 and x_2.**

1) Evaluate $(x_1 - 3)(x_2 - 3)$. Deduce that the value of $\frac{1}{x_1-3} + \frac{1}{x_2-3}$ is $\frac{3}{2}$.
2) Write a second degree equation in z, such that its roots are $z_1 = \frac{1}{x_1-3}$ and $z_2 = \frac{1}{x_2-3}$.
3) ABC is a right triangle at A such that $AB = \frac{x_1^2+1}{x_1-3}$ and $AC = \frac{x_2^2+1}{x_2-3}$.
   Calculate the area of triangle ABC.

III- (4 points)

Remark: The three parts of this question are independent.

1) Given that $\cos x = -\frac{1}{3}$ and $\sin y = -\frac{\sqrt{5}}{3}$ such that $x \in \left[\frac{\pi}{2}, \pi\right]$ and $y \in \left]-\pi, -\frac{\pi}{2}\right[$.
   Prove that $\cos 2x = -\frac{7}{9}$ and $\cos 2y = -\frac{1}{9}$.
2) Given that $\frac{\pi}{2} < a < \pi$ and $-\frac{\pi}{2} < b < 0$ such that $\tan a = -\frac{4}{3}$ and $\tan b = -7$.
   Verify that $\tan(a + b) = 1$ then deduce the value of $a + b$.
3) ABC and ADB are two direct right triangles at A and D respectively such that $(\overrightarrow{BC}; \overrightarrow{BA}) = \frac{\pi}{6} (2\pi)$ and $(\overrightarrow{BA}; \overrightarrow{BD}) = 2\frac{\pi}{9} (2\pi)$.
   Determine the measure of the angles $(\overrightarrow{AB}; \overrightarrow{BC})$, $(\overrightarrow{BA}; \overrightarrow{CA})$, and $(\overrightarrow{AD}; \overrightarrow{BC})$.



IV- (3 points)

Remark: The two parts of this question are independent.

1) Let f be a function defined by $f(x) = \begin{cases} \frac{1}{2}x^2 - 2 & \text{if } x < 4 \\ ax + b & \text{if } x \geq 4 \end{cases}$ where a and b are two real numbers.

Find **a** and **b** such that f is continuous and differentiable at $\boldsymbol{x = 4}$.

2) Find the derivative $f'(x)$ in each of the following cases.

a) $f(x) = \left(\frac{x-1}{3x+2}\right)^4$ **b)** $f(x) = \sqrt{-x + \sin(3x)}$.

V- (4 points)

Consider the sequence (U_n) defined by: $U_0 = -1$ and $U_{n+1} = 1 - 2U_n$ for all $n \in \mathbb{N}$.

1) Calculate U_1 and U_2. Deduce that the sequence (U_n) is neither arithmetic nor geometric.

2) Consider the sequence (V_n) defined by $V_n = -3U_n + 1$ for all $n \in \mathbb{N}$.

a) Show that (V_n) is a geometric sequence whose common ratio **r** and its first term $\mathbf{V_0}$ are to be determined

b) Express V_n in terms of n.

c) Verify that $U_n = -\frac{1}{3}V_n + \frac{1}{3}$ then deduce U_n in terms of n.

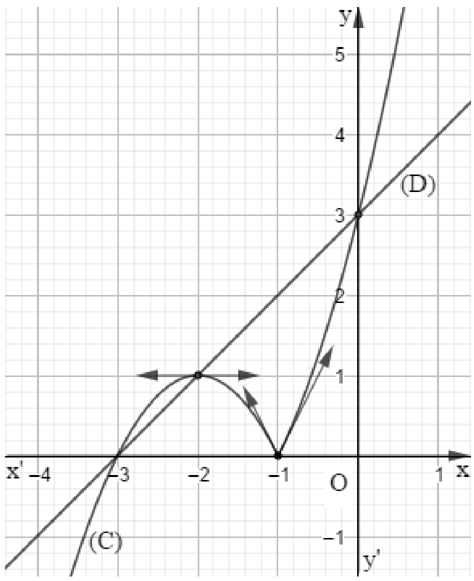
d) Let $S = V_0 + \ldots + V_n$ and $S' = U_0 + \ldots + U_n$.

Express S in terms of n then deduce that $S' = -\frac{4}{9}(1-(-2)^{n+1}) + \frac{1}{3}(n+1)$.

VI- (5 points)

In the adjacent figure, we have:

- **(C)** is the representative curve of a function f defined on $\mathbb{R}$.
- **(D)** is the straight line of equation $y = x + 3$.



1) Determine the values of $f(0), f(-1)$ and $f'(-2)$.

2) Determine the limits of f at $-\infty$ and $+\infty$.

3) Is f differentiable at -1? Justify.

4) Solve graphically the following inequalities:

a) $f(x) < 0$. **b)** $f(x) - x - 3 > 0$.

5) Reproduce then complete the following table of variations of f:

x	$-\infty$	-2	-1	$+\infty$
$f'(x)$		0	▨	
$f(x)$			0	

VII- (7 points)

Consider the function g defined on $\mathbb{R}$, by $g(x) = x^4 - 2x^3 + 2x$.

Let (G) be the representative curve of g in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) Calculate $\lim\limits_{x \to -\infty} g(x)$ and $\lim\limits_{x \to +\infty} g(x)$.

2) Verify that $g'(x) = (x-1)^2(4x+2)$ and deduce that g is strictly increasing over $]-0.5\,; +\infty[$.

3) Set up the table of variations of g.

4) **a)** Show that $g(x) = 0$, has over $\mathbb{R}$, only two roots, one of them is 0 and the other is a real number α.

b) Verify that $\alpha \in\,]-0.84\,; -0.83[$.

5) Prove that (G) has two points of inflection whose coordinates are to be determined.

6) Write the equation of the tangent (t) to (G) at its point of abscissa 1.

7) Draw (t) and (G).