## Midyear common exam (07-08)

Duration: 150 minutes

The plane, when needed, is referred to an orthonormal system $(\mathrm{O} ; \vec{i} ; \vec{j})$

## I- (4 points)

In the table below, only one of the proposed answers to each question is correct. Write down the number of each question and give, with justification, the answer corresponding to it.

| № | Questions | Answers |  |  |
| :---: | :--- | :---: | :---: | :---: |
|  |  | $\mathbf{B}$ | $\mathbf{C}$ |  |
| $1^{\circ}$ | If $\overrightarrow{\mathrm{AB}}=\overrightarrow{C A}$, then | ABC is an <br> isosceles <br> triangle | $\mathrm{A}, \mathrm{B}$, and C <br> are collinear | $(\mathrm{AB})$ and <br> $(\mathrm{BC})$ are <br> parallel |
| $2^{\circ}$ | The direction vector $\vec{R}$ of <br> (d): $3 \mathrm{x}-\mathrm{y}+5=0$ is | $\vec{R}(3,-1)$ | $\vec{R}(1,-3)$ | $\vec{R}(1,3)$ |
| $3^{\circ}$ | Given: $(\mathrm{D}):\left\{\begin{array}{l}x=2 t-1 \\ y=3-5 t \\ \text { The director coefficient of (D) } \\ \text { is: }\end{array}\right.$ <br> $4^{\circ}$ <br> $\cos ^{2} 180^{\circ}+\sin ^{2} 179^{\circ}$ is$\quad \frac{2}{3}$ | $\frac{-5}{2}$ | $\frac{-1}{3}$ |  |

## II- (3.5 points)

Given: $\mathrm{E}=\{\mathrm{x} \in \mathrm{IN} * / \mathrm{x}<10\}, \mathrm{A}=\{1,2,3,4\}, \mathrm{B}=\{2,3,5,7\}$ and $\overline{\mathrm{C}}=\{2,5,6,9\}$, where C is a subset of $E$.

1) Write $B$ in comprehension and $C$ in extension.
2) Determine, in extension, $A \cap C$ and $\overline{B \cup C}$.
3) How many 3-different-digit numbers can we form using the elements of $E$ ?

## III- (3 points)

Given the two straight-lines (D) and (D'):
(D) : $2 \mathrm{x}-\mathrm{y}-4=0 \quad$ and $\quad$ (D') : $\left\{\begin{array}{l}x=1-t \\ y=3 t+1\end{array}\right.$

1) Prove that (D) and ( $\mathrm{D}^{\prime}$ ) are concurrent; find the coordinates of their point of intersection.
2) Find a cartesian equation of the straight-line that passes through $A(1 ;-2)$ and parallel to ( $\mathrm{D}^{\prime}$ ).

## IV- (4.5 points)

1) Simplify: $A=\cos (3 \pi+x)+\cos \left(\frac{5 \pi}{2}-x\right)+\sin \left(\frac{3 \pi}{2}-x\right)+\sin (-4 \pi-x)$.
2) Show that: $\frac{\sin ^{2} x+\sin x \cos x}{\sin ^{2} x-\cos ^{2} x}=\frac{\tan x}{\tan x-1}$
3) Prove that: $\sin ^{2} 22^{\circ}+\cos ^{2} 20^{\circ}+\sin ^{2} 68^{\circ}+\cos ^{2} 70^{\circ}=2$

## V-(5 points)

1) Solve in IR: $\left\{\frac{x+1}{3}-\frac{x-1}{4}>\frac{x}{2}-\frac{2}{3}\right.$

$$
\left\{\frac{(x-2)^{2}-(2 x-1)^{2}}{x-2} \leq 0\right.
$$

2) Solve, graphically: $\left\{\begin{array}{l}x-y-1<0 \\ 2 x+y+2 \leq 0\end{array}\right.$

## VI- (5 points)

Consider the points $\mathrm{A}(-3 ; 2), \mathrm{B}(-1 ; 1)$, and $\mathrm{C}(5 ; 2)$.

1) Express $\overrightarrow{\mathrm{AB}}$ in terms of $\vec{i}$ and $\vec{j}$.
2) Let $D$ be the point in the plane such that $A B C D$ is a parallelogram of center $I$. Calculate the coordinates of D and I .
3) Let $\vec{U}(9 ;-0.5)$ and $\vec{V}=\overrightarrow{\mathrm{AB}}+2 \overrightarrow{\mathrm{AC}}$ be two vectors in the plane.
a) Calculate the coordinates of $\vec{V}$.
b) Show that $\vec{U}$ and $\vec{V}$ have the same direction.

## VII- (5 points)

Let $\mathrm{A}, \mathrm{B}, \mathrm{C}$, and D be four given points in the plane and let E be the point defined by:
$\overrightarrow{\mathrm{EA}}+\overrightarrow{\mathrm{EB}}+\overrightarrow{\mathrm{EC}}+\overrightarrow{\mathrm{ED}}=\overrightarrow{0}$

1) Prove that, for every point O in the plane, $\overrightarrow{\mathrm{OA}}+\overrightarrow{\mathrm{OB}}+\overrightarrow{\mathrm{OC}}+\overrightarrow{\mathrm{OD}}=4 \overrightarrow{\mathrm{OE}}$
2) Let $G$ be the point defined by: $3 \overrightarrow{O G}=\overrightarrow{\mathrm{OB}}+\overrightarrow{\mathrm{OC}}+\overrightarrow{\mathrm{OD}}$. Prove that $G$ is the center of gravity of triangle BCD.
3) Show that A, E, and G are collinear and precise the position of E.

## Good Work

