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The Islamic Institution For Education & Teaching
Al-Mahdi Schools

Mid Year Exam

Math Department
February 2010

Subject: Mathematics

Grade 11 S

Duration: 150 minutes

I. (1.5 pts)

Each of the following questions has exactly one correct answer. Write down the number of the question then indicate, with justification, the correct answer.

Questions	Answers		
	A	B	C
1) If $x \in [-1;0[$ then	$x^3 \le x \le x^2$	$x \le x^3 \le x^2$	$x \le x^2 \le x^3$
2) If $y = x - \frac{\pi}{2}$ then	$\sin x = -\cos y$	$\sin x = \sin y$	$\sin^2 x + \sin^2 y = 1$
3) Given $\vec{u}(5;-4)$ and $\vec{v}(-4;5)$ then $\vec{u} - \vec{v} =$	$-\vec{i} - \vec{j}$	$9(\vec{i} - \vec{j})$	$-9\vec{i} + 9\vec{j}$
4) $2x - 1 = m$ has a solution in Z if	m is even	m is odd	$m \in N$

II. (3.5 pts)

1. Solve the following system of inequations and represent the solution using axis:

$$\begin{cases} \dfrac{(x-1)^2 - 16}{(-x+1)(x^2+2)} > 0 \\ \dfrac{x+1}{6} - \dfrac{x-1}{4} \ge \dfrac{x}{2} - \dfrac{1}{6} \end{cases}$$

2. Given the two strictly positive real numbers x and y.

   Show that $\frac{x+y}{2} \ge \sqrt{xy}$.

III. (1 pts)

Given the two intervals $A =]-\infty;-2]$ and $B = [-3;3]$

1) Find $A \cap B$ and $A \cup B$.

2) Write in interval form: $\overline{A}$ and $\overline{B}$, then deduce $\overline{A} \cap \overline{B}$.

IV. (3 pts)

(The 2 parts of this question are independent)

1) Given: $\cos x = \frac{-\sqrt{5}}{3}$ and $\frac{\pi}{2} < x < \pi$.

   a) Show that $3\sin x + \sqrt{5}\tan x = 0$.

   b) Find the value of: $\sin\left(\frac{11\pi}{2} - x\right) + \cos(-9\pi + x) + \tan\left(\frac{3\pi}{2} - x\right)$.

2) Show that: $(1 + \cos^2 x)\tan^2 x - \frac{1}{\cos^2 x} = -\cos^2 x$.

V. (4 pts)

Given the 2 lines $(D_1): 3x + 2y - 1 = 0$ and $(D_2): \begin{cases} x = -k + 1 \\ y = -2k + 3 \end{cases}$ and the two points $A(1;2)$ and $B(2m-1;4)$.

1) Give a direction vector $\vec{R_1}$ and $\vec{R_2}$ of (D_1) and (D_2) respectively.
   a) Calculate m if $(AB)//(D_2)$
2) Write (D_2) in its cartesian form.
3) Prove that (D_1) and (D_2) are intersecting then find the coordinates of their point of intersection I.
4) Plot (D_1) & (D_2) in an orthonormal system $(O, \vec{i}, \vec{j})$
5) The line parallel to *y*-axis and passing through the point *M* (1, 0) cuts (D_1) & (D_2) in *E* and *F* respectively. Determine a system of inequations having its solution represented by the region inside the triangle *IEF*.

VI. (1.5 pts)

In a system $(O, \vec{i}, \vec{j})$ given the points *A*(-3;0);*B*(5;4); *C*(10;0) and *D*(2;-4)

1) Show that *ABCD* is a parallelogram.
2) Let *N*(*x* ; *y*) be a point such that : $xy - 2x - 2y + 1 = 0$
   Suppose that *X* and *Y* are the coordinates of *N* in the new system which is the image of $(O; \vec{i}, \vec{j})$ by the translation of the vector $\overrightarrow{OO'} = 2\vec{i} + 2\vec{j}$ .Show that $XY = 3$.

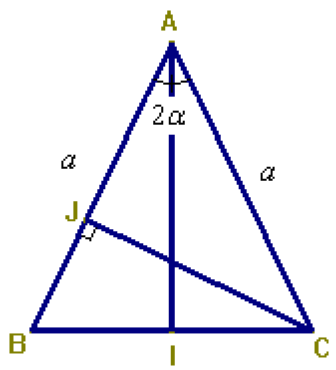
VII. (3.5 pts)

Given a triangle ABC and let D be a point defined by $\overrightarrow{DA} + 3\overrightarrow{DB} + 2\overrightarrow{DC} = \vec{0}$.

1) Show that $\overrightarrow{BD} = \frac{1}{6}\left(\overrightarrow{BA} + 2\overrightarrow{BC}\right)$.
2) Let *M* be a point in the plane of this triangle , suppose that:
   $\vec{v} = \overrightarrow{MA} - 3\overrightarrow{MB} + 2\overrightarrow{MC}$. Show that $\vec{v} = \overrightarrow{BA} + 2\overrightarrow{BC}$.
3) Show that the points *B*, *D* and *E* are collinear, where *E* is defined by $\overrightarrow{BE} = \vec{v}$.
4) Draw the figure and plot the points *E* and *D*.
5) Let *F* be a point defined by $\overrightarrow{CF} = \frac{1}{3}\overrightarrow{CA}$.Show that *D* is the midpoint of [*BF*].

VIII. (2 pts)

Consider the isosceles triangle *ABC* such that $AB = AC = a$ and $B\hat{A}C = 2\alpha$, where $\alpha \in \left]0, \frac{\pi}{4}\right[$ .Let [*AI*] be the median segment relative to [*BC*] and let [CJ] be the height issued from *C*.



1) Calculate *BI* in terms of *a* and $\sin\alpha$.
2) Deduce that $BC = 2a\sin\alpha$.
3) Let *S* be the area of triangle *ABC* .Show that:
   a) $S = a^2 \sin\alpha\cos\alpha$
   b) $S = \frac{1}{2}a^2 \sin 2\alpha$.
4) Deduce that $\sin 2\alpha = 2\sin\alpha\cos\alpha$.

GOOD WORK