In his name

Mid Year Exam

The Islamic Institution
For Education & Teaching
Al-Mahdi Schools

Math Department
February 2013

Subject: Mathematics

Grade 10

Duration: 150 minutes

I- <u>(6 points)</u>

In the table below, only one of the proposed answers to each question is correct. Write down the letter corresponding to the proper response , **<u>with justification</u>**.

		Answers		
N°	**Questions**	**a**	**b**	**C**
1	**If** $x > 2$ **then** $\frac{\lvert x-2 \rvert}{x-2} =$	$\frac{x+2}{x-2}$	**1**	**–1**
2	$[-3;4[\cap]-1;6]=$	$[-3;6]$	$[-1;4]$	$]-1;4[$
3	$\theta = -\frac{121\pi}{7}$**. The principal measure of** θ **is**	$\frac{5\pi}{7}$ **rad**	$-\frac{2\pi}{7}$ **rad**	$-\frac{\pi}{7}$ **rad**
4	If $\lvert -2x + 3 \rvert > 5$, *then*	$x < -1$	$-1 < x < 4$	$x < -1$ **or** $x > 4$
5	$\frac{\sqrt[3]{4\sqrt{2}}}{\sqrt{2}} =$	$\sqrt{2}$	$\sqrt[3]{2}$	$\sqrt[3]{4}$
6	How many 4- digit numbers can be formed from the digits 0, 1, 2 and 3 (no digit may be used twice and 0 may not be used as a 1st digit.)	**18**	**24**	**27**

II- (5 points)

Remark: The four parts of this problem are independent.

1) Solve: $\lvert 2x - 3 \rvert = \lvert x - 2 \rvert$.
2) Find x so that: $\lvert x \rvert + 5 > 2\lvert x \rvert + 3$.
3) Frame $E = 2x+3$, knowing that $-1 \le x \le 0$.
4) Simplify :

a) $\frac{5\sqrt{2} + 2\sqrt[4]{4} - 4\sqrt[8]{16}}{\sqrt[6]{81} \times \sqrt[3]{3}}$ b) $\frac{(ab^2)^2 \times (ab^{-1})^3 \times (a^2b)^{-1}}{(a^2b^3)^{-2} \times a^3 b^5}$

III- (3 points)

Consider the three sets E, A, and B defined by:

$E = \{0, 1, 2, 3, 4, 5, 6, 7\}$, $A = \{1, 3, 7\}$ $B = \{2, 4, 6\}$, where A and B are two subsets of E.

1) Write in extension sets $\overline{A}$, $\overline{\overline{A} \cup B}$, and $A \cap \overline{B}$.
2) Compare the two sets $\overline{\overline{A} \cup B}$ and $A \cap \overline{B}$.

IV- (4 points)

1) Given below the tables of signs of A(x) and B(x) respectively:

x	$-\infty$	2	$+\infty$
A(x)	+	0	-

x	$-\infty$	1	$+\infty$
B(x)	-	0	+

Let **Q(x) = A(x) × B(x)** and **R(x)** = $\frac{B(x)}{A(x)}$

a) Study the signs of Q(x).

b) Solve the inequation: $R(x) \leq 0$.

2) Solve the following system of inequalities: $\begin{cases} (-x+1) > 0 \\ \dfrac{x+1}{x-2} \leq 0 \end{cases}$

V- (7 points)

Remark :The four parts of this problem are independent .

1) Simplify: $A=\cos(x-3\pi)+3\sin(-x)+3\sin(5\pi-x)+\tan(-7\pi+x)+\cos\left(-x+\frac{13\pi}{2}\right)$

2) Calculate, *without using the calculator* : $B=\sqrt{3}\cos\left(\frac{5\pi}{6}\right)-\sqrt{2}\sin\left(\frac{5\pi}{4}\right)+\cos\left(\frac{5\pi}{3}\right)$

3) Let y be an angle such that : $y \in \left[\frac{\pi}{2};\pi\right]$

a) Verify that: $(2\sin y+3)(2\sin y-1) = -4\cos^2 y+4\sin y+1$

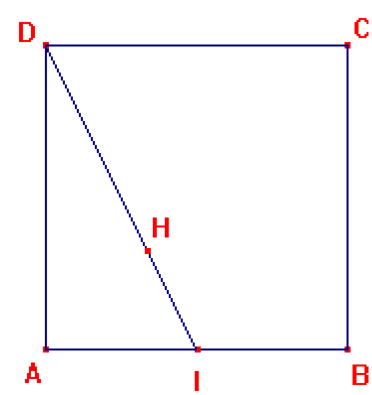
b) If $-4\cos^2 y+4\sin y+1=0$, Find **sin y**, then deduce the value of cos y and tan y.

c) Deduce the value of y.

VI- (5 points)

Part A :

ABCD is a square, I is the midpoint of [AB] and H is a point defined by $\overrightarrow{DH}=\frac{2}{3}\overrightarrow{DI}$.



1) Express $\overrightarrow{AC}$ in terms of $\overrightarrow{AB}$ and $\overrightarrow{AD}$.

2) Prove that: $\overrightarrow{AH}=\frac{1}{3}\overrightarrow{AB}+\frac{1}{3}\overrightarrow{AC}$.

3) Deduce that A, C and H are collinear.

<u>Part B</u> :

Consider the orthonormal system $(A,\overrightarrow{AB};\overrightarrow{AD})$.

1) Determine the coordinates of the points A, B, C, D and I in the system $(A,\overrightarrow{AB};\overrightarrow{AD})$.

2) Knowing that: $\overrightarrow{AH}=k\overrightarrow{AC}$. Calculate in terms of k, the coordinates of point H.

3) Use the collinearity of the points D, H and I to calculate the value of k, and then deduce that the coordinates of the point H are $\left(\frac{1}{3},\frac{1}{3}\right)$.

4) Let T be the midpoint of [AD]. Prove that the points T ,H and B are collinear.

Good Work!!