IN HIS NAME

The Islamic Institution For Education & Teaching Al-Mahdi Schools		Math Department February 2015
Subject: Mathematics		Mark: 50 points
Grade : 10	Mid-Year Exam	Duration: 150 minutes

I- (4 points)
In the table below, only one of the proposed answers is correct. Write the number of each question and give, with justification, its correct answer.

	Questions	Proposed Answers		
		A	B	C
1)	The principal measure of $\frac{-33\pi}{6}$ is	$\frac{\pi}{2}$	$\frac{-3\pi}{2}$	$\frac{\pi}{6}$
2)	$\sqrt[3]{-54}-\sqrt[3]{250}+5\sqrt[3]{16}=$	$7\sqrt[3]{2}$	$\sqrt[3]{2}$	$2\sqrt[3]{2}$
3)	If $-3 < x < 2$, then	$x^2 \in [0\,,9[$	$x^2 \in [4\,,9]$	$x^2 \in]4\,,9[$
4)	If $\vec{u}=\overrightarrow{CD}-\overrightarrow{EA}+\overrightarrow{DA}-\overrightarrow{BE}$ then $\vec{u}=$	$\overrightarrow{AB}$	$\overrightarrow{CB}$	$\vec{0}$

II- (6 points)
Simplify each of the following expressions.

1) $A=\frac{6\times2^{-4}+3\times2^{-3}}{3\times2^{-2}-2^{-3}}$.

2) $B=\frac{\sqrt[5]{9}\times\left(\sqrt[5]{\sqrt[3]{9}}\right)^2\times\sqrt{27}}{\sqrt[6]{3}}$.

3) $C=\frac{3^{n+4}-2\times3^{n+1}}{15\times3^n}$, where $n\in\mathbb{N}$.

4) $D=2-5\sqrt{x^2}-\sqrt{(7x-2)^2}+\sqrt{(3-x)^2}$, where $x\in\,]-\infty;0]$.

III- (5 points)
Given: I= [1; 3[, J= [4; 5] and $K=3|x-y|+|3x-2y+8|$

1) Knowing that $x\in I$ and $y\in J$, frame $x-y$ and $3x-2y+8$.

2) Write K without the symbol of absolute value.

IV- (5 points)
In an orthonormal system $(O;\ \vec{i},\vec{j})$, consider the points A (1, 2), B (–1, 3), C (–1, –1), and M(x, y).

1) Show that the three points A, B, and C form a triangle.

2) Find the coordinates of point M, knowing that $\overrightarrow{OM}=\overrightarrow{MA}+2\overrightarrow{MB}-3\overrightarrow{MC}$

3) Find the coordinates of point M, knowing that B is the centroid (center of gravity) of triangle AMC.

4) Let M(a, –2a), where a is a real number. Determine a so that the three points M, A, and B are collinear

5) Consider the system $(A;\ \vec{i},\vec{j})$. Let D (–2, –3) be a point in the system. Calculate the coordinates of D in the system $(O;\ \vec{i},\vec{j})$.

V- (5 points)

1) Solve in $\mathbb{R}$, each of the following inequalities.

a) $(4x^2-9)(x^2+4x+4)\leq 0$

b) $\frac{(x^2+1)(x-1)}{2x-6}<0$.

2) Deduce the solution of the system: $\begin{cases}(4x^2-9)(x^2+4x+4)\leq 0\\ \frac{(x^2+1)(x-1)}{2x-6}<0\end{cases}$

VI- (6 points)
A, B, and C are three points, in an orthonormal system $(O;\ \vec{i},\vec{j})$, such that:

$$(AB): y = \frac{-3}{2}x + \frac{11}{2}, \quad (AC): \begin{cases} x = 3t - 5 \\ y = t + 2 \end{cases} \text{ and the point } C(-5,2)$$

1) Determine the coordinates of A, the point of intersection of the two lines (AB) and (AC).
2) Find the abscissa of point B, knowing its ordinate is 1.
3) Write a system of parametric equations of line (OB).
4) Write a cartesian equation of line (AC).
5) Let (d) be a line of equation: 2x + ky – 1 = 0, where k is a real number. Determine k in each of the following cases:
   a) The slope of (d) is equal to 2.
   b) The line (d) is parallel to line (AC).

VII- (8 points)
ABCD is a rectangle. O is the meeting point of the diagonals [AC] and [BD]. I and J are two points such that: $\overrightarrow{AI} = \frac{1}{4}\overrightarrow{AC}$ and $\overrightarrow{AJ} = \frac{3}{4}\overrightarrow{AC}$. M and N are the midpoints of [DC] and [BC] respectively.
1) Plot the two points I and J.
2)
   a) Write each of the following two vectors $\overrightarrow{DI}$ and $\overrightarrow{JB}$ in terms of $\overrightarrow{AD}$ and $\overrightarrow{AC}$.
   b) Deduce that the quadrilateral DIBJ is a parallelogram.
3) Prove that: $\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{BC}$.
4) Prove that: $\overrightarrow{AC} + \overrightarrow{DB} = 2\overrightarrow{AB}$.
5) Show that: $\overrightarrow{OA} + \overrightarrow{OM} + \overrightarrow{ON} = \vec{0}$. What do you conclude?
6) $\overrightarrow{W} = \overrightarrow{HA} + \overrightarrow{HM} + \overrightarrow{HN}$. Find the locus of point H when $\|\overrightarrow{W}\| = 3$

VIII- (11 points)
Remark: the four parts of this question are independent.
1) Verify each of the identities.
   a) $\dfrac{1-\tan^2 x}{1+\tan^2 x} = \cos^2 x - \sin^2 x$ b) $\cot^2 x - \cos^2 x = \cos^2 x \times \cot^2 x$
2) Given $\sin a = \dfrac{1}{5}$, where $\frac{\pi}{2} < a < \pi$.
   a) Calculate $\cos a$ and $\tan a$.
   b) Deduce the values of $\sin\left(\dfrac{5\pi}{2} + a\right)$ and $\tan\left(\dfrac{3\pi}{2} - a\right)$
3) Let $f(x) = \dfrac{4\sin\left(\dfrac{\pi}{6}\right) \times \sin(13\pi + x) + \sin(15\pi - x)}{2\cos\pi \times \cos(-9\pi + x) + \sin\left(\dfrac{7\pi}{2} + x\right)}$. Show that $f(x) = -\tan x$, then calculate $f\left(\dfrac{5\pi}{4}\right)$.
4) Calculate: $F = \cos^2\left(\dfrac{7\pi}{8}\right) + \cos^2\left(\dfrac{5\pi}{8}\right) + \cos^2\left(\dfrac{3\pi}{8}\right) + \cos^2\left(\dfrac{\pi}{8}\right)$.

Bonus: The product $\left(1-\dfrac{1}{50}\right)\left(1-\dfrac{2}{50}\right)\left(1-\dfrac{3}{50}\right)\ldots\ldots\left(1-\dfrac{99}{50}\right)\left(1-\dfrac{100}{50}\right)$ is equal to a) -1 ; b) 0 ; c) 1

Good Work