IN HIS NAME

The Islamic Institution for Education & Teaching Al-Mahdi Schools		**Mathematics Department** **Scholastic Year**: 2015-2016 **Date**: / 02 / 2016
Grade 10		**Duration**: 150 minutes
Name: ____________________	**Mid-Year Exam**	**Mark**: 30 points

I- (4 points)

In the table below, only one of the proposed answers is correct. Write the number of each question and give, **with justification**, its correct answer.

Questions		Proposed Answers A	B	C
1)	Given: $\alpha = -\frac{27\pi}{7}$. The principal measure of α is:	$-\frac{\pi}{7}$	$\frac{\pi}{7}$	$-\frac{6\pi}{7}$
2)	If we rationalize the denominator of $K = \frac{2}{\sqrt[3]{4}}$, then	$K = \sqrt[3]{4}$	$K = \sqrt[3]{2}$	$K = \sqrt{2}$
3)	$\lvert -(x+1)2 + 2x \rvert =$	$x^2 - 1$	$1 + x^2$	$(x+1)^2 + 2x$
4)	If $2 \leq \frac{1}{x-1} \leq 3$, then	$x \in \left[\frac{1}{4} ; \frac{1}{3}\right]$	$x \in \left[\frac{3}{2} ; \frac{4}{3}\right]$	$x \in \left[\frac{4}{3} ; \frac{3}{2}\right]$

II- (3 points)

Given the two numbers x and y such that $\lvert x - 5 \rvert \leq 1$ and $1 < \sqrt{y+1} < 2$.

1) Show that $4 \leq x \leq 6$ and that $0 < y < 3$.
2) Bound $x + y$ and $x - y$. Deduce the boundaries of $x^2 - y^2$.
3) Compare $\sqrt{x^2 - y^2}$ and $\frac{1}{x^2 - y^2}$.

III- (4 points)

Simplify.

1) $A = \frac{\sqrt[3]{24} \times \sqrt[4]{64} \times \sqrt[4]{\sqrt[3]{6}}}{\sqrt[12]{6^5 \times 2^7}}$.
2) $B = (\sqrt[4]{5} - 1)(\sqrt[4]{5} + 1)(\sqrt[4]{25} + 1)$.
3) $C = \sqrt[6]{(\sqrt{3} - 2)^6} + \sqrt[3]{(\sqrt{3} - 2)^3} + \lvert -2\sqrt{3} - 1 \rvert$.

IV- (4 points)

1) Let $E = (2x - 1)(x + 3) - (x + 3)$.
   a) Prove that $E = 2(x + 3)(x - 1)$.
   b) Let $F = \frac{E}{x+3}$, where $x \neq -3$. Simplify F, then solve the inequality $\lvert F \rvert > 2$.
2) Solve the system of inequalities: $\begin{cases} \frac{x^2 + 9}{3 - x} < 0 \\ x^2 \geq (2x - 1)^2 \end{cases}$.

V- (5 points)

Given, in an orthonormal system $(O; \vec{i}, \vec{j})$, the three points A(2 ; –3), B(1 ; 4), and C(3 ; 2) and the vector $\overrightarrow{AM} = (2a-1)\vec{i} - (b-2)\vec{j}$, where a and b are two real numbers.

1) Show that the three points A, B, and C form a triangle (non-collinear).

2) Find the coordinates of point J, the fourth vertex of the parallelogram ABCJ.

3)

a) Prove that the coordinates of point M are (2a + 1 ; –b – 1).

b) Determine a and b so that O is the center of gravity of triangle ABM.

4) Consider the point E such that $E(4\cos\alpha + 2 ; 4\sin\alpha - 3)$, where α is real number. Show that AE = 4.

5) Find the coordinates of A in the system $(C; \vec{i}, \vec{j})$.

VI- (5 points)

Consider a parallelogram ABCD with BC = 3 cm, and AB = 5 cm.

E and F are two points in the plane such that: $\overrightarrow{CE} = \frac{1}{3}\overrightarrow{CD}$ and $\overrightarrow{AF} = \frac{3}{2}\overrightarrow{AE}$.

1) Draw the figure.

2) Show that $\overrightarrow{BF} = \overrightarrow{CD} + \frac{3}{2}\overrightarrow{AE}$.

3) Show that $\overrightarrow{BC} = \frac{2}{3}\overrightarrow{CD} + \overrightarrow{AE}$.

4) Deduce that the three points B, F, and C are collinear.

5) Consider the vector $\vec{V} = \overrightarrow{MD}$, where M is any point in the plane.

a) Show that BF = 4.5 cm.

b) Deduce the locus of point M if $\|\vec{V}\| = \|2\overrightarrow{BF}\|$.

VII- (5 points)

Remark: The two parts of this question are independent.

1) Let α be an angle such that $\frac{-\pi}{2} < \alpha < 0$, with $\tan\alpha = \frac{-1}{2}$.

a) Calculate cosα and sinα .

b) Calculate the value of the expression: $E = \tan\left(\frac{13\pi}{2} + \alpha\right) \times \cos\left(117\pi + \alpha\right)$.

2) Simplify each of the following identities.

a) $F = \frac{1-\tan^2 x}{1+\tan^2 x} - \cos^2 x$.

b) $G = \sin(\pi + x) + \sin(8\pi + x) - \sin(5\pi - x) - \cos\left(\frac{3\pi}{2} + x\right)$.

Good work