IN HIS NAME

The Islamic Institution for Education & Teaching
Al-Mahdi Schools

Mathematics Department
Scholastic Year: 2016-2017
Date: / 02 / 2017

Class: Grade 10 | **Duration**: 150 minutes

Name: ____________________ | **Mid-Year Exam** | **Mark**: 30 points

I- (5 points)

In the table below, only one of the proposed answers is correct. Write the number of each question and give, **with justification**, its correct answer.

	Questions	Proposed Answers		
		A	**B**	**C**
1)	The principal measure of $\frac{31\pi}{4}$ is	$\frac{\pi}{4}$	$-\frac{\pi}{4}$	$\frac{3\pi}{4}$
2)	If $\lvert 1-2t \rvert \leq 3$, then	$t \in [-1\ ,\ 2]$	$t \in [-2\ ,\ 1]$	$t \in [-1\ ,\ +\infty[$
3)	If EFG is any triangle and I is the midpoint of [FG], then $\lVert \overrightarrow{EF}+\overrightarrow{EG} \rVert =$	EF + EG	$2\,\overrightarrow{EI}$	2 EI
4)	If $\vec{u}=3\overrightarrow{AB}-2\overrightarrow{AC}$ and $\vec{v}=3\overrightarrow{AB}+2\overrightarrow{BC}$, then the components of $\vec{u}$ and $\vec{v}$ in the system $(A;\overrightarrow{AB},\overrightarrow{AC})$ are	$\vec{u}(3\ ,-2)$ $\vec{v}(3\ ,2)$	$\vec{u}(1\ ,-1)$ $\vec{v}(0\ ,1)$	$\vec{u}(3\ ,-2)$ $\vec{v}(1\ ,2)$
5)	Let x be a non-zero real number. which statement is true?	$\frac{2x}{x^2+1} \leq \frac{2x-1}{x^2}$	$\frac{2x}{x^2+1} \geq \frac{2x-1}{x^2}$	$\frac{2x}{x^2+1} = \frac{2x-1}{x^2}$

II- (10 points)

Consider an equilateral triangle ABC. Let I be the midpoint of [BC] and P, Q, and G be three points defined by: $\overrightarrow{PB}+3\overrightarrow{PC}=\vec{0}$, $\overrightarrow{QA}+2\overrightarrow{QC}=\vec{0}$, and $\overrightarrow{GA}-\overrightarrow{GB}-\overrightarrow{GC}=\vec{0}$.

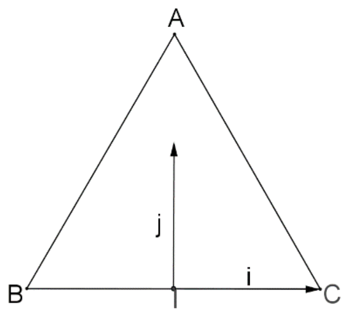
Part A

1) Show that $\overrightarrow{CP}=\frac{1}{4}\overrightarrow{CB}$ and $\overrightarrow{CQ}=\frac{1}{3}\overrightarrow{CA}$.
2) Show that $\overrightarrow{AG}=\overrightarrow{AB}+\overrightarrow{AC}$.
3) Construct the three points P, Q, and G.
4) Show that $\overrightarrow{QG}=\overrightarrow{AB}+\frac{1}{3}\overrightarrow{AC}$ and $\overrightarrow{QP}=\frac{1}{4}\overrightarrow{AB}+\frac{1}{12}\overrightarrow{AC}$.
5) Deduce that the three points Q, P, and G are collinear.

Part B

Consider the orthonormal system $(I;\ \vec{i}\ ,\ \vec{j})$, where $\vec{i}=\overrightarrow{IC}$ as shown in the figure at right.

1) Show that the coordinates of A are $(0\ ,\sqrt{3})$, then find the coordinates of the points B, C, P, Q, and G.
2) Show that $\overrightarrow{QG}\left(-\frac{2}{3},-\frac{4\sqrt{3}}{3}\right)$ and $\overrightarrow{PG}\left(-\frac{1}{2},-\sqrt{3}\right)$.
3) Deduce that the three points Q, P, and G are collinear.
4) Let M(x , y) be any point in the plane.
   What relation must exist between x and y so that the two lines (CM) and (AB) are parallel?



III- (6 points)

Remark: The three parts of this question are independent.

1) Simplify $\frac{\sqrt[3]{4}\times\sqrt{3}}{\sqrt{\sqrt[3]{2^5}}}$.

2) Let E = $\left(\sqrt{3+2\sqrt{2}}\right)(1-\sqrt{2})$. Calculate E^2, then deduce the simplest form of E.

3) Given the two real numbers a and b such that $a \in [2\,;3]$ and $b \in [1\,;4]$. Let F = $\frac{3a-b}{2a^2+b^2}$.

   a) Frame $3a - b$ and $2a^2 + b^2$.

   b) Deduce that $\frac{1}{17} \le F \le \frac{8}{9}$, then order the three numbers F, F^2, and $\sqrt{F}$ in increasing order.

IV- (5 points)

Given: $A = \cos\left(\frac{7\pi}{3}\right) + \sqrt{2}\sin\left(\frac{7\pi}{4}\right) - \sqrt{3}\cos\left(\frac{5\pi}{6}\right)$

$B = \sin\left(x - \frac{9\pi}{2}\right) \times \cos(\pi + x) + \sin(7\pi - x) \times \cos\left(x - \frac{\pi}{2}\right)$

$C = 3\sin\alpha + 3\cos\alpha - 2\tan\alpha - 2$

1) Simplify A and B.

2) Simplify C, knowing that $2\sin^2\alpha + \cos^2\alpha = \frac{14}{9}$ and $\alpha \in \left]-\frac{\pi}{2}, 0\right[$.

3) Verify that A = B + C.

V- (4 points)

Remark: The two parts of this question are independent.

1) Solve the system of inequalities: $\begin{cases} \frac{(2-x)(-4x)}{x-1} < 0 \\ (x-3)^2 \ge (x-3) \end{cases}$.

2) Let M = $\frac{2x-2}{(x-1)^2+(3+2x)(1-x)}$. Solve the inequality $|\,M\,| > 2$.

Good work