IN HIS NAME

The Islamic Institution for Education & Teaching **Al-Mahdi Schools**		**Mathematics Department** **Scholastic Year**: 2017-2018 **Date**: / 02 / 2018
Class: Grade 10		**Duration**: 150 minutes
Name: ____________________	**Mid-Year Exam**	**Mark**: 25 points

ملاحظة: يسمح باستخدام آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
يستطيع التلميذ الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (3 points)

In the table below, only one of the proposed answers is correct.
Write the number of each question and give, **with justification**, its correct answer.

Nº	Questions	Proposed Answers		
		A	**B**	**C**
1)	Let $x < 5$. If $E = \left\lvert \frac{4x}{x-5} - 4 \right\rvert$, then E =	$\frac{20}{5-x}$	$\frac{20}{x-5}$	$\frac{20}{x+5}$
2)	The principal measure of $\frac{17\pi}{3}$ is	$\frac{\pi}{3}$	$-\frac{\pi}{3}$	$\frac{2\pi}{3}$
3)	Let A = [–2 ; 6] and B =]0 ; +∞[. $A \cap \overline{B}$ =	[–2 ; 0]	]–∞ ; 6]	]0 ; 6]
4)	Given the two vectors $\vec{U}(2a ; 6)$ and $\vec{V}(1 ; 3)$, where a is a real number. If $\vec{U}$ and $\vec{V}$ are collinear, then a =	1	$\frac{1}{2}$	–1

II- (4 points)

Remark: The two parts of this question are independent.

Part A *(Show all the steps of calculations)*

Given the following numbers.

$A = \left[(81)^{-\frac{1}{4}} \times 18^{0.5} + (5.6)^0\right] \times \left[1 - \frac{2}{\sqrt{2}}\right]$; $B = \frac{12-4\sqrt{3}}{\sqrt{3}-1}$; $C = \frac{\sqrt{3} \times \sqrt[3]{12} \times \sqrt[4]{8}}{\sqrt[12]{3^8} \times 2^{\frac{5}{12}} \times \sqrt[9]{3^{-3}}}$

1) Prove that A is an integer.
2) Rationalize the denominator of B.
3) Show that $\frac{B}{C}$ is a natural number.

Part B

Solve, in ℝ.

1) $|x^2 - 3| = x^2 + 1$
2) $\frac{(x-1)(4-2x)}{4x^2-36} \geq 0$

III- (4 points)

In an orthonormal system $(O ; \vec{i}, \vec{j})$, consider the three points A(1 ; –2), B(3 ; –4), and C(2 ; 3) and the two vectors $\vec{v} = -2\vec{i} + 3\vec{j}$ and $\overrightarrow{OM} = x\vec{i} + y\vec{j}$, where x and y are two real numbers,

1) Find the coordinates of the point D defined by $\overrightarrow{AD} = -2\overrightarrow{BC} + 3\overrightarrow{BD}$.
2) Find a relation between x and y so that the three points A, B, and M are collinear.
3) Calculate x and y when $\overrightarrow{AM} = -2\vec{v}$.
4) Calculate the coordinates of point G, the center of gravity of triangle ABC.
5) Calculate the coordinates of point A in the system $(B ; \vec{i}, \vec{j})$.

IV- (4 points)
Given the polynomial $P(x) = ax^2 + bx – 6$, where a and b are two real numbers.

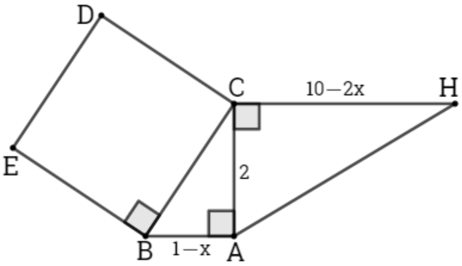
1)
- **a)** Find a relation between a and b if $x = 2$ is a root of $P(x)$.
- **b)** Find a relation between a and b if the remainder of the division of $P(x)$ by $(x – 3)$ is 6.
- **c)** Deduce a and b, and then $P(x)$.

In what follows, suppose that a = b = 1.

2) Use the Euclidian division to show that $P(x) = (x + 3)(x – 2)$.

3) Solve, in $\mathbb{R}$, the inequality $P(x) > 0$.

4) In the adjacent figure, BCDE is a square, ABC and ACH are two right-angled triangles at A and C respectively such that $AB = 1 – x$, $AC = 2$, and $CH = 10 – 2x$, where $x < 1$.
- **a)** Let S_1, S_2, and S_3 be the respective areas of BCDE, ABC, and ACH.
  Show that $S_1 – (S_2 + S_3) = P(x)$.
- **b)** For what values of x do we have $S_1 > S_2 + S_3$?



V- (5 points)
Part A
Given: $A = \sin\alpha.\cos\alpha(1 + \cot^2\alpha)$, where α is an arc such that $-\frac{\pi}{2} < \alpha < 0$.

1) Simplify A.

2) Suppose that $\tan\alpha = -\frac{1}{2}$
- **a)** Calculate $\cos\alpha$.
- **b)** Deduce the value of $E = \sin\left(\frac{5\pi}{2} - \alpha\right) \times \cos(-7\pi + \alpha)$.

Part B

1) Show that $\frac{\cos\beta}{1-\cos^2\beta} - \frac{1}{1-\cos\beta} = \frac{-1}{\sin^2\beta}$.

2) Let $u = \sin\theta – \cos\theta$ and $v = \sin\theta + \cos\theta$.
Prove that $u^2 + v^2 = 2$.

VI- (5 points)
Part A
In the adjacent figure, ABCD is a square of side 6 cm, E is the symmetric of B with respect to C, and F is a point such that $2\overrightarrow{DF} – \overrightarrow{FB} = \vec{0}$.

1) Prove that $\overrightarrow{DF} = \frac{1}{3}\overrightarrow{DB}$.

2) Show that $\overrightarrow{AF} = \frac{1}{3}\overrightarrow{AB} + \frac{2}{3}\overrightarrow{AD}$ and that $\overrightarrow{AE} = \overrightarrow{AB} + 2\overrightarrow{AD}$.

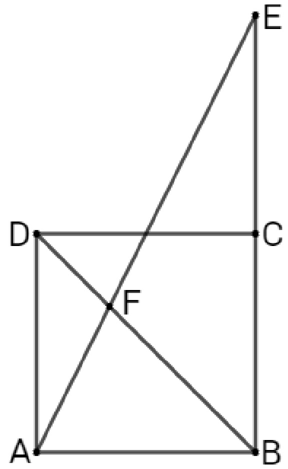
3) Deduce that the three points A, F, and E are collinear.

Part B
Let $\vec{V} = \overrightarrow{MA} – 3\overrightarrow{MC} + \overrightarrow{MB} + \overrightarrow{MD}$, where M is any point in the plane.

1) Show that $\vec{V} = 2\overrightarrow{CA}$.

2) Deduce the value of $\|\vec{V}\|$.

3) Let G be the center of gravity of ABD.
Prove that $\vec{V} = 3\overrightarrow{CG}$.



Part C
The plane is referred to the orthonormal system $(A ; \overrightarrow{AB}, \overrightarrow{AD})$.

1) Find the coordinates of the three points A, E, and F.

2) Prove, again, that the three points A, F, and E are collinear.