IN HIS NAME

The Islamic Institution for Education & Teaching Al-Mahdi Schools		**Mathematics Department** **Scholastic Year**: 2019-2020 **Date**: January 2020
Class: Grade 10		**Duration**: 150 minutes
Name: ___________________	**Mid-Year Exam**	**Mark**: 30 points

ملاحظة: يسمح باستخدام آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.

يستطيع التلميذ الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (3.5 points)

In the table below, only one of the proposed answers to each question is correct. Write the number of each question and give, **with justification**, its correct answer.

Nº	Questions	Proposed Answers A	B	C
1)	x is a real number different of zero. If $\frac{1}{9} < \frac{1}{x} < \frac{1}{4}$ then $\sqrt{x} \in$	$]-\infty ; 2[\cup]3 ; +\infty[$	$]2 ; 3[$	$\left]\frac{1}{3} ; \frac{1}{2}\right[$
2)	In an orthonormal system $(O; \vec{i}, \vec{j})$, consider the points A(–1 ; 7) and B(3 ; –4). The coordinates of B in the new system $(A; \vec{i}, \vec{j})$ are	(2 ; –3)	(4 ; –1)	(4 ; –11)
3)	x and y are two real numbers such that $x > y$. If $A = (\sqrt{2} - 3)x$ and $B = (\sqrt{2} - 3)y$ then	$A > B$	$A < B$	$A = B$
4)	If $(2x + 5)^3 = -27$ then $x =$	1	– 4	– 1

II- (4 points)

The two parts in this question are independent.

Part A

Given $A = \frac{4^3 \times 7^{-2} \times 35}{49^{-1} \times 56 \times 2^3}$ and $B = 5\sqrt[3]{54} + 3\sqrt[3]{-16} - \sqrt[3]{128}$.

1) Show that A = 5.
2) Verify that $B = A\sqrt[3]{m}$ where m is an integer to be determined.

Part B

Solve, in $\mathbb{R}$, the following system of inequations $\begin{cases} (3 - x)^2 \leq 4 \\ \frac{(x^2+9)(x-5)}{x-1} \geq 0 \end{cases}$.

III- (4 points)

Consider the following sets:

$E = \{x \in \mathbb{R} / -10 \leq x \leq 10\}$

$C = \{x \in \mathbb{R} / x \in E \text{ and } 2x - 8 \geq 4\}$

$D = \{x \in \mathbb{R} / x \in E \text{ and } (x - 10)^4 = 81\}$.

1) **a)** Write E in the form of an interval and verify that C = [6 ; 10].
   b) Give a representation, on an axis, of the intervals obtained.
2) Show that D is a singleton set.
3) Determine an interval X so that $X \cap E = C$.
4) $\overline{E}$ and $\overline{C}$ are the respective complements of E and C in $\mathbb{R}$.
   Determine an interval Y so that $Y \cup \overline{E} = \overline{C}$.

IV- (5 points)

Consider the following expressions: E $= \left|3\sqrt{2} - 4\right| - 3\left|1 - \sqrt{2}\right|$, A = $\dfrac{2x-1}{x+3}$ and B = $|-x^2 + 4\ |$ where x is a real number such that $2 < x < 5$.

1) Show, by bounding, that $\frac{3}{8} < \text{A} < \frac{9}{5}$ and $-21 < -x^2 + 4 < 0$.
2) Write B without the absolute value.
3) Show that E = –1.
4) Solve the following equations:
   a) $|\text{A}| = |\text{E}|$.
   b) 3B = E $-$ 14.

V- (3.5 points)

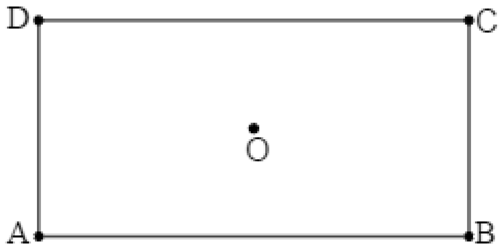
Given the polynomial P(x) = (m – 1)x^3 + (2m – 1)x^2 – (m + 1)x – 3 where m is a real parameter.

1) Determine the value of m if (x – 1) is a factor of P(x).
2) In this part consider that m = 3 and P(x) = $2x^3 + 5x^2 - 4x - 3$.
   a) Write P(x) in the form of (x – 1)(ax^2 + bx + c) where a, b and c are real numbers to be determined.
   b) Write P(x) in the form of the product of three factors of first degree.

VI- (10 points)

In the adjacent figure, we have:

- *ABCD* is a rectangle of center *O*
- *AB* = 6 cm and *AD* = 3 cm.



E and F are two points defined by:
$3\,\overrightarrow{EB} - \overrightarrow{EA} = \vec{0}$ and $\overrightarrow{CF} + 2\,\overrightarrow{BF} = \vec{0}$.

Part A

1) Show that $\overrightarrow{AE} = \frac{3}{2}\overrightarrow{AB}$ and $\overrightarrow{CF} = \frac{2}{3}\overrightarrow{CB}$.
2) Reproduce the figure and place the points E and F.
3) Show that $\overrightarrow{DE} = \frac{3}{2}\overrightarrow{AB} - \overrightarrow{AD}$ and $\overrightarrow{DF} = \overrightarrow{AB} - \frac{2}{3}\overrightarrow{AD}$.
4) Deduce that *D*, *E* and *F* are collinear.
5) Let *I* be a point defined by: $3\,\overrightarrow{IB} - \overrightarrow{IA} + 2\,\overrightarrow{IC} = \vec{0}$.
   Show that *I* is the midpoint of [*CE*].

Part B

Consider the system (A; $\frac{1}{2}\overrightarrow{\text{AB}}$, $\overrightarrow{\text{AD}}$).

1) Find the coordinates of points *B*, *C*, *D*, *O*, *E* and *F*.
2) Let *L* be the symmetric of *C* with respect to *E*.
   Verify that the coordinates of *L* are (4 ; –1).
3) Show that *O*, *B* and *L* are collinear.
4) Show that *B* is the center of gravity of triangle *CAL*.
5) Let *M*(x ; y) where x and y are two real numbers.
   Find the coordinates of *M* such that *ACLM* is a parallelogram.