c) In right ΔABD

$\sin B\hat{A}D = \frac{opp}{hyp}$ $\left(B\hat{A}D = \frac{mes\ \overset{\frown}{BD}}{2} = \frac{30}{2} = 15^\circ \right)$

$\sin 15^\circ = \frac{BD}{AB}$ [AB] is a diameter of (C)

$\sin 15^\circ = \frac{BD}{2r}$ $\sin 15^\circ = \frac{\sqrt{6}-\sqrt{2}}{4}$ (using calculator)

Thus, $BD = 2r \sin 15^\circ$

$= r(\sqrt{6}-\sqrt{2})$ cm

$\cos B\hat{A}D = \frac{adj}{hyp}$

$\cos 15^\circ = \frac{AD}{AB}$

$\cos 15^\circ = \frac{\sqrt{6}+\sqrt{2}}{4}$ (using calculator)

$\cos 15^\circ = \frac{AD}{2r}$

Thus, $AD = 2r \cos 15^\circ$

$= r(\sqrt{6}+\sqrt{2})$ cm

2) a) In Δ's ACI & BDI we have:

(BC) cuts (AD) at I (given)

So, $A\hat{I}C = D\hat{I}D$ (vertical opp. angles formed between two st. lines)

$C\hat{A}I = I\hat{B}D = \frac{1}{2} mes(\overset{\frown}{CD})$ (inscribed angles intercepting the same arc)

Thus Δs are similar by A.A-postulate (two angles in one triangle are resp. equal to two angle in another)

Ratio of similitude: $\begin{array}{c|c} \Delta ACI & \frac{AC}{BD} = \frac{AI}{BI} = \frac{CI}{DI} = cst. \\ \Delta BDI & \ \ (1) \quad\ (2) \quad\ (3) \end{array}$

b) Using ratios (1) & (2)

$\frac{AC}{BD} = \frac{AI}{BI}$ Thus, $AC \times BI = AI \times BD$.